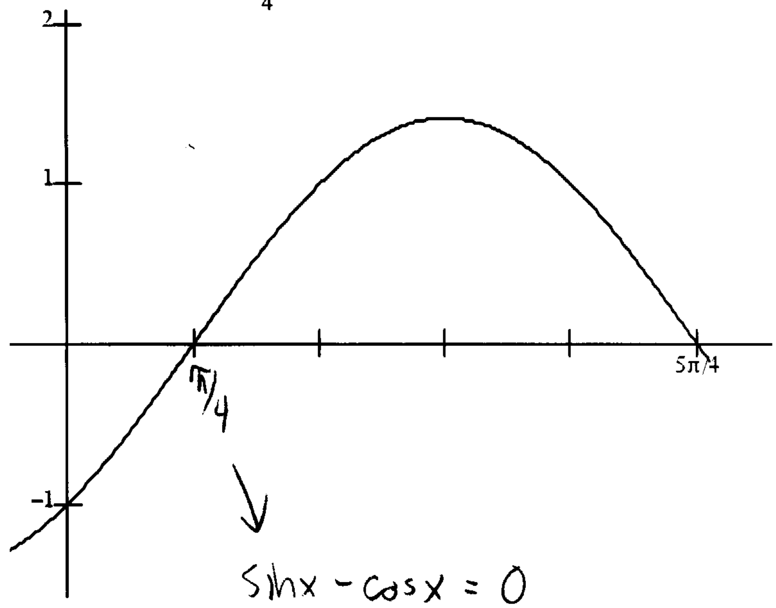
Name Solutions

Chapter 5 Review 2

Below is the graph of $f(x) = \sin x - \cos x$ on the interval $[0, \frac{5\pi}{4}]$



1) Evaluate $\int_0^{5\pi/4} f(x)\, dx$

$$\int_0^{5\pi/4} \sin x - \cos x \, dx$$

$$-\cos x - \sin x \Big]_0^{5\pi/4}$$

$$-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4} - \left[-\cos 0 - \sin 0\right]$$

$$-\left(-\frac{\sqrt{2}}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right) - (-1)$$

$$\sqrt{2} + 1$$

$$\sin x - \cos x = 0$$

$$\sin x = \cos x$$

$$\tan x = 1 \Rightarrow x = \frac{\pi}{4}$$

2) Find the *total area between the curve* of *f(x) and the x-axis* over the interval $[0, \frac{5\pi}{4}]$

$$\int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx - \int_0^{\pi/4} \sin x - \cos x \, dx$$

$$\left[-\cos x - \sin x \right]_{\pi/4}^{5\pi/4} - \left[-\cos x - \sin x\right]_0^{\pi/4}$$

$$\underbrace{-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}}_{} - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right] - \left[-\cos\frac{\pi}{4} - \sin\frac{\pi}{4} - \left(-\cos 0 - \sin 0\right)\right]$$

$$\sqrt{2} - (-\sqrt{2}) - \left[-\sqrt{2} + 1\right]$$

$$\sqrt{2} + \sqrt{2} + \sqrt{2} - 1$$

$$\boxed{3\sqrt{2} - 1}$$

3) Water is being pumped into a tank at a rate of 10 gallons per minute while, because Mussman and Garrett were talking too much and not paying attention to a leak that is getting bigger by the minute, some of it is also flowing out. When the leak began, the tank already contained 50 gallons of water. Because of their negligence, water leaks out of the tank at $5\sqrt{t}$ gallons per minute where t is measured in minutes measured from the time that the leak started.

a) If $R(t)$ is a function of t that measures the rate at which the amount of water in the tank *changes* in gallons per minute, write an expression for $R(t)$.

$$R(t) = \underbrace{10}_{\text{going in}} - \underbrace{5\sqrt{t}}_{\text{leaking out}}$$

b) Using the graph of $R(t)$, apply the trapezoid rule with 4 subintervals to approximate the water's highest level in gallons to two decimal places. Show the work that leads to your answer.

$R(t) = 0$ at $t = 4$ minutes $\Rightarrow$ Water level max $= 50 + \int_0^4 R(t)\,dt$

Trapezoid Approx: $\frac{1}{2} \cdot 1\left(R(0) + 2R(1) + 2R(2) + 2R(3) + R(4)\right)$

$\frac{1}{2} \cdot 1\left(10 + 10 + 2(10 - 5\sqrt{2}) + 2(10 - 5\sqrt{3}) + 0\right)$

$\approx 14.27 \Rightarrow$ max level $\approx 50 + 14.27 = \boxed{64.27 \text{ gallons}}$

c) Find the exact value of the water's highest level.

$$50 + \int_0^4 10 - 5\sqrt{t}\,dt = 50 + \left[10t - 5\tfrac{2}{3}t^{3/2}\right]_0^4$$

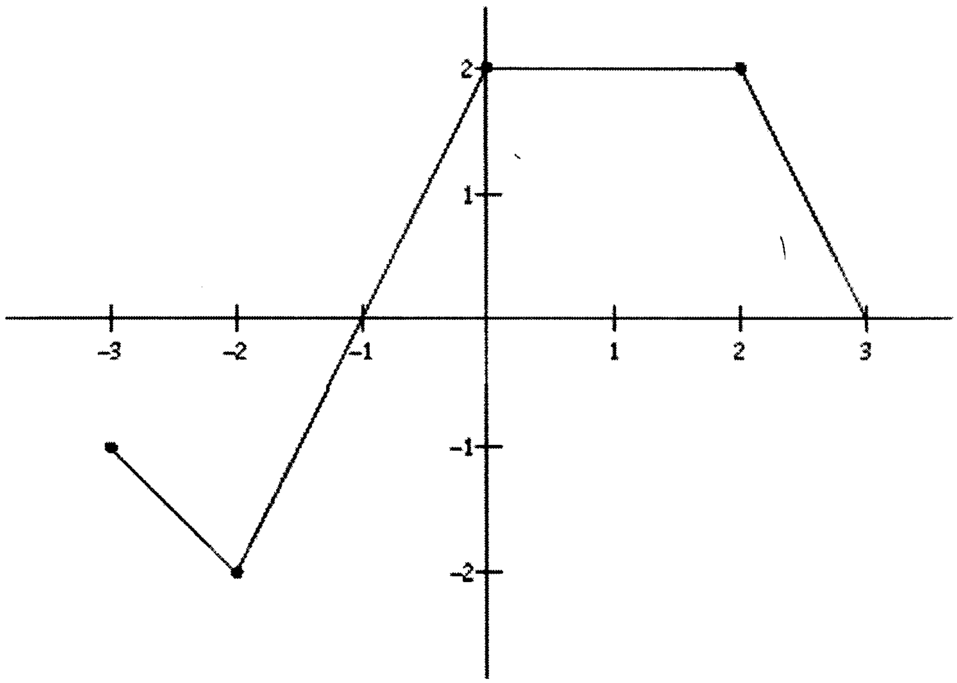
$$= 50 + \left[40 - \tfrac{10}{3}(8)\right]$$

$$= 50 + 40 - \tfrac{80}{3} = \boxed{63\tfrac{1}{3} \text{ gallons}}$$

d) Use your graphing calculator to determine how much water is left in the tank after 15 minutes.

$$50 + \int_0^{15} 10 - 5\sqrt{t}\,dt \approx 6.35 \text{ gallons}$$

4) The function $f(x)$ is differentiable on the interval $[-3, 3]$ and contains the point $(2, 1)$. The derivative f' is graphed below.



(a) Over what intervals of x is the graph of f concave up? Justify your answer.

$-2 < x < 0$

f' slope is $+$ therefore $f'' > 0$

(b) Find $f(-2)$ and $f(3)$. Show the work that leads to your answer.

$$f(-2) = f(2) + \int_2^{-2} f'(x)\,dx = f(2) - \int_{-2}^{2} f'(x)\,dx = 1 - 4 = -3$$

$$f(3) = f(2) + \int_2^3 f'(x)\,dx = f(2) + 1 = 1 + 1 = 2$$

(c) Are there any other values of x for which $f(x) = f(-2)$? Why or why not?

$$f(x) = f(2) + \int_2^x f'(t)\,dt$$

$$f(-2) = -3 = f(2) + \int_2^x f'(t)\,dt \rightarrow \int_2^x f'(t)\,dt = -4$$

$x = 0$