Calculus Chapter 3 Review Sheet 1 Name_______________

Show all your work

1) $f(x) = \begin{cases} 1-2x, & x<0 \\ 1-2\tan x, & x \geq 0 \end{cases}$ Show whether $f(x)$ is differentiable at $x = 0$. If it is, find $f'(0)$. ($\frac{d}{dx}\tan(x) = \sec^2 x$)

Is $f(x)$ continuous?

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 1-2x = 1$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 1-2\tan x = 1$

$f(0) = 1$ ∴ $f(x)$ is continuous at $x=0$

Is $f(x)$ differentiable at $x=0$

$f'(x) \begin{cases} -2 & x<0 \\ -2\sec^2 x & x>0 \end{cases}$

$\lim_{x\to 0^+} -2 = -2$

$\lim_{x\to 0^-} -2\sec^2 x = -2$

∴ f is differentiable at $x=0$

$f'(0) = -2$

2) Given $f(x) = x^2 - 5x$, use either difference quotient formula to find the *equation* of the tangent line at $x = 2$. $f(2) = -6$ so the point is $(2, -6)$

$\lim_{h\to 0} \frac{(x+h)^2 - 5(x+h) - x^2 + 5x}{h}$

$\lim_{h\to 0} \frac{x^2 + 2xh + h^2 - 5x - 5h - x^2 + 5x}{h}$

$\lim_{h\to 0} \frac{2xh + h^2 - 5h}{h}$

$\lim_{h\to 0} 2x + h - 5 = 2x - 5 \Rightarrow f'(2) = -1$

or

$\lim_{x\to 2} \frac{x^2 - 5x + 6}{x-2} = \frac{(x-3)(x-2)}{(x-2)} = x - 3 = -1$

$f'(2) = -1$

equation of tangent line

$y + 6 = -1(x-2)$

Use the product rule to find $\frac{dy}{dx}$.

3) $y = (5x+7)(4-3x^2)$

$f \quad g$

$f' = 5 \quad g' = -6x$

$y' = 5(4-3x^2) + (-6x)(5x+7)$

$= 20 - 15x^2 - 30x^2 - 42x$

Use the quotient rule to find $\frac{dy}{dx}$.

4) $y = \frac{7x^3 - 3}{2 - \tan x}$ f g $f' = 21x^2$ $g' = -\sec^2 x$

$$y' = \frac{21x^2(2-\tan x) - (7x^3-3)(-\sec^2 x)}{(2-\tan x)^2}$$

5) Max, Christine, and Martine are standing on a balcony that is 10 feet above the plaza. They let go of a helium balloon in the air and observe that it rises for a few seconds and then falls because they didn't tie it tight enough. Meanwhile, Annie and Brian are tracking the balloon and have determined the equation for its height off the ground s (in feet) to be $s(t) = 10 + 18t + 6t^2 - 2t^3$.

a) Find the balloons initial velocity.

$v(t) = 18 + 12t - 6t^2$

$v(0) = 18$ ft/sec

b) When does the balloon reach its highest point?

$v(t) = 0 = 18 + 12t - 6t^2$

$= 3 + 4t - t^2$

$= (3-t)(1+t)$

$t = -1, 3 \Rightarrow t \neq -1$ seconds

$t = 3$ seconds

c) What is the balloons highest point?

$s(3) = 10 + 18(3) + 6(9) - 2(27)$

$= 10 + 54 + 54 - 54 = 64$ ft

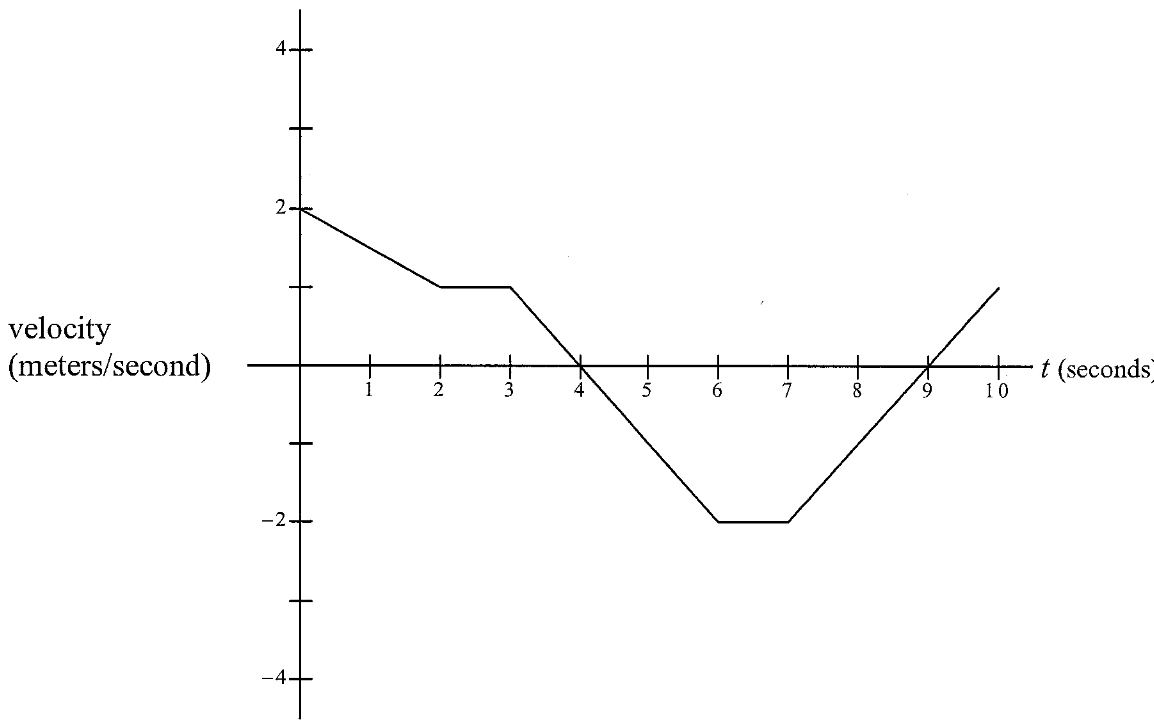
d) When is the balloon's *acceleration* downward?

$a(t) < 0$

$12 - 12t < 0$

$t > 1$ second.

The graph of the velocity of a moving particle in meters/second is shown below.



6) Over what intervals of t is the particle moving to the left? Justify your answer.

$4 < t < 9$ because velocity is negative

7) When is the particle's acceleration positive? Justify your answer.

$7 < t < 10$ because the slope of the velocity curve is positive

8) Over which intervals is the particle slowing down? Justify your answer.

$0 < t < 2$ because $v > 0$, $a < 0$

$3 < t < 4$ " $v > 0$, $a < 0$

$7 < t < 9$ " $v < 0$, $a > 0$