Chapter 3 Test 2 Review Name Solutions

1) Arielle is solving calculus problems at a rate of 1.5 problems every minute. Meanwhile, right behind her, Nichole is telling Liezl about all of her drama. She is talking at a rate of 120 words for every problem that Arielle solves. How fast is Nichole talking in words per minute?

$\frac{dW}{dP} = 120$

$\frac{dP}{dt} = 1.5$

$\frac{dW}{dt} = ?$

$\frac{dW}{dt} = \frac{dW}{dP} \cdot \frac{dP}{dt} = 120 \cdot 1.5 =$ 180 words/minute

2) Michael Gray and Dominick Lau are airline pilots who would like to take off but can't because Brendan is not doing a very good job of fueling their plane because there is a leak that he is too distracted to notice. The amount of fuel in the tank in gallons at this time is given by $Q(t) = 2t^3 - 39t^2 + 180t + 170$ where t is measured in minutes from the time that they began fueling.

a) When did the fuel level first begin to drop? → when $Q'(t) = 0$ (Hint: Graph $Q(t)$)

$Q(t) = 2t^3 - 39t^2 + 180t + 170$

$Q'(t) = 6t^2 - 78t + 180 = 0 = 6(t^2 - 13t + 30) = 0$

$= (t-10)(t-3) = 0$

$t = 3, 10$

$t = 3$ minutes

b) When did the fuel level finally start to level off and rise again?

$t = 10$ minutes ← find the other point where $Q'(t) = 0$

c) What was the fuel level at this time?

$Q(10) = 70$ gallons

Find the derivative of each expression.

3) a) $\cos^2\left(\frac{1}{\sqrt{x^3-2x}}\right) = \left[\cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]^2$

$\Rightarrow 2\left[\cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]\left[-\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]\left(-\frac{1}{2}\right)\left(x^3-2x\right)^{-\frac{3}{2}}\left(3x^2-2\right)$

$\Rightarrow \cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)\left(3x^2-2x\right)\left(x^3-2x\right)^{-3/2}$

b) $\cos\left(e^{x^3-2x}\right) \Rightarrow \left[-\sin\left(e^{x^3-2x}\right)\right]\left(e^{x^3-2x}\right)\cdot\left(3x^2-2\right)$

$\left[-\sin\left(e^{x^3-2x}\right)\right]\left(3x^2-2\right)e^{x^3-2x}$

See last pg for more on #3

4) $\tan^{-1}(\sec x)$

$\Rightarrow \frac{1}{1+\sec^2 x}\sec x\tan x$

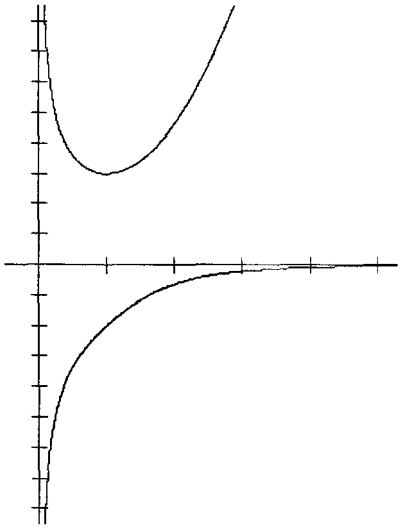
$= \frac{\sec x\tan x}{1+\sec^2 x}$

5) $\sin^{-1}\left(5^{3x}\right)$

$\Rightarrow \frac{1}{\sqrt{1-5^{6x}}}\left(5^{3x}\right)\left(3\ln 5\right)$

$\frac{5^{3x}(3\ln 5)}{\sqrt{1-5^{6x}}}$

6) Given the equation $xy^2 - x^3y = 6$ graphed below



a) Find the slope of the line tangent to the graph at the point (1, 3)

$y^2 + 2yy'x - 3x^2y - x^3y' = 0$

$9 + 6y' - 9 - y' = 0$

$5y' = 0$

$m = 0$

b) Find the other point on the curve where the x coordinate is 1 and find the equation of the tangent line there.

plug in 1 for X

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3, -2 \Rightarrow (1,-2)$ is the other point

plug $(1,-2)$ into the derivative $\Rightarrow 4 - 4y' + 6 - y' = 0$

$-5y' = -10$

$y' = 2$

$y + 2 = 2(x-1)$

7) $y = x^{\frac{1}{x}}$ Find y'

$$\ln y = \ln x^{\frac{1}{x}} = \frac{1}{x}\ln x = \frac{\ln x}{x}$$

$$\frac{1}{y}y' = \frac{\frac{1}{x}x - \ln x}{x^2}$$

$$\frac{1}{y}y' = \frac{1 - \ln x}{x^2}$$

$$y' = y\,\frac{1 - \ln x}{x^2}$$

$$y' = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right) \quad \text{or}$$

$$x^{\frac{1}{x}-2}(1-\ln x)$$

$$\cos^2\left(\frac{1}{\sqrt{x^3-2x}}\right) =$$

There are 4 "babies" here. Make sure to take each derivative one at a time

$$\left[\cos\left(x^3-2x\right)^{-\frac{1}{2}}\right]^2$$

$$2\left[\cos\left(x^3-2x\right)^{-\frac{1}{2}}\right]\cdot\left[-\sin\left(x^3-2x\right)^{-\frac{1}{2}}\right]\cdot$$

$$-\frac{1}{2}\left(x^3-2x\right)^{-\frac{3}{2}}\cdot\left(3x^2-2\right)$$

$$= \cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)\left(\frac{3x^2-2}{2\left(x^3-2x\right)^{3/2}}\right)$$