Chapter 6/7 Review Name Solutions

Integrate using either substitution or integration by parts.

1) $\int_1^e \frac{\ln x}{x}\,dx$

$u = \ln x$ $\quad x=1 \quad u=0$

$du = \frac{dx}{x}$ $\quad x=e \quad u=1$

$\int_0^1 u\,du = \frac{u^2}{2}\Big]_0^1 = \frac{1}{2}$

2) $\int 2x \ln x\,dx$

$u = \ln x \qquad dv = 2x$

$du = \frac{dx}{x} \qquad v = x^2$

$x^2 \ln x - \int x^2 \frac{dx}{x} = x^2 \ln x - \int x\,dx$

$\boxed{x^2 \ln x - \frac{x^2}{2} + C}$

3) $\int \frac{\sqrt{2+\frac{1}{x^2}}}{x^3}\,dx$

$u = 2 + \frac{1}{x^2}$

$du = -\frac{2}{x^3}\,dx$

$-\frac{1}{2}\,du = \frac{dx}{x^3}$

$-\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2} + C = \boxed{-\frac{1}{3}\left(2+\frac{1}{x^2}\right)^{3/2} + C}$

4) $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} \qquad du = \frac{dx}{2\sqrt{x}}$

$x=1 \quad u=1 \qquad 2\,du = \frac{dx}{\sqrt{x}}$

$x=4 \quad u=2$

$2\int_1^2 e^u\,du = e^u\Big]_1^2 = 2(e^2 - e)$

5) $\int x^2 \cos x\,dx$

u	dv
x^2	$\cos x$
$2x$	$\sin x$
2	$-\cos x$
0	$-\sin x$

$x^2 \sin x + 2x\cos x - 2\sin x + C$

6) $\int_0^1 \frac{\tan^{-1} x}{1+x^2}\,dx$

$u = \tan^{-1} x \qquad x=0 \quad u=0$

$du = \frac{dx}{1+x^2} \qquad x=1 \quad u=\frac{\pi}{4}$

$\int_0^{\pi/4} u\,du = \frac{u^2}{2}\Big]_0^{\pi/4} = \frac{\frac{\pi^2}{16}}{2} = \frac{\pi^2}{32}$

7) $\int \frac{dx}{\sqrt{x}(2+\sqrt{x})}$

$u = 2 + \sqrt{x}$

$du = \frac{dx}{2\sqrt{x}}$

$2\,du = \frac{dx}{\sqrt{x}}$

$2\int \frac{du}{u} = 2\ln|u| + C$

$= \boxed{2\ln|2+\sqrt{x}| + C}$

8) $\int_0^1 \frac{x}{x+1}\,dx$

$u = x+1 \Rightarrow x = u-1$

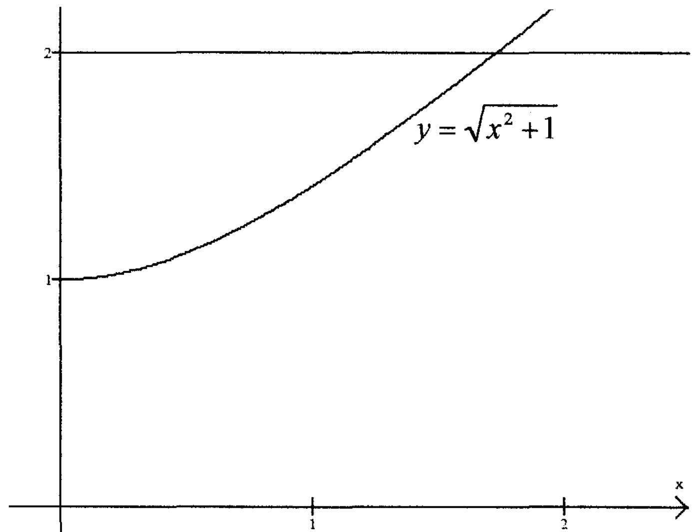
$du = dx \qquad x=0 \quad u=1$

$x=1 \quad u=2$

$\int_1^2 \frac{u-1}{u}\,du = \int_1^2 1 - \frac{1}{u}\,du \longrightarrow$

$\Rightarrow u - \ln u\Big]_1^2 = 2 - \ln 2 - (1 - \ln 1)$

$= 1 - \ln 2$



Read each question carefully.

1) Identify the region bounded by the curve $y = \sqrt{x^2+1}$, the line $y = 2$, and the y-axis. Indicating the method that you use each time, set up the integral to find

(a) the area of the region

$$\int_0^{\sqrt{3}} 2 - \sqrt{x^2+1} \quad dx$$

How is the upper limit $\sqrt{3}$?

Set curves equal to each other

$y = 2 = \sqrt{x^2+1} \Rightarrow 4 = x^2+1 \Rightarrow \pm\sqrt{3} = x$

(b) the volume when the region is rotated about the y-axis

Shell dx

$$2\pi\int_0^{\sqrt{3}} x\left(2-\sqrt{x^2+1}\right) dx$$

$r = x$

$h = 2 - \sqrt{x^2+1}$

OR $\rightarrow$ Disk dy

$$\pi\int_1^2 \left(\sqrt{y^2-1}\right)^2 dy$$

r = curve (in terms of y)

$y = \sqrt{x^2+1} \Rightarrow y^2 = x^2 +$

$y^2 - 1 = x^2 \Rightarrow \boxed{x = \sqrt{y^2-}}$

(c) the volume when the region is rotated about x-axis

Shell dy

$$2\pi\int_1^2 y\left(\sqrt{y^2-1}\right) dy$$

$r = y$

$h = \sqrt{y^2-1}$

OR $\rightarrow$ Washer dx

$$\pi\int_0^{\sqrt{3}} 2^2 - \left(\sqrt{x^2+1}\right)^2 dx$$

$R = 2$

$r = \sqrt{x^2+1}$

(d) the volume when the region is rotated about the line $y = 2$

Shell dy

$$2\pi\int_1^2 (2-y)\sqrt{y^2-1} \quad dy$$

$r = 2 - y$

$h = \sqrt{y^2-1}$

OR $\rightarrow$ Disk dx

$$\pi\int_0^{\sqrt{3}} \left(2 - \sqrt{x^2+1}\right)^2 dx$$

$r = 2 - \sqrt{x^2+1}$

2) Find each numeric answer for #1 using your calculator

(a) Area = 1.074

(d) Volume = 2.608

(b) Volume = 4.189

(c) Volume = 10.883

3) The region in #1 is the base of a solid. Set up the integral to find the volume of the solid if the cross-sections perpendicular to the *x*-axis(sliced along the *y*-axis) are

(a) squares with a side on the *xy* plane

side of square $= 2-\sqrt{x^2+1}$ Area of square $= s^2 = \left(2-\sqrt{x^2+1}\right)^2$

$$\text{Volume} = \int_0^{\sqrt{3}} \left(2-\sqrt{x^2+1}\right)^2 dx$$

(b) rectangles in which the base is half the height

base $= 2-\sqrt{x^2+1}$ height $= 2\left(2-\sqrt{x^2+1}\right)$ Area of rectangle $= 2\left(2-\sqrt{x^2+1}\right)^2$

$$\text{Volume} = \int_0^{\sqrt{3}} 2\left(2-\sqrt{x^2+1}\right)^2 dx = 2\int_0^{\sqrt{3}} \left(2-\sqrt{x^2+1}\right)^2 dx$$

(c) isosceles right triangles in which one side is on the *xy* plane

side $= 2-\sqrt{x^2+1}$ Area $= \frac{1}{2}s^2 = \frac{1}{2}\left(2-\sqrt{x^2+1}\right)^2$

$$\text{Volume} = \frac{1}{2}\int_0^{\sqrt{3}} \left(2-\sqrt{x^2+1}\right)^2 dx$$

(d) isosceles right triangles in which the hypotenuse is on the *xy* plane

hypotenuse $= 2-\sqrt{x^2+1}$ Area $= \frac{1}{2}s^2 = \frac{1}{2}\left(\frac{2-\sqrt{x^2+1}}{\sqrt{2}}\right)^2$

each side $= \dfrac{2-\sqrt{x^2+1}}{\sqrt{2}}$

$$\text{Volume} = \int_0^{\sqrt{3}} \frac{1}{2}\left(\frac{2-\sqrt{x^2+1}}{\sqrt{2}}\right)^2 dx = \frac{1}{4}\int_0^{\sqrt{3}} \left(2-\sqrt{x^2+1}\right)^2 dx$$

(e) Circles with the diameter on the *xy* plane

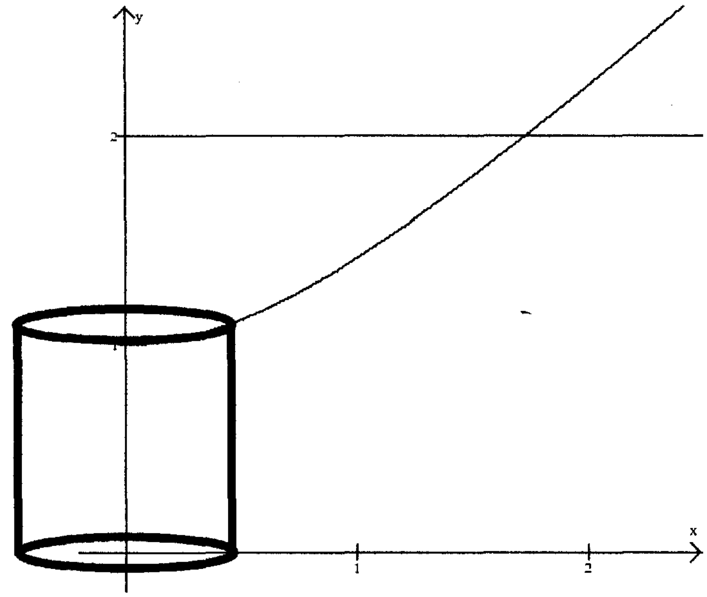
diameter $= 2-\sqrt{x^2+1}$ Area $= \pi r^2 = \pi\left[\frac{1}{2}\left(2-\sqrt{x^2+1}\right)\right]^2 = \pi\frac{1}{4}\left(2-\sqrt{x^2+1}\right)^2$

radius $= \frac{1}{2}\left(2-\sqrt{x^2+1}\right)$

$$\text{Volume} = \frac{\pi}{4}\int_0^{\sqrt{3}} \left(2-\sqrt{x^2+1}\right)^2 dx$$

4) Identify the region in the first quadrant bounded by the curve $y = \sqrt{x^2+1}$ and the line $x = 1$.

(a) Using the shell method, find the volume obtained when the region is rotated about the y axis. Do not use your calculator to find this answer.



Shell = $2\pi r h \, dx$

$r = x$, $h = \sqrt{x^2+1}$

$$2\pi \int_0^1 x\sqrt{x^2+1}\, dx$$

$u = x^2+1$ $x=0$ $u=1$

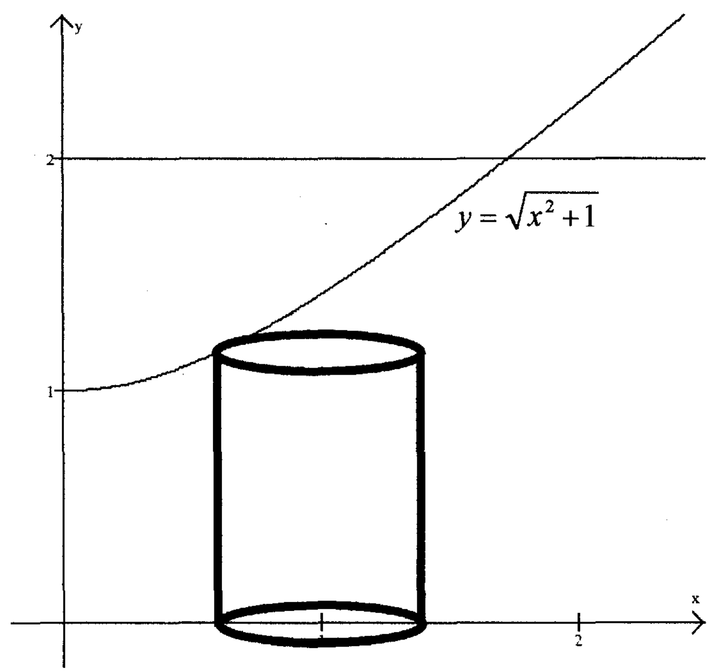
$du = 2x\,dx$ $x=1$ $u=2$

$\frac{1}{2}du = x\,dx$

$$2\pi \int_1^2 u^{1/2}\left(\tfrac{1}{2}du\right) = \pi \int_1^2 u^{1/2}\,du = \pi\left[\tfrac{2}{3}u^{3/2}\right]_1^2 \rightarrow$$

$$\rightarrow = \frac{2\pi}{3}\left[2^{3/2} - 1\right] = \boxed{\frac{2\pi}{3}\left(2\sqrt{2}-1\right)}$$

(b) How will the shell method differ when rotating the region about the line $x = 1$? Set up this integral and use your calculator to find the volume.



Shell = $2\pi r h \, dx$

$r = 1-x$, $h = \sqrt{x^2+1}$

radius was x in part a

In part b radius is $1-x$

$$2\pi \int_0^1 (1-x)\sqrt{x^2+1}\, dx$$

$= 3.382$