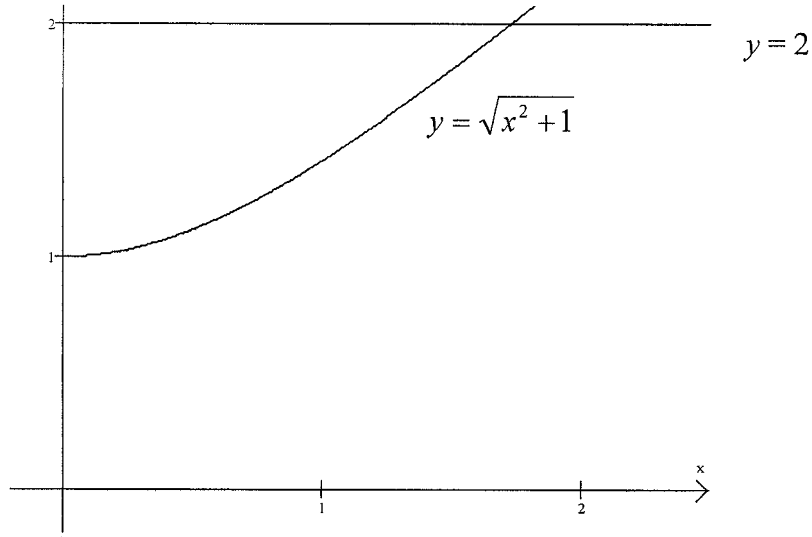
Chapter 7 Review Name Solutions



Read each question carefully.

1) Identify the region bounded by the curve $y=\sqrt{x^2+1}$, the line $y = 2$, and the y-axis. Indicating the method that you use each time, set up the integral to find

(a) the area of the region

$$\boxed{\int_0^{\sqrt{3}} 2-\sqrt{x^2+1}\,dx}$$

limits of integration: why $\sqrt{3}$?

$\sqrt{x^2+1}=2$

$x^2+1=4$

$x^2=3 \Rightarrow x=\pm\sqrt{3}$

(b) the volume when the region is rotated about the y-axis

Horizontal rectangles (dy)

Disk method $r=\sqrt{y^2-1}$

$y=\sqrt{x^2+1}$

$y^2=x^2+1$

$y^2-1=x^2 \rightarrow x=\pm\sqrt{y^2-1}$

$$\boxed{\pi\int_1^2\left(\sqrt{y^2-1}\right)^2 dy}$$

(c) the volume when the region is rotated about x-axis

Washer $R=2$ $r=\sqrt{x^2+1}$

$$\boxed{\pi\int_0^{\sqrt{3}} 4-(x^2+1)\,dx}$$

(d) the volume when the region is rotated about the line $y = 2$

Disk $r=2-\sqrt{x^2+1}$

$$\boxed{\pi\int_0^{\sqrt{3}}\left(2-\sqrt{x^2+1}\right)^2 dx}$$

2) Find each numeric answer for #1 using your calculator

a) 1.074

b) $\frac{4\pi}{3} \approx 4.189$

c) 10.883

d) 2.608

3) The region in #1 is the base of a solid. Set up the integral to find the volume of the solid if the cross-sections perpendicular to the *x*-axis(sliced along the *y*-axis) are

(a) squares with a side on the *xy* plane

side of square $= 2 - \sqrt{x^2+1}$

Volume $= (\text{side})^2\,dx \Rightarrow \boxed{\int_0^{\sqrt{3}} \left(2 - \sqrt{x^2+1}\right)^2 dx}$

(b) rectangles in which the base is half the height (the base is on the *xy* plane)

$b = \frac{1}{2}h$

$2b = h \Rightarrow$ base $= 2 - \sqrt{x^2+1}$

height $= 2\left(2 - \sqrt{x^2+1}\right)$

Volume $= b\,h\,(dx) = 2b^2\,dx$

$\boxed{2\int_0^{\sqrt{3}} \left(2 - \sqrt{x^2+1}\right)^2 dx}$

(c) isosceles right triangles in which one side is on the *xy* plane

$V = \frac{1}{2}s^2\,dx$

side $= 2 - \sqrt{x^2+1}$

$\boxed{\frac{1}{2}\int_0^{\sqrt{3}} \left(2 - \sqrt{x^2+1}\right)^2 dx}$

(d) isosceles right triangles in which the hypotenuse is on the *xy* plane

hyp $= 2 - \sqrt{x^2+1}$

side$\cdot\sqrt{2}$ = hyp $\Rightarrow$ side $= \frac{\text{hyp}}{\sqrt{2}} = \frac{2 - \sqrt{x^2+1}}{\sqrt{2}}$

$\boxed{\frac{1}{2}\int_0^{\sqrt{3}} \left(\frac{2 - \sqrt{x^2+1}}{\sqrt{2}}\right)^2 dx}$

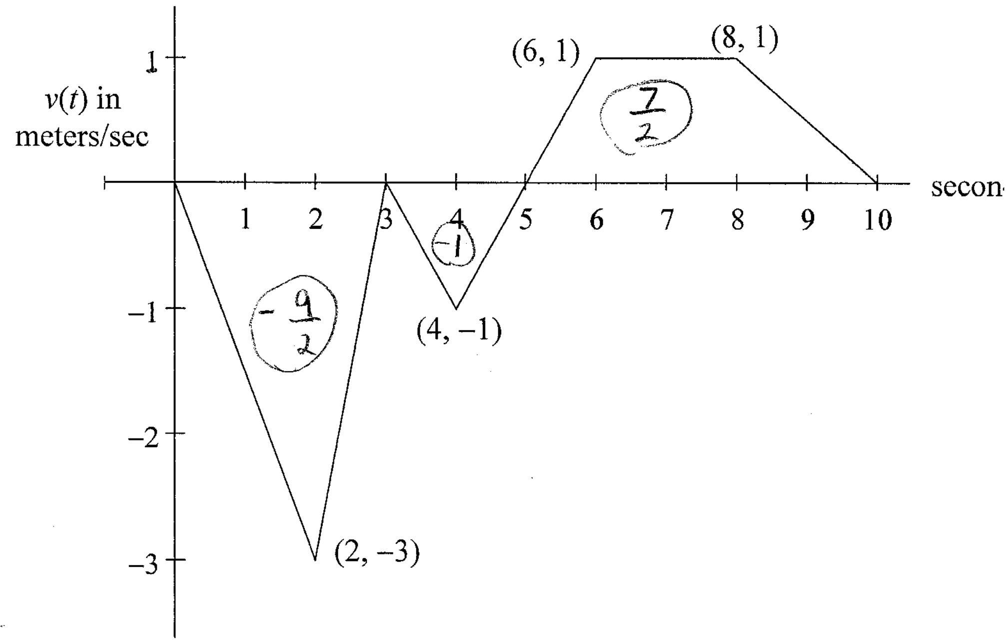
(e) Circles with the diameter on the *xy* plane (note that in this case the *xy* plane is not the base anymore)

diameter $= 2 - \sqrt{x^2+1}$

radius $= \frac{1}{2}\left(2 - \sqrt{x^2+1}\right)$

$V = \pi r^2\,dx = \boxed{\pi\int_0^{\sqrt{3}} \left(\frac{2 - \sqrt{x^2+1}}{2}\right)^2 dx}$

4) Abby is spazzing out while taking her drivers test because Nica had her believing that it wasn't for three more days. Over the course of 10 seconds, she struggles just to get the car in the right gear as it goes both backward and forward. The car's velocity is shown below.



a) In what gear was she initially? Justify your answer.

reverse because the velocity is negative initially

b) What was the total distance travelled by the car over the 10 second period?

$$\int_0^{10} |v(t)|\,dt = \frac{9}{2} + 1 + \frac{7}{2}$$

$= 9$ meters

c) How far and in what direction was the car's final position from its initial one?

Find displacement

$$\int_0^{10} v(t)\,dt = -\frac{9}{2} - 1 + \frac{7}{2} = -2$$

2 meters behind where she started