9.2 #5 $\cos(x+2)$ at $\underline{x=0}$

$^{0}\, f(0) = \cos(2)$ $\qquad \cos(2)$

$^{1}\, f'(0) = -\sin(2) = \cos\left(2-\frac{\pi}{2}\right)$ $\qquad -\cos\left(2-\frac{\pi}{2}\right)$

$^{2}\, f''(0) = -\cos(2)$ $\qquad -\frac{\cos(2)}{2}$

$^{3}\, f'''(0) = \sin(2) = \cos\left(2-\frac{\pi}{2}\right)$ $\qquad \frac{\cos\left(2-\frac{\pi}{2}\right)}{3!}$

$$\cos(2) - \cos\left(2-\frac{\pi}{2}\right)x - \frac{\cos(2)}{2}x^2 + \frac{\cos\left(2-\frac{\pi}{2}\right)}{3!}x^3$$

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1)

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad \sin 2x = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n+1}}{(2n+1)!}$$

Ratio test: $\underline{\lim_{n\to\infty}} \left| \frac{(2x)^{2n+3}}{(2n+3)!} \cdot \frac{(2n+1)!}{(2x)^{2n+1}} \right| = \left| \frac{(2x)^2}{(2n+3)(2n+2)} \right| = 0$

$0 < 1$ for all x so $\boxed{\text{Interval of Convergence} = -\infty < x < \infty}$

13) (b) $f(x) = x^3 - 2x + 4$ $a = 1$ $\sum_{n=0}^{\infty} \frac{f^n(1)}{n!}(x-1$

$f(1) = 3$ 3

$f'(1) = 1$ $(x-1)^1$

$f''(1) = 6$ $\frac{6}{2!}(x-1)^2$

$f'''(1) = 6$ $\frac{6}{3!}(x-1)^3$

$3 + (x-1) + 3(x-1)^2 + (x-1)^3$

$3 + x - 1 + 3x^2 - 6x + 3 + x^3 - 3x^2 + 3x - 1$

$x^3 - 2x + 4$

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13) $f(x) = x^3 - 2x + 4$

a) $a = 0$

$f(0) = 4 \rightarrow 4$ 4

$f'(0) = -2 \rightarrow -2x$ $= -2x$

$f''(0) = 0 \rightarrow 0x^2/2! = 0$

$f'''(0) = 6 \rightarrow \frac{6x^3}{3!} = x^3$

$\boxed{x^3 - 2x + 4}$

19) $g(x) = \dfrac{\sum_{n=0}^{\infty} \frac{x^n}{n!} - 1}{x} = \dfrac{x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} \ldots}{x}$

$g(x) = 1 + \frac{x}{2} + \frac{x^2}{6} + \frac{x^3}{24} \ldots \quad \boxed{\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}}$

$g'(x) = \dfrac{xe^x - (e^x - 1)}{x^2}$

$g'(x) = \frac{1}{2} + \frac{2x}{6} + \frac{3x^2}{24} + \frac{4x^3}{120}$

$= \sum_{n=1}^{\infty} \frac{n}{(n+1)!} x^{n-1}$

$g'(1) = \dfrac{e - e + 1}{1} = 1$

$g'(x) = \sum_{n=1}^{\infty} \frac{n}{(n+1)!} x^{n-1} \Rightarrow g'(1) = \sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$

from when
we found
$g'(1)$