Implicit Differentiation

Think of y as a function of x so that applying the chain rule(outside-inside) would give us these results:

a) $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$

outside - inside

b) $\frac{d}{dx}(\sin y) = (\cos y)\frac{dy}{dx}$

outside - inside

1) Find the equation of the line tangent to the circle $x^2 + y^2 = 9$ at the point $(2, -\sqrt{5})$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{dy}{dx} = -\frac{2}{-\sqrt{5}}$$

$$= \frac{2}{\sqrt{5}}$$

$$y + \sqrt{5} = \frac{2}{\sqrt{5}}(x-2)$$

Find y'(Remember $y' = \frac{dy}{dx}$)

2) $\sqrt{x} - \sqrt{y} = 5$

$$\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{y}}y' = 0$$

$$-\frac{1}{2\sqrt{y}}y' = -\frac{1}{2\sqrt{x}}$$

$$y' = \frac{2\sqrt{y}}{2\sqrt{x}}$$

$$y' = \frac{\sqrt{y}}{\sqrt{x}}$$

$$y' = \sqrt{\frac{y}{x}}$$

3) $x^3 + xy + y^3 = xy^2$

$$3x^2 + y + xy' + 3y^2y' = y^2 + 2xyy'$$

$$xy' + 3y^2y' - 2xyy' = y^2 - y - 3x^2$$

$$y'(x + 3y^2 - 2xy) = y^2 - y - 3x^2$$

$$y' = \frac{y^2 - y - 3x^2}{x + 3y^2 - 2xy}$$

4) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ where a is a constant

$$\frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}y' = 0$$

$$\frac{2}{3}y^{-\frac{1}{3}}y' = -\frac{2}{3}x^{-\frac{1}{3}}$$

$$y' = -\frac{x^{-\frac{1}{3}}}{y^{-\frac{1}{3}}}$$

$$= -\frac{y^{1/3}}{x^{1/3}}$$

$$= -\sqrt[3]{\frac{y}{x}}$$

5) $\sin^2 y = x^2 + 2$

$$2\sin y(\cos y)y' = 2x$$

$$y' = \frac{x}{\sin y\cos y}$$

6) $\sin(xy) = 2x + 5$

$$\cos(xy)(y + xy') = 2$$

$$y + xy' = \frac{2}{\cos(xy)}$$

$$xy' = \frac{2}{\cos(xy)} - y$$

$$y' = \frac{2}{x\cos(xy)} - \frac{y}{x}$$

7) For #1, find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = -\frac{x}{y}$$

$$\frac{d^2y}{dx^2} = -\frac{(1)y - xy'}{y^2}$$

$$= -\frac{y - x\left(-\frac{x}{y}\right)}{y^2}$$

$$= -\frac{y + \frac{x^2}{y}}{y^2} \cdot \frac{y}{y}$$

$$= -\frac{y^2 + x^2}{y^3} = \frac{-9}{y^3}$$