Name Solutions

The Chain Rule

Find $f'(x)$

1) $(4x^2 + 1)^7$

$7(4x^2+1)^6 8x$

$56x(4x^2+1)^6$

2) $\cos\sqrt{3x^2+5x-2}$

$-\left(\sin\sqrt{3x^2+5x-2}\right)\left(\frac{1}{2\sqrt{3x^2+5x-2}}\right)(6x+5)$

$$\frac{-(6x+5)\sin\sqrt{3x^2+5x-2}}{2\sqrt{3x^2+5x-2}}$$

3) $\frac{1}{x^2+x^4} = (x^2+x^4)^{-1}$

$-(x^2+x^4)^{-2}(2x+4x^3)$

$$-\frac{2x+4x^3}{(x^2+x^4)^2}$$

4) $\frac{\sin x}{(1-x)^3}$

$$\frac{(\cos x)(1-x)^3-(\sin x)3(1-x)^2(-1)}{(1-x)^6}$$

$$\frac{(1-x)^3\cos x+3(1-x)^2\sin x}{(1-x)^6}$$

$$\frac{(1-x)\cos x+3\sin x}{(1-x)^4}$$

5) $\frac{x}{(\sqrt{x}-1)^3}$

$$\frac{(\sqrt{x}-1)^3-(x)3(\sqrt{x}-1)^2\frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^6}$$

$$\frac{(\sqrt{x}-1)-\frac{3x}{2\sqrt{x}}}{(\sqrt{x}-1)^4} = \frac{\sqrt{x}-1-\frac{3}{2}\sqrt{x}}{(\sqrt{x}-1)^4} = \frac{-1-\frac{1}{2}\sqrt{x}}{(\sqrt{x}-1)^4} = -\frac{2+\sqrt{x}}{2(\sqrt{x}-1)^4}$$

Use the Chain Rule to find the derivatives for #6 and 7

6) $\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$-(\cos x)^{-2}(-\sin x)$

$\frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x}$

$= \boxed{\sec x \tan x}$

7) $\csc x = \frac{1}{\sin x} = (\sin x)^{-1}$

$-(\sin x)^{-2}\cos x = -\frac{\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$\boxed{= -\csc x \cot x}$

8) If you are driving 65 miles per hour $\left(\frac{dM}{dt} = 65\, miles/hr\right)$ in a car that is burning a gallon of gas every 20 miles $\left(\frac{dG}{dM} = \frac{1}{20}\, gallons/mile\right)$, how many gallons is your car burning every hour?

$\frac{dG}{dt} = \frac{dG}{dM} \cdot \frac{dM}{dt}$

$= \frac{1}{20} \cdot \frac{65}{1} = \frac{65}{20} = \frac{13}{4} = \boxed{3.25\ gal/hr}$

9) Suppose the length, L cm, of a steel bar depends on the air temperature, H° Celcius, which itself depends on time t, measured in hours. If the length increases by 2 cm for every degree increase in temperature and the temperature is increasing at 3° per hour, how fast is the length of the bar increasing?

$\frac{dL}{dH} = 2\,cm/{}^\circ C$ $\qquad$ $\frac{dH}{dt} = 3^\circ/hr$ $\qquad$ $\boxed{\frac{dL}{dt}}$

$\frac{dL}{dt} = \frac{dL}{dH} \cdot \frac{dH}{dt}$

$= 2 \cdot 3 = \boxed{6\ cm/hr}$