Review for Chapter 9 Test 1 Name____________________

You may do your work on a separate sheet of paper

Test the series for absolute convergence, conditional convergence, or divergence.

1) $\sum_{n=1}^{\infty}(-1)^n \frac{n^2-1}{n^2+n}$ $\lim_{n\to\infty} \frac{n^2-1}{n^2+n} = 1 \neq 0$ $\therefore$ Diverges by Test for Divergence

2) $\sum_{n=1}^{\infty}(-1)^n \frac{n^4-1}{n^5+n}$ Limit Comparison Test $\lim_{n\to\infty} \frac{\frac{n^4-1}{n^5+n}}{\frac{1}{n}} = 1$

Alternating Series converges but Absolute value does not $\therefore$ Conditional Convergence

3) $\sum_{n=1}^{\infty}\left(\frac{3n}{1-8n}\right)^n$ Root Test $\lim_{n\to\infty}\left|\sqrt[n]{\left(\frac{3n}{1-8n}\right)^n}\right| = \frac{3}{8} < 1$ $\therefore$ Absolute Convergence

4) $\sum_{k=1}^{\infty}\frac{2^k k!}{(k+2)!}$ Ratio Test $\lim_{n\to\infty}\left|\frac{2^{k+1}(k+1)!}{(k+3)!}\,\frac{(k+2)!}{2^k k!}\right| = \left|\frac{2(k+1)}{k+3}\right| = 2 > 1$

$\therefore$ Diverges

5) $\sum_{n=1}^{\infty}\frac{3^n n^2}{n!}$ Ratio Test $\lim_{n\to\infty}\left|\frac{3^{n+1}(n+1)^2}{(n+1)!}\cdot\frac{n!}{3^n n^2}\right| = \lim_{n\to\infty}\left|\frac{3}{n+1}\right| = 0 < 1$

$\therefore$ Absolute Convergence

6) $\sum_{n=1}^{\infty}(-1)^n 2^{1/n}$ $\lim_{n\to\infty} 2^{1/n} = 1 \neq 0$ $\therefore$ Diverges by Divergence Test

7) $\sum_{n=1}^{\infty}(-1)^n \frac{\ln n}{\sqrt{n}}$ $\lim_{n\to\infty}\frac{\ln n}{\sqrt{n}} = 0$ by L'Hopitals Rule but $\frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}} \leftarrow$ p series $p = \frac{1}{2}$

$\therefore$ Conditional Convergence

8) $\sum_{k=1}^{\infty}\frac{k+5}{5^k}$ Ratio Test $\lim_{n\to\infty}\left|\frac{k+6}{5^{k+1}}\cdot\frac{5^k}{k+5}\right| = \left|\frac{1}{5}\right| < 1$ $\therefore$ Converges

9) $\sum_{k=1}^{\infty}\frac{(-2)^{2k}}{k^k}$ Root Test $\lim_{k\to\infty}\sqrt[k]{\left|\frac{(-2)^{2k}}{k^k}\right|} = \left|\frac{4}{k}\right| = 0 < 1$

$\therefore$ Absolute Convergence

Limit Comparison Test

10) $\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$ Compare with $\frac{\sqrt{n^2}}{n^3} = \frac{n}{n^3} = \frac{1}{n^2}$

Converges

11) $\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$ Ratio Test $\lim_{n\to\infty} \left| \frac{(n+1)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n!} \right| = \left| \frac{n+1}{e^{2n+1}} \right| = 0 < 1$

Use L'Hopital's Rule

$\therefore$ Converges

12) $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ Limit Comparison $\lim_{n\to\infty} \frac{\frac{e^{1/n}}{n^2}}{\frac{1}{n^2}} = e^{1/n} = 1$ $\therefore$ converges

13) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n\sqrt{n}}$ $\tan^{-1} n < \frac{\pi}{2}$

L.C. Test $\longrightarrow$ $\lim_{n\to\infty} \left(\frac{\frac{\tan^{-1} n}{n^{3/2}}}{\frac{1}{n^{3/2}}} \right) = \lim_{n\to\infty} \tan^{-1} n = \frac{\pi}{2}$ converges

14) $\sum_{n=1}^{\infty} \frac{(2n)^n}{n^{2n}}$ Root Test $\lim_{n\to\infty} \left| \sqrt[n]{\frac{(2n)^n}{(n^2)^n}} \right| = \lim_{n\to\infty} \left| \frac{2n}{n^2} \right| = 0 < 1$ $\therefore$ converges

15) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$ Divergence Test $\lim_{n\to\infty} \left[\left(\frac{n}{n+1}\right)^n \right]^2 = \lim_{n\to\infty} \left[\frac{1}{\left(\frac{n+1}{n}\right)^n} \right]^2 \rightarrow$

$\rightarrow \lim_{n\to\infty} \left[\frac{1}{\left(1+\frac{1}{n}\right)^n} \right]^2 = \left(\frac{1}{e}\right)^2 = \frac{1}{e^2} \neq 0$

$\therefore$ Diverges

Find the sum of the given series.

16) $\sum_{n=1}^{\infty} \frac{3}{2^n}$ Geo series

$\sum_{n=1}^{\infty} \frac{3}{2}\left(\frac{1}{2}\right)^{n-1}$ $\frac{a}{1-r} = \frac{3/2}{1-1/2} = \frac{3/2}{1/2} = 3$

17) $\sum_{k=0}^{\infty} \frac{1}{k^2+5k+6}$

$AK + 2A + BK + 3B = 1$ $B = 1$ $A = -1$

$= \sum_{K=0}^{\infty} \frac{1}{(K+3)(K+2)} = \frac{A}{K+3} + \frac{B}{K+2}$

$A + B = 0$

$2A + 3B = 1$

$= \sum_{K=0}^{\infty} \frac{1}{K+2} - \frac{1}{K+3} = \left(\frac{1}{2} - \cancel{\frac{1}{3}}\right) + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}\right) + \left(\cancel{\frac{1}{4}} - \frac{1}{5}\right) \ldots \left(\frac{1}{K+1} - \frac{1}{K+2}\right) + \left(\frac{1}{K+2} - \frac{1}{K+3}\right)$

$= \frac{1}{2}$