Calculus Chapter 10 Review Name___________________

Show all your work. You may use your calculator unless otherwise instructed.

1) Without using your calculator, find the length of the parametric curve $x = e^t \cos t$ and $y = e^t \sin t$ defined over the interval $0 \leq t \leq \pi$

$$\frac{dx}{dt} = e^t\cos t - e^t\sin t \qquad \frac{dy}{dt} = e^t\sin t + e^t\cos t$$

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{\left(e^t\cos t - e^t\sin t\right)^2 + \left(e^t\sin t + e^t\cos t\right)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{\left(e^{2t}\cos^2 t - e^{2t}\cos t\sin t + e^{2t}\sin^2 t\right) + \left(e^{2t}\sin^2 t + e^{2t}\sin t\cos t + e^{2t}\cos^2 t\right)}\, dt$$

$$= \int_0^{\pi} \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t}\, dt$$

$$= \int_0^{\pi} \sqrt{2e^{2t}\left(\cos^2 t + \sin^2 t\right)}\, dt = \sqrt{2}\int_0^{\pi} e^t\, dt$$

$$= \sqrt{2}\left(e^{\pi} - e^0\right) = \sqrt{2}\left(e^{\pi} - 1\right)$$

2) A particle moves in the xy-plane so that its position at any time t, $0 \le t \le 3$ is given by $x(t) = \frac{t^2}{2} - 3\ln(2+t)$ and $y(t) = 3\sin \pi t$.

a) At what time t does $x(t)$ attain its minimum value? What is the position $(x(t), y(t))$ of the particle at this time? Show the work that leads to your answer.

$x(t) =$ min value when $x'(t) = 0$

$x'(t) \quad t - \frac{3}{(2+t)} = 0$

$t = \frac{3}{2+t}$

$2t + t^2 = 3$

$t^2 + 2t - 3 = 0$

$\Rightarrow (t-1)(t+3) = 0$

$t = 1, -3$ not in domain

$t = 1$

position $\langle \frac{1}{2} - 3\ln(3), 0 \rangle$

b) What is the speed of the particle at this time? Show the work that leads to your answer. You may use your graphing calculator to evaluate.

speed $= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$\frac{dx}{dt} = x'(1) = 0$

$\frac{dy}{dt} = 3\pi \cos \pi t = 3\pi \cos \pi = -3\pi$

c) At what values of t over the interval $0 < t < 3$ is the particle is on the x-axis? Show the work that leads to your answer.

$y(t) = 0 = 3\sin \pi t$

$\sin \pi t = 0 \quad t = 0, 1, 2, 3, 4 \ldots$

$t = 1, 2$

d) Set up the equation that will determine the total distance traveled by the particle over the interval $0 \le t \le 3$. Evaluate this equation using your graphing calculator.

$$\int_0^3 \sqrt{\left(t - \frac{3}{2+t}\right)^2 + (3\pi \cos \pi t)^2}\, dt$$

≈ 18.56

3) A curve is defined by the parametric equations $x = 3t - t^3$, $y = 3t^2$ over the interval $0 \leq t \leq 3$.

a) At what value(s) of t is the tangent line vertical?

$\frac{dx}{dt} = 0 = 3 - 3t^2 \quad t = \pm 1 \quad t = 1$

-1 is not in the domain

b) Find the equation of the tangent line at $t = 2$.

$\frac{dy}{dx} = \frac{6t}{3-3t^2} = \frac{12}{-9} = -\frac{4}{3} \quad x = 3(2) - 8 = -2 \quad y = 3(4) = 12$

$\boxed{y - 12 = -\frac{4}{3}(x+2)}$

c) Find d^2y/dx^2.

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(dy/dx)}{dx/dt} = \frac{\frac{(3-3t^2)6 - 6t(-6t)}{3-3t^2}}{3-3t^2} = \boxed{\frac{18 + 18t^2}{(3-3t^2)^3}}$$

d) Without using your calculator, find the length of the parametric curve over the interval $0 \leq t \leq 3$.

$$\int_0^3 \sqrt{36t^2 + 9 - 18t^2 + 9t^4}\, dt$$

$$\int_0^3 \sqrt{9t^4 + 18t^2 + 9}\, dt = \int_0^3 \sqrt{(3t^2+3)^2}\, dt$$

$$= \int_0^3 3t^2 + 3\, dt = \left[t^3 + 3t\right]_0^3 = 27 + 9 = 36$$