Chapter 6 Review 2 Name Solutions

If you need more room, you may do these problems on a separate sheet of paper.
Evaluate the integral.

1) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{du}{\sqrt{u}} = \int_1^4 u^{-1/2}\,du$

let $u = \ln x$

$du = \frac{dx}{x}$

$x = e \quad u = \ln(e) = 1$

$x = e^4 \quad u = \ln(e^4) = 4$

$= \left.\frac{u^{1/2}}{\frac{1}{2}}\right]_1^4 = 2(2) - 2(1)$

$= 2$

2) $\int_0^{\frac{\pi}{2}} e^{\sin x}\cos x\,dx = \int_0^1 e^u\,du$

$u = \sin x$

$du = \cos x$

$x = 0 \quad u = 0$

$x = \frac{\pi}{2} \quad u = 1$

$= \left. e^u \right]_0^1$

$= e - 1$

3) $\int_1^4 \ln\sqrt{x}\,dx = x\ln\sqrt{x} - \int_1^4 x\frac{1}{\sqrt{x}}\frac{dx}{2\sqrt{x}}$

$u = \ln\sqrt{x} \quad dv = dx$

$du = \frac{1}{\sqrt{x}}\frac{dx}{2\sqrt{x}} \quad v = x$

$x\ln\sqrt{x} - \int \frac{1}{2}\,dx$

$\left. x\ln\sqrt{x} - \frac{1}{2}x \right]_1^4$

$= 4\ln\sqrt{4} - \frac{1}{2}(4) - \left(1\ln(1) - \frac{1}{2}(1)\right)$

$= 4\ln 2 - 2 + \frac{1}{2} = 4\ln 2 - \frac{3}{2}$

4) $\int_0^{\frac{\sqrt{5}}{2}} \frac{x\,dx}{\sqrt{9-4x^2}} = -\frac{1}{8}\int_9^4 \frac{du}{\sqrt{u}}$

$u = 9 - 4x^2$

$du = -8x\,dx$

$x = 0 \quad u = 9 - 4(0) = 9$

$x = \frac{\sqrt{5}}{2} \quad u = 9 - 5 = 4$

$= \frac{1}{8}\int_4^9 u^{-1/2}\,du$

$= \left.\frac{1}{8}2u^{1/2}\right]_4^9$

$= \frac{1}{4}\sqrt{9} - \frac{1}{4}\sqrt{4}$

$= \frac{3}{4} - \frac{2}{4} = \frac{1}{4}$

5) $\int x\sec^2 x\,dx = x\tan x - \int \tan x\,dx$

$u = x \quad dv = \sec^2 x\,dx$

$du = dx \quad v = \tan x$

$x\tan x - \int \tan x\,dx$

$= x\tan x - \int \frac{\sin x}{\cos x}\,dx = x\tan x + \int \frac{du}{u}$

let $u = \cos x$, $du = -\sin x\,dx$

$= x\tan x + \ln|\sec x| + C$

6) $\int_0^{\frac{\pi}{4}} \frac{\tan^3\theta}{\cos^2\theta} = \int_0^{\pi/4} \tan^3\theta\sec^2\theta\,d\theta$

let $u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$\theta = 0 \quad u = 0$

$\theta = \frac{\pi}{4} \quad u = 1$

$\int_0^1 u^3\,du$

$\left.\frac{u^4}{4}\right]_0^1 = \frac{1}{4}$

7) $\int_1^{\sqrt{6}} \sqrt{x^4+3x^2}\,dx = \int_1^{\sqrt{6}} \sqrt{x^2(x^2+3)}\,dx$

$u = x^2+3$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x=1 \quad u=4$

$x=\sqrt{6} \quad u=9$

$= \int_1^{\sqrt{6}} x\sqrt{x^2+3}\,dx$

$= \frac{1}{2}\int_4^9 u^{1/2}\,du$

$= \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_4^9$

$= \frac{1}{2}\left[18 - \frac{16}{3}\right]$

$= 9 - \frac{8}{3} = \frac{19}{3}$

8) $\int x\sqrt{x+7}\,dx$ use both methods

Substitution

$u = x+7 \Rightarrow x = u-7$

$du = dx$

$\int (u-7)\sqrt{u}\,du$

$\int u^{3/2} - 7u^{1/2}\,du$

$\frac{2}{5}u^{5/2} - \frac{14}{3}u^{3/2} + C$

$\boxed{\frac{2}{5}(x+7)^{5/2} - \frac{14}{3}(x+7)^{3/2} + C}$

9) The number of minutes that Naomi sleeps in class increases exponentially every day during the second semester. Carolyn and Carisa guess that as Naomi gets less and less sleep, the number of sleepy-time minutes becomes an exponential growth equation. They figure the differential equation

for her sleepy-time minutes to be given by $\frac{dy}{dx} = ky$ where y is the number of minutes that

Naomi sleeps on a given day and t is the number of days after March 1. If Naomi sleeps for 5 minutes on March 1 and her time has doubled three weeks later, how many minutes will Naomi sleep on May 1 (day 61)?

$y = y_0 e^{kt}$

$y_0 = 5$

$y = 5e^{kt}$

$10 = 5e^{k(21)}$ (Doubled three weeks later)

$2 = e^{21k} \Rightarrow \frac{\ln 2}{21} = k$

May 1

$y = 5e^{\left(\frac{\ln 2}{21}\right)(61)}$

$y \approx 37.445$ minutes

#8 Integration by Parts

$$\int x\sqrt{x+7}\,dx$$

$u = x \qquad dv = \sqrt{x+7}$

$du = dx \qquad v = \frac{2}{3}(x+7)^{3/2}$

$$uv - \int v\,du$$

$$\frac{2}{3}x(x+7)^{3/2} - \frac{2}{3}\int (x+7)^{3/2}\,dx$$

$$\frac{2}{3}x(x+7)^{3/2} - \frac{2}{3}\,\frac{2}{5}(x+7)^{5/2} + C$$

$$\frac{2}{3}x(x+7)^{3/2} - \frac{4}{15}(x+7)^{5/2} + C$$

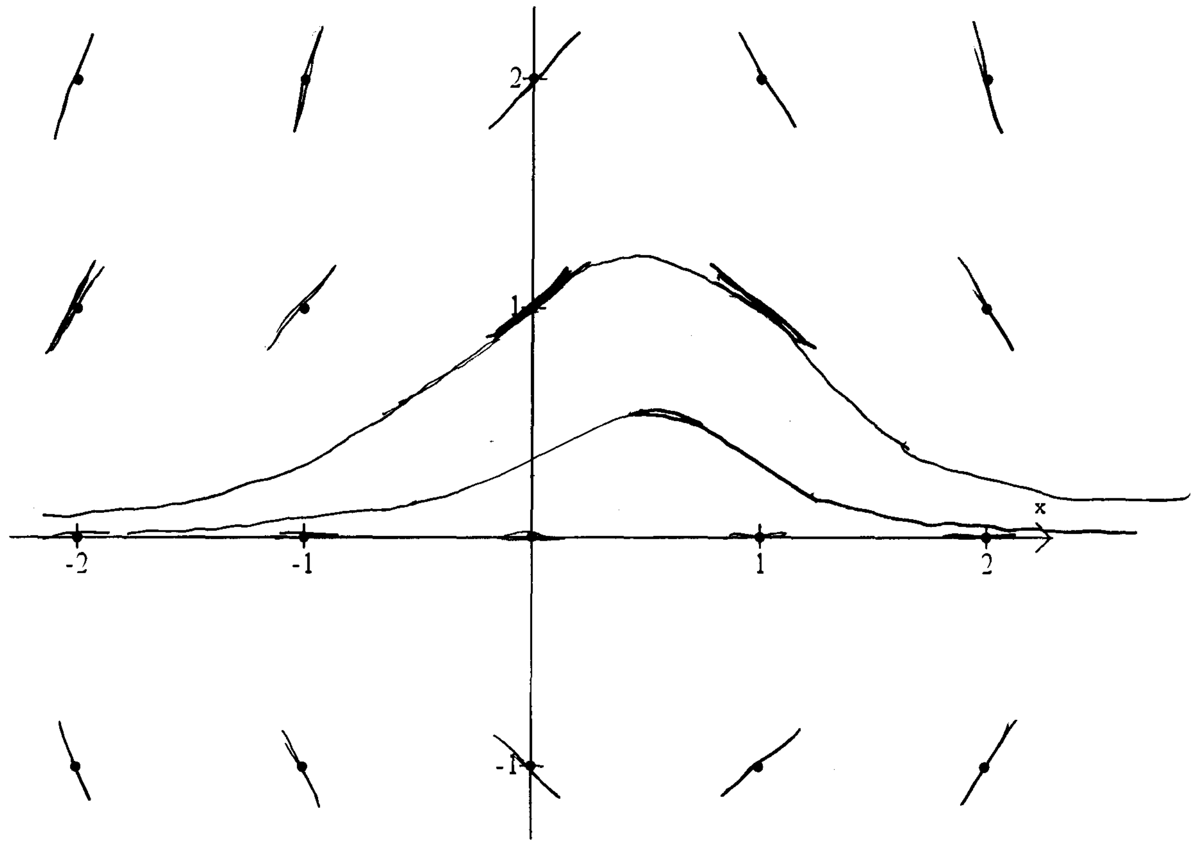
While this answer differs from the one using substitution, both can be differentiated to get $x\sqrt{x+7}$

10) Consider the differential equation $\dfrac{dy}{dx} = y(1-2x)$

a) On the axes below, sketch the slope field for the given differential equation at the twenty points indicated.

b)
Upper curve:

c)
lower curve



b) Sketch an approximate graph of y with an initial point of $(1, 1)$.

c) Sketch an approximate graph of y with an initial point of $(0, \frac{1}{e})$.

d) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = \frac{1}{e}$.

$$\frac{dy}{y} = 1-2x \, dx$$

$$\int \frac{dy}{y} = \int 1-2x \, dx$$

$$\ln|y| = x - x^2 + C$$

$$y = Ae^{x-x^2}$$

$$\frac{1}{e} = Ae^0$$

$$\frac{1}{e} = A$$

$$y = \frac{1}{e}e^{x-x^2}$$

$$\boxed{y = e^{x-x^2-1}}$$

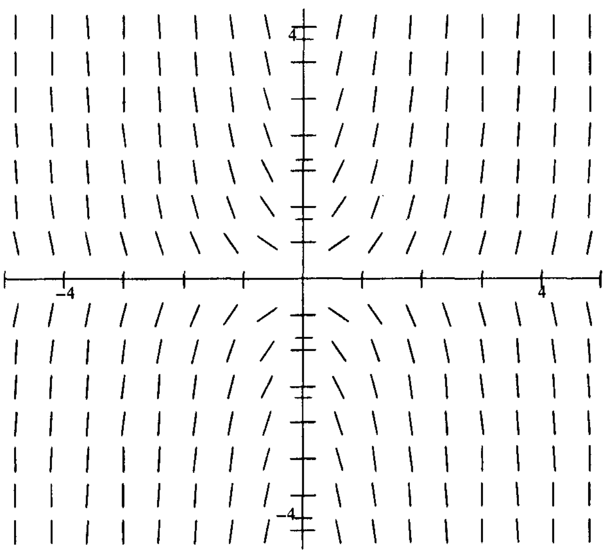
Note: You may use your slopefield program to check answers to #11, 12, and 13 but remember that your calculators will not be allowed on the test

11) $\frac{dy}{dx} = x - y$ C

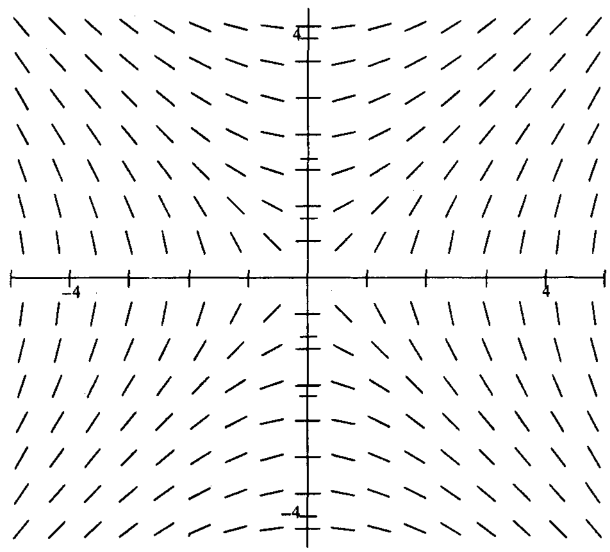
12) $\frac{dy}{dx} = 2xy$ A

13) $\frac{dy}{dx} = \frac{x}{y}$ B

A.



B.



C.