Solutions

Euler's Method of Approximation

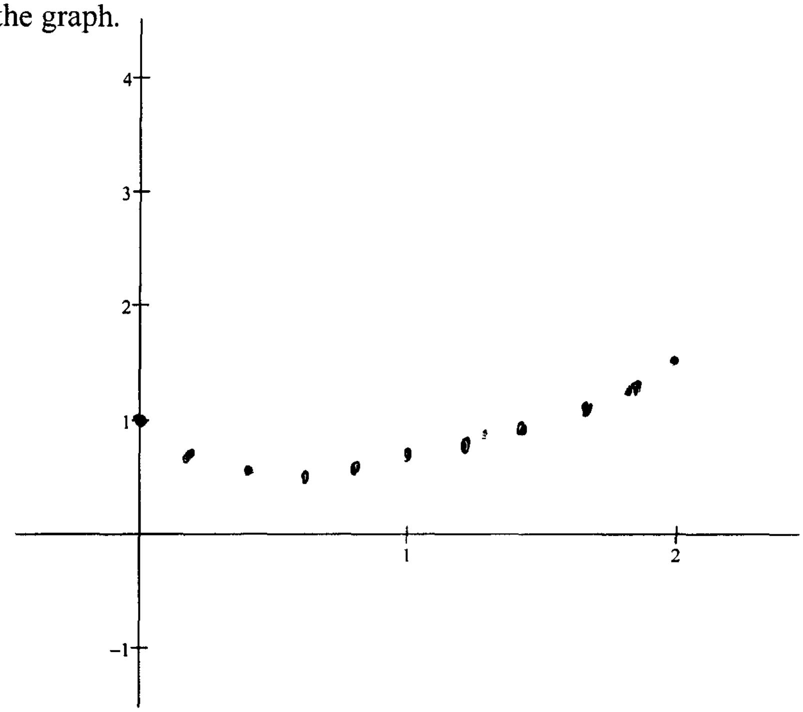
A simplified version of Euler's method works like this:

Starting with the difference quotient approximation:

$f'(x) \approx \dfrac{f(x+h)-f(x)}{h}$ where h is very close to 0. Multiplying both sides by h and adding $f(x)$ to both sides yields $f(x+h) \approx f(x) + f'(x)h$

1) Using this formula approximate the graph of y for the given differential equation. $y' = x - y$, $y(0) = 1$ from x = 0 to x = 2 using h = .2. Record your results in the table and plot the approximation on the graph.

x	y
0	1
.2	0.8
.4	0.68
.6	0.624
.8	0.6192
1	0.655
1.2	0.725
1.4	0.819
1.6	0.936
1.8	1.068
2	1.215



$$f(0.2) \approx f(0) + f'(0)(0.2)$$

$$\approx 1 + (-1)(0.2) = 0.8$$

$$f(0.4) \approx 0.8 + (-0.6)(0.2) = 0.68$$

$$f(0.6) \approx 0.68 + (0.4 - 0.68)(0.2) = 0.624$$

Separable Equations

An example of a separable differential equation would be $y' = y$. This can be solved by rewriting the equation as

$\frac{dy}{dx} = y$ and separating the x's and the y's putting each on one side of the equation.

By doing so, we get

$\frac{dy}{y} = dx$. Integrating both sides gives us

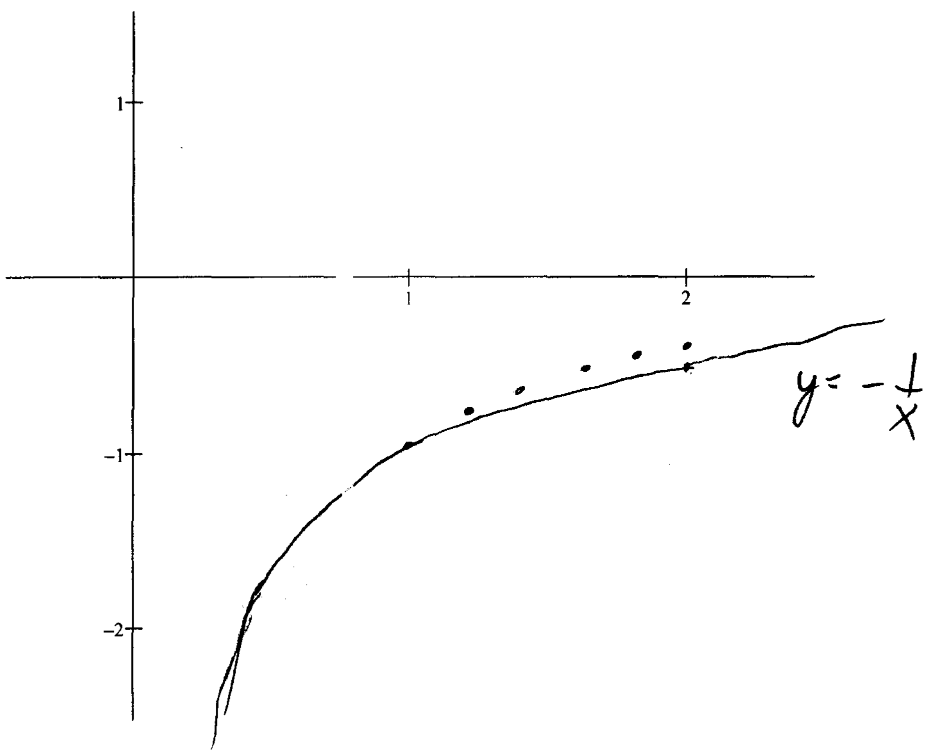
$\ln y = x + C$ and solving for y gives us

$y = e^{x+c}$ or just $y = Ce^x$

Approximate the graphs of y for the given differential equations. Show your calculations. Then use separation of variables to solve for y.

2) $y' = y^2$ $\quad y(1) = -1$ $\quad h = .2$ $\quad$ over the interval [1, 2]

x	y
1	-1
1.2	-0.8
1.4	-0.672
1.6	-0.582
1.8	-0.514
2	-0.461



$\frac{dy}{dx} = y^2$

$\int \frac{dy}{y^2} = \int dx$

$-\frac{1}{y} = x + C$

$y = -\frac{1}{x+C}$

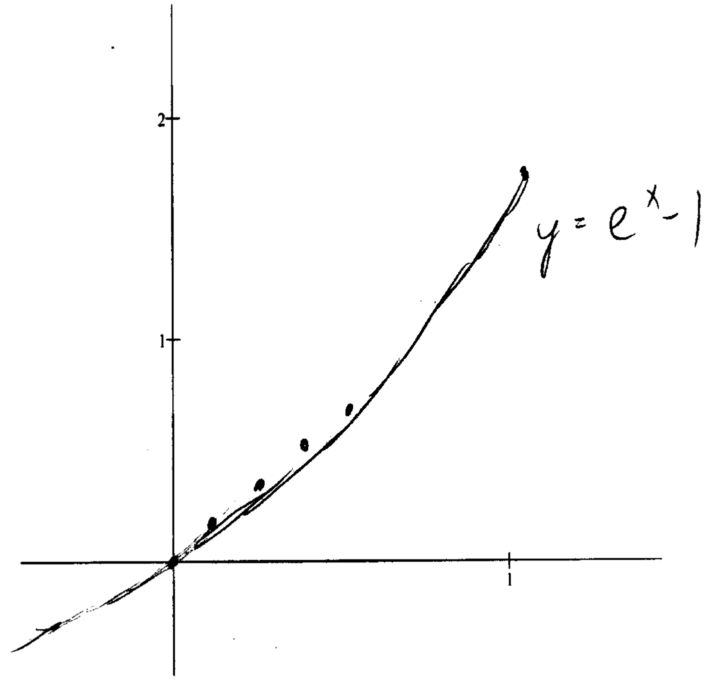
$-1 = -\frac{1}{1+C}$

$C = 0$

$\boxed{y = -\frac{1}{x}}$

3) $y' = 1 + y \qquad y(0) = 0 \qquad h = .1 \qquad$ over the interval [0, 0.5]

x	y
0	0
0.1	0.1
0.2	0.21
0.3	0.331
0.4	0.464
0.5	0.611



$$\frac{dy}{dx} = 1 + y$$

$$\int \frac{dy}{1+y} = \int dx$$

$$\ln|1+y| = x + C$$

$$1 + y = \pm e^{x+C}$$

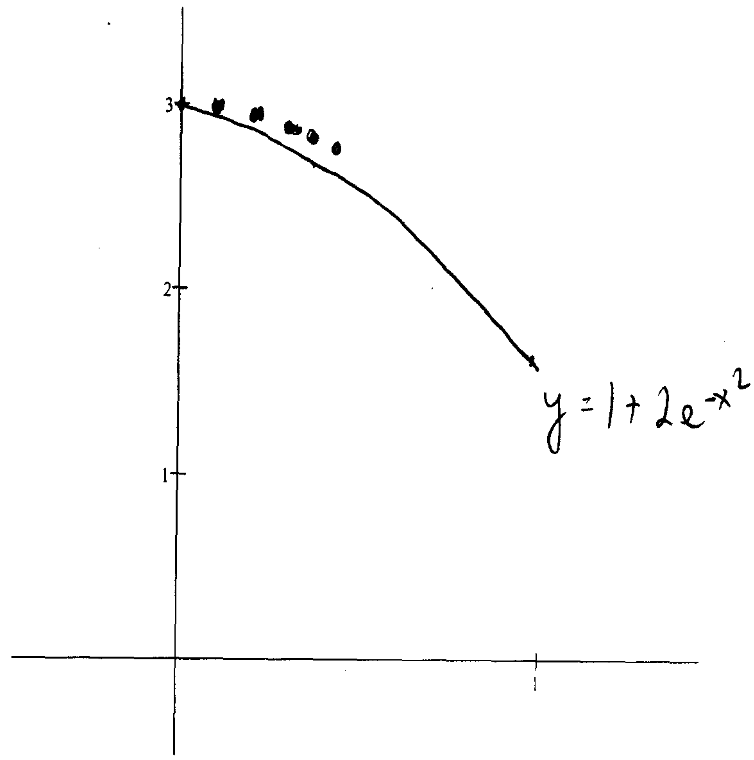
$$y = Ae^x - 1$$

$$0 = Ae^0 - 1$$

$$y = e^x - 1$$

4) $y' = 2x - 2xy$ $y(0) = 3$ $h = .1$ over the interval [0, 0.5]

x	y
0	3
0.1	3
0.2	2.96
0.3	2.881
0.4	2.769
0.5	2.627



$$\frac{dy}{dx} = 2x - 2xy$$

$$\int \frac{dy}{1-y} = \int 2x\,dx$$

$$-\ln|1-y| = x^2 + C$$

$$\ln|1-y| = C - x^2$$

$$1 - y = Ae^{-x^2}$$

$$y = 1 - Ae^{-x^2}$$

$$3 = 1 - Ae^0$$

$$-2 = A$$

$$\boxed{y = 1 + 2e^{-x^2}}$$