

Chapter 3 Test 2 Review

Name Solutions

- 1) Arielle is solving calculus problems at a rate of 1.5 problems every minute. Meanwhile, right behind her, Nichole is telling Liezl about all of her drama. She is talking at a rate of 120 words for every problem that Arielle solves. How fast is Nichole talking in words per minute?

$$\frac{dW}{dP} = 120$$

$$\frac{dP}{dt} = 1.5$$

$$\frac{dW}{dt} = ?$$

$$\frac{dW}{dt} = \frac{dW}{dP} \cdot \frac{dP}{dt} = 120 \cdot 1.5 = 180 \text{ words/minute}$$

- 2) Michael Gray and Dominick Lau are airline pilots who would like to take off but can't because Brendan is not doing a very good job of fueling their plane because there is a leak that he is too distracted to notice. The amount of fuel in the tank in gallons at this time is given by $Q(t) = 2t^3 - 39t^2 + 180t + 170$ where t is measured in minutes from the time that they began fueling.

- a) When did the fuel level first begin to drop? → when $Q'(t) = 0$ (Hint: Graph $Q(t)$)

$$Q(t) = 2t^3 - 39t^2 + 180t + 170$$

$$Q'(t) = 6t^2 - 78t + 180 = 0 \Rightarrow 6(t^2 - 13t + 30) = 0$$

$$= (t-10)(t-3) = 0$$

$$t = 3, 10 \quad t = 3 \text{ minutes}$$

- b) When did the fuel level finally start to level off and rise again?

$t = 10 \text{ minutes}$ ← find the other point where $Q'(t) = 0$

- c) What was the fuel level at this time?

$$\underline{Q(10) = 70 \text{ gallons}}$$

Find the derivative of each expression.

$$3) \text{ a) } \cos^2\left(\frac{1}{\sqrt{x^3-2x}}\right) = \left[\cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]^2$$

$$\Rightarrow 2\left[\cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]\left[-\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)\right]\left(-\frac{1}{2}\right)(x^3-2x)^{-\frac{3}{2}}(3x^2-2)$$

$$\Rightarrow \cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)(3x^2-2)(x^3-2x)^{-\frac{3}{2}}$$

$$\text{ b) } \cos(e^{x^3-2x}) \Rightarrow [-\sin(e^{x^3-2x})](e^{x^3-2x})$$

$$(3x^2-2)$$

$$[-\sin(e^{x^3-2x})](3x^2-2)e^{x^3-2x}$$

See last page
for more on #3

$$4) \tan^{-1}(\sec x)$$

$$\Rightarrow \frac{1}{1+\sec^2 x} \sec x \tan x$$

$$= \frac{\sec x \tan x}{1+\sec^2 x}$$

$$5) \sin^{-1}(5^{3x})$$

$$\Rightarrow \frac{1}{\sqrt{1-5^{6x}}}(5^{3x})(3 \ln 5)$$

$$\frac{5^{3x}(3 \ln 5)}{\sqrt{1-5^{6x}}}$$

6) Given the equation $xy^2 - x^3y = 6$ graphed below

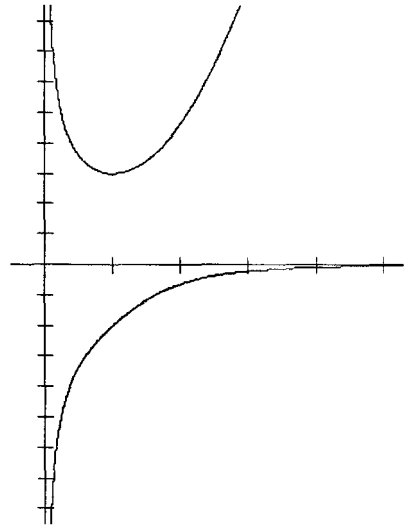
a) Find the slope of the line tangent to the graph at the point (1, 3)

$$y^2 + 2yy'x - 3x^2y - x^3y' = 0$$

$$9 + 6y' - 9 - y' = 0$$

$$5y' = 0$$

$$m = 0$$



b) Find the other point on the curve where the x coordinate is 1 and find the equation of the tangent line there.

plug in 1 for x

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$y = 3, -2 \Rightarrow (1, -2)$ is the other point

plug (1, -2) into the derivative $\Rightarrow 4 - 4y' + 6 - y' = 0$

$$-5y' = -10$$

$$y' = 2$$

$$y + 2 = 2(x - 1)$$

7) $y = x^{\frac{1}{x}}$ Find y'

$$\ln y = \ln x^{\frac{1}{x}} = \frac{1}{x} \ln x = \frac{\ln x}{x}$$

$$\frac{1}{y} y' = \frac{\frac{1}{x} - \ln x}{x^2}$$

$$\frac{1}{y} y' = \frac{1 - \ln x}{x^2}$$

$$y' = y \frac{1 - \ln x}{x^2}$$

$$y' = x^{\frac{1}{x}} \left(\frac{1 - \ln x}{x^2} \right) \text{ or}$$

$$x^{\frac{1}{x}-2} (1 - \ln x)$$

$$\cos^2\left(\frac{1}{\sqrt{x^3-2x}}\right) =$$

$$\left[\cos(x^3-2x)^{-\frac{1}{2}} \right]^2$$

There are 4
"babies" here. Make
sure to take each
derivative one at
a time

$$2 \left[\cos(x^3-2x)^{-\frac{1}{2}} \right] \cdot \left[-\sin(x^3-2x)^{-\frac{1}{2}} \right]$$

$$-\frac{1}{2} (x^3-2x)^{-\frac{3}{2}} \cdot (3x^2-2)$$

$$= \cos\left(\frac{1}{\sqrt{x^3-2x}}\right) \sin\left(\frac{1}{\sqrt{x^3-2x}}\right) \left(\frac{3x^2-2}{2(x^3-2x)^{3/2}} \right)$$