Fall Review 1 Name Solutions

1) Find the limit if it exists: $\lim_{x\to 0}\frac{2x(\cot x-\csc x)}{\cos x-1}$ (Hint: write all trig functions in terms of sine and cosine)

$$\lim_{x\to 0}\frac{2x(\cot x-\csc x)}{\cos x-1}=\frac{0}{0}\Rightarrow \lim_{x\to 0}\frac{2x\csc x(\cos x-1)}{\cos x-1}=\lim_{x\to 0}\frac{2x}{\sin x}=\lim_{x\to 0}2\frac{x}{\sin x}$$

$$\frac{x}{\sin x}\to 1$$

$$=2$$

2) Is the function $f(x)=\begin{cases}x^3+1 & \text{when } x\le 1\\ 3x-1 & \text{when } x>1\end{cases}$ differentiable at $x = 1$? Show the work that leads to your answer.

$$\left.\begin{aligned}\lim_{x\to 1^-} x^3+1 &= 2\\ \lim_{x\to 1^+} 3x-1 &= 2\end{aligned}\right\}\text{ continuous}\qquad \left.\begin{aligned}\lim_{x\to 1^-} 3x^2 &= 3\\ \lim_{x\to 1^+} 3 &= 3\end{aligned}\right\}\text{ Differentiable}$$

3) For the function in #2, find the *equation* of the tangent line at

a) $x = 0$

$m=3(0)^2=0 \quad f(0)=1$

$y-1=0(x-0)$

$y=1$

b) $x = 2$

$m=3 \quad f(2)=5$

$y-5=3(x-2)$

c) $x = 1$ (if the function is differentiable at 1)

$m=3 \quad f(1)=2$

$y-2=3(x-1)$

4) Given $xy^3 = 1$, use implicit differentiation to find y'.

$$y^3+x\,3y^2y'=0$$

$$x3y^2y'=-y^3$$

$$y'=-\frac{y^3}{3y^2x}=-\frac{y}{3x}$$

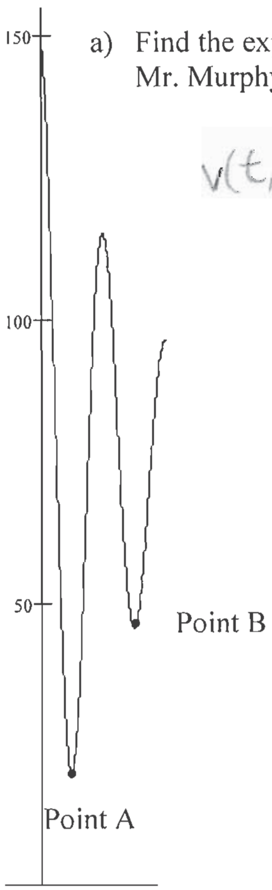
5) Given $y^2 - x^3 - 2x = 0$, use implicit differentiation to find y''.

$$2yy' - 3x^2 - 2 = 0$$
$$2yy' = 3x^2 + 2$$
$$y' = \frac{3x^2+2}{2y}$$

$$y'' = \frac{6x(2y) - 2y'(3x^2+2)}{4y^2} = \frac{12xy - 2y'(3x^2+2)}{4y^2}$$

$$y'' = \frac{12xy - 2\frac{3x^2+2}{2y}(3x^2+2)}{4y^2} = \boxed{\frac{12xy^2 - (3x^2+2)^2}{4y^3}}$$

6) Trying to have the most creative physics project, Brendan and Sherman dangle Mr. Murphy from the top of the SLC on the end of a bungee cord. They calculate the equation for Mr. Murphy's position as being $s(t) = 75+ 75$ over a period of 12 seconds approximating the SLC to be about 150 feet high. The diagram of the motion is displayed below.



a) Find the expression for Mr. Murphy's velocity and use it and your calculator to find when and where Mr. Murphy is dangling at his lowest point (Point A) → First time $v(t) = 0$

$$v(t) = s'(t) = -15e^{-t/5}\cos(2t) + 75e^{-t/5}(-2\sin 2t)$$
$$= -15e^{-t/5}\cos(2t) - 150e^{-t/5}\sin 2t = 0$$

Calculator

$$t \approx 1.521 \text{ seconds}$$
$$s(1.521) \approx 19.946 \text{ feet}$$

b) Use $v(t)$ found in part a) to find out Mr. Murphy's second lowest point (Point B) Third time $v(t) = 0$

$$t \approx 4.663 \text{ seconds}$$
$$s(4.663) \approx 45.629 \text{ feet}$$

7) As they pull Mr. Murphy back up to the ledge, the cord breaks on the way up and Mr. Murphy plunges 128 feet onto a huge airbag that Herman and Evan put down to break his fall. Naomi, who is supposed to be there helping out wonders, "I wonder when his instantaneous velocity will equal his average velocity on the way down." Given that the equation for his free fall is $s(t) = 128 - 16t^2$

a) Find the answer to Naomi's question.

$$s'(c) = \frac{s(b)-s(a)}{b-a}$$

$a = 0$

$b = 2\sqrt{2}$

$s(t) = 0 = 128 - 16t^2$

$t = 2\sqrt{2}$

$$-32t = \frac{0-128}{2\sqrt{2}}$$

$$-32t = -\frac{64}{\sqrt{2}}$$

$$t = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ seconds}$$

b) While Naomi is pondering this question, Gio wonders "I wonder when his instantaneous acceleration will equal his average acceleration? Find the answer to Gio's question.

$$s''(c) = \frac{s'(b)-s'(a)}{b-a}$$

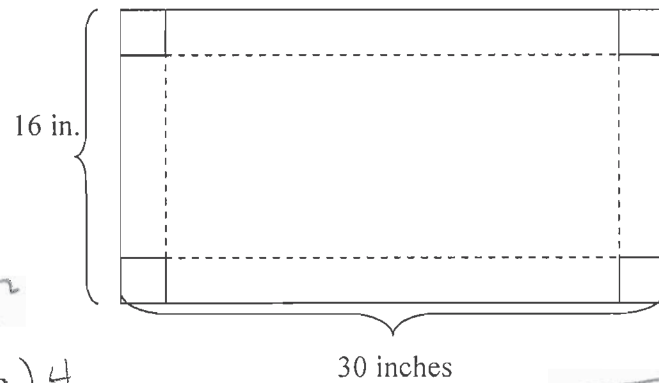
$$-32 = \frac{-64\sqrt{2} - 0}{2\sqrt{2}}$$

$$-32 = -32$$

always

Fall Review 2 Name Solutions

1) Adrienne needs to make an open box from a 16 inch by 30 inch piece of cardboard by cutting out square of equal size from the four corners and bending up the sides. But instead, she just sits at the table whining that it can't be done. Since Lauren is no help either, Samantha now have to do it for her. What dimensions should she use to obtain a box with the largest possible volume?



$l = 16-2x$

$h = x$

$w = 30-2x$

$V = (16-2x)x(30-2x)$

$V = 480x - 92x^2 + 4x^3$

$V' = 480 - 184x + 12x^2$

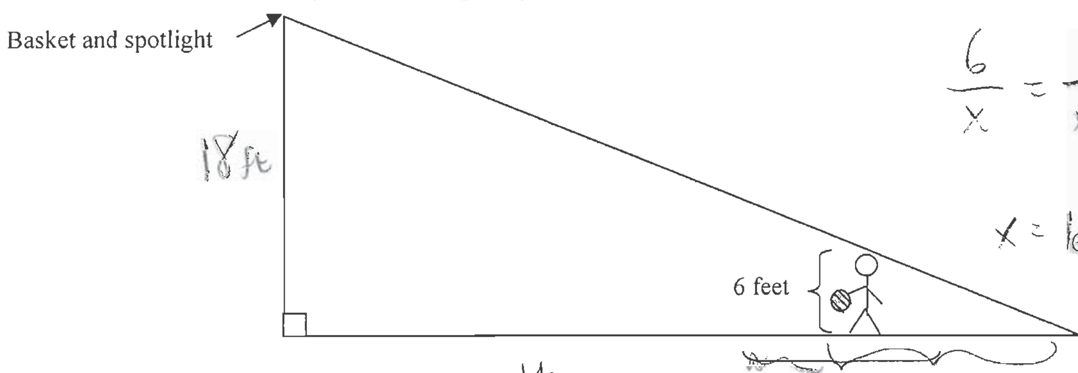
$0 = (3x-10)(x-12)4$

$x = \frac{10}{3}, 12$

Test intervals or use concavity to show

$x = 3\frac{1}{3}$ inches $V = 725.926$ in³

2) Mussman, claiming to be 6 ft, hopes to dunk a ball over Hamidou in a basket that has been moved from 10 feet high to 18 feet high. Alvin and Matt, having bet against him, mount a camera with a spotlight at the rim of the basket to film his attempt. Mussman runs toward the basket at 6 ft/sec with the spotlight casting a shadow behind him. The diagram of the spotlight, Mussman, and his shadow is shown below.



$\frac{6}{x} = \frac{18}{x+y}$ $\frac{dy}{dt} = -6$ ft/sec

x = length of shadow

y = distance to basket

a) At what rate is the size of his shadow changing? (This rate should be negative)

$6x + 6y = 18x$

$6y = 12x$ $\Rightarrow 6\frac{dy}{dt} = 12\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = -3$ ft/sec (his shadow is shrinking)

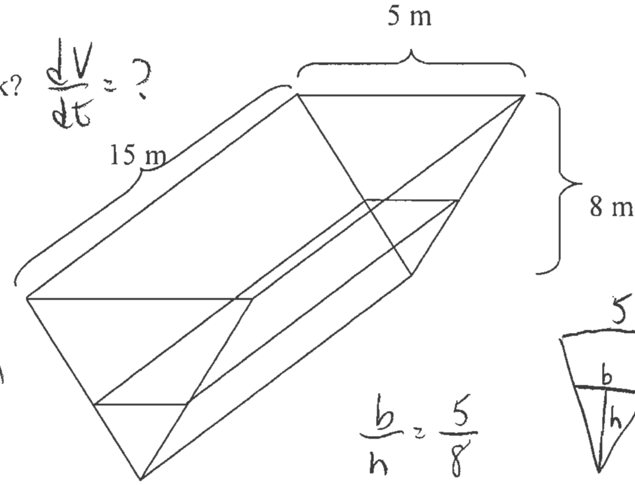
b) How fast is the tip of his shadow moving along the ground?

His shadow is both shrinking and moving with him so...

$\frac{dx}{dt} + \frac{dy}{dt} = \boxed{-9 \text{ ft/sec}}$

3) While Garrett is whining about having too much calculus homework, Natalie and Jennica have to observe the filling of a trough 15 meters long, 5 meters wide, and 8 meters deep all by herself. The water is flowing into the trough at a constant rate. When the height of the water reaches 4 meters, she notices that the height is increasing at a rate of 2 m/min. ($V = \frac{1}{2}\, bhl$)

a) At what rate is the fuel flowing into the tank? $\frac{dV}{dt} = ?$



$V = 15bh$

$V = 15\left(\frac{5}{8}h\right)h$

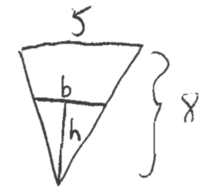
$= \frac{75}{8}h^2$

$\frac{dh}{dt} = 2$ m/min

$h = 4$

$\frac{b}{h} = \frac{5}{8}$

$b = \frac{5}{8}h$



$\frac{dV}{dt} = \frac{75}{4}h\frac{dh}{dt}$

$= \frac{75}{4}(4)(2) = \boxed{150\ m^3/min}$

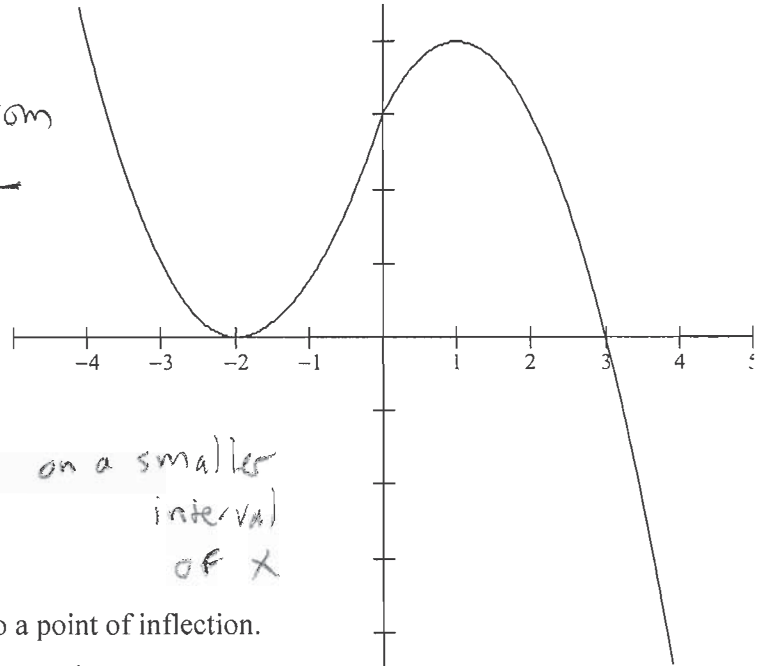
b) How long will it take to fill the tank?

$V = 15(5)(8) = 600\ m^3/min$

$150 \cdot t = 600$

$\boxed{t = 4 \text{ minutes}}$

4) The graph of $f'(x)$ on the interval $-4 \le x \le 4$ is shown below. Find *and justify* all values of x for which $f(x)$



a) has a relative maximum

$x=3$ f' goes from $+$ to $-$

b) has a relative minimum

none although a case could be made for $x=-2$ on a smaller interval of x

c) is concave down.

$-4<x<-2$
$1<x<4$
because $f''<0$

d) Has a critical point that is also a point of inflection.

$x=-2$ because $f'=f''=0$

e) Given that the graph of f crosses the origin, sketch a graph of f on the axes below.