Substitution

Name Solutions

1) $\int_0^{\frac{1}{2}} \sqrt{1-2x}\, dx$

$u = 1-2x$

$du = -2\,dx$

$-\frac{1}{2}du = dx$

$x = 0 \quad u = 1-2(0) = 1$

$x = \frac{1}{2} \quad u = 1-2(\frac{1}{2}) = 0$

$$= -\frac{1}{2}\int_1^0 \sqrt{u}\, du$$

$$= \frac{1}{2}\int_0^1 u^{\frac{1}{2}}\, du$$

$$= \frac{1}{2}\,\frac{2}{3}\,u^{3/2}\Big]_0^1$$

$$= \frac{1}{3}(1)^{3/2} - \frac{1}{3}(0)^{3/2}$$

$$= \frac{1}{3}$$

2) $\int \sin(7-3x)\, dx$

$u = 7-3x$

$du = -3\,dx$

$-\frac{1}{3}du = dx$

$$\int \sin(7-3x)\, dx = -\frac{1}{3}\int \sin u\, du$$

$$= \frac{1}{3}\cos u + C$$

$$= \frac{1}{3}\cos(7-3x) + C$$

3) $\int x(x^2-1)^{99}\, dx$

$u = x^2-1$

$du = 2x\, dx$

$\frac{1}{2}du = x\, dx$

$$\int x(x^2-1)^{99}\, dx = \frac{1}{2}\int u^{99}\, du$$

$$= \frac{1}{2}\,\frac{u^{100}}{100} + C$$

$$= \frac{u^{100}}{200} + C$$

$$= \frac{(x^2-1)^{100}}{200} + C$$

4) $\int \frac{x^2}{\sqrt{2+x^3}}\, dx$

$u = 2+x^3$

$du = 3x^2\, dx$

$\frac{1}{3}du = x^2\, dx$

$$\int \frac{x^2}{\sqrt{2+x^3}}\, dx = \frac{1}{3}\int \frac{du}{\sqrt{u}} = \frac{1}{3}\int u^{-\frac{1}{2}}\, du$$

$$= \frac{1}{3}\,\frac{u^{1/2}}{1/2} + C$$

$$= \frac{2}{3}u^{1/2} + C$$

$$= \frac{2}{3}\sqrt{2+x^3} + C$$

5) $\int \frac{x+3}{(x^2+6x)}\,dx$

$u = x^2+6x$

$du = (2x+6)\,dx$

$du = 2(x+3)\,dx$

$\frac{1}{2}du = (x+3)\,dx$

$\int \frac{x+3}{x^2+6x}\,dx = \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln(x^2+6x) + C$

6) $\int x^3(1-x^4)^5\,dx$

$u = 1-x^4$

$du = -4x^3\,dx$

$-\frac{1}{4}du = x^3\,dx$

$\int x^3(1-x^4)^5\,dx = -\frac{1}{4}\int u^5\,du$

$= -\frac{1}{4}\frac{u^6}{6} + C$

$= -\frac{(1-x^4)^6}{24} + C$

7) $\int \sin x \cos x\,dx$

$u = \sin x$

$du = \cos x\,dx$

$\int \sin x \cos x\,dx = \int u\,du$

$= \frac{u^2}{2} + C$

$= \frac{\sin^2 x}{2} + C$

8) $\int_0^{\pi/4} \tan^3 x \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$x = 0 \quad u = \tan 0 = 0$

$x = \frac{\pi}{4} \quad u = \tan\frac{\pi}{4} = 1$

$\int_0^1 u^3\,du = \frac{u^4}{4}\Big]_0^1$

$= \frac{1}{4} - \frac{0}{4} = \frac{1}{4}$

9) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-\frac{1}{2}}\,dx = \frac{1}{2\sqrt{x}}\,dx$

$2\,du = \frac{dx}{\sqrt{x}}$

$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx = 2\int \cos u\,du$

$= 2\sin u + C$

$= 2\sin\sqrt{x} + C$

10) $\int_{\sqrt[3]{\pi/2}}^{\sqrt[3]{\pi}} t^2\cos(\pi - t^3)\,dt$

$u = \pi - t^3$

$du = -3t^2\,dt$

$-\frac{1}{3}du = t^2\,dt$

Change limits

$t = \sqrt[3]{\pi/2}$

$u = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

$t = \sqrt[3]{\pi}$

$u = \pi - \pi = 0$

$-\frac{1}{3}\int_{\frac{\pi}{2}}^{0} \cos u\,du = \frac{1}{3}\int_{0}^{\pi/2} \cos u\,du$

$= \frac{1}{3}\sin u\Big]_0^{\frac{\pi}{2}} = \frac{1}{3}\sin\frac{\pi}{2} - \frac{1}{3}\sin 0 = \frac{1}{3}$

11) $\int_0^1 (x^4+x)^5(4x^3+1)\,dx$

$u = x^4 + x$

$du = (4x^3+1)\,dx$

Change limits

$x = 0$

$u = 0 + 0 = 0$

$x = 1$

$u = 1 + 1 = 2$

$\int_0^2 u^5\,du = \frac{u^6}{6}\Big]_0^2$

$= \frac{32}{3}$

12) $\int_1^4 \frac{1}{x^2}\sqrt{1+\frac{1}{x}}\,dx$

$u = 1 + \frac{1}{x}$

$du = -\frac{1}{x^2}\,dx$

$-du = \frac{1}{x^2}\,dx$

Change limits

$x = 1 \quad u = 1 + \frac{1}{1} = 2$

$x = 4 \quad u = 1 + \frac{1}{4} = \frac{5}{4}$

$-\int_2^{5/4} \sqrt{u}\,du = \int_{5/4}^{2} u^{1/2}\,du$

$= \frac{u^{3/2}}{3/2}\Big]_{5/4}^{2}$

$= \frac{2}{3}\left(\sqrt[3]{8} - \sqrt[3]{\frac{5}{4}}\right)$

13) $\int_0^1 \frac{e^x}{e^x+1}\,dx$ let $u = e^x+1$

$du = e^x\,dx$

$x = 0 \quad u = e^0+1 = 2$

$x = 1 \quad u = e+1$

$$\int_2^{e+1} \frac{du}{u} = \ln u \Big]_2^{e+1}$$

$$= \ln(e+1) - \ln 2$$

or

$$\ln\left(\frac{e+1}{2}\right)$$

14) $\int_1^2 x\sqrt{x-1}\,dx$ $u = x-1$ $x=1$ $u=0$

$du = dx$ $x=2$ $u=1$

$$\int_0^1 (u+1)\sqrt{u}\,du$$

$$\int_0^1 u\sqrt{u} + \sqrt{u}\,du$$

$$\int_0^1 u^{3/2} + u^{1/2}\,du$$

$$\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\Big]_0^1$$

$$\frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$$