Determine if the given series converges or diverges. Refer to page 505 for this worksheet

Name Solutions

Important Limits: $\lim_{n\to\infty} \sqrt[n]{n} = 1$ $\quad \lim_{n\to\infty} \frac{\ln n}{n} = 0$ $\quad \lim_{n\to\infty} \frac{x^n}{n!} = 0$

$\lim_{n\to\infty} (1+\frac{x}{n})^n = e^x$ $\quad \lim_{n\to\infty} x^{1/n} = 1 \; x > 0$ $\quad \lim_{n\to\infty} x^n = 0 \; |x| < 1$

1) $\sum_{n=1}^{\infty} \frac{1}{\ln n}$

$\ln n < n$

for all $n > 2$

$\frac{1}{\ln n} > \frac{1}{n}$ $\therefore$

$\sum \frac{1}{\ln n}$ Converges by Direct Comparison

2) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$

Limit Comparison Test

$\lim_{n\to\infty} \frac{\frac{1}{1+\ln n}}{\frac{1}{\ln n}} = \frac{\ln n}{\ln n + 1}$

$= 1$ $\therefore$

Series Diverges

3) $\sum_{n=1}^{\infty} \frac{10n+1}{n(n+1)(n+2)}$ L.C. Test with $\sum \frac{1}{n^2}$

$\lim_{n\to\infty} \frac{\frac{10n+1}{n(n+1)(n+2)}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{10n^3+1}{n(n+1)(n+2)}$

$= 10$

$\therefore$ since $\sum \frac{1}{n^2}$ converges so does this series

4) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

Compare with convergent p-series $\sum \frac{1}{n^{3/2}}$

L.C. Test

$\lim_{n\to\infty} \frac{\frac{\sqrt{n}}{n^2+1}}{\frac{1}{n^{3/2}}} = \frac{n^2}{n^2+1} = 1$

$\therefore$ Series Converges

5) $\sum_{n=1}^{\infty} \frac{1}{n(\sqrt[n]{n})}$

Compare with Harmonic Series

$\lim_{n\to\infty} \frac{\frac{1}{n\sqrt[n]{n}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n\underbrace{\sqrt[n]{n}}_{1}} = 1$

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$\therefore$ Series diverges

6) $\sum_{n=1}^{\infty} \frac{(-2)^n}{3^n} = \sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n$

$r = -\frac{2}{3}$

$|r| < 1$ $\therefore$ geometric series Converges

or

try the Ratio or Root Tests

7) $\sum_{n=1}^{\infty} \frac{n^3(-2)^n}{3^n}$

Ratio Test: $\lim_{n\to\infty} \left| \frac{(n+1)^3(-2)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n^3(-2)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)^3}{n^3} \frac{(-2)}{(3)} \right| = \frac{2}{3} < 1$

∴ Series Converges

8) $\sum_{n=1}^{\infty} \frac{2+(-1)^n}{(1.25)^n}$

$= \sum_{n=1}^{\infty} \frac{2}{(1.25)^n} + \sum_{n=1}^{\infty} \left(-\frac{1}{1.25}\right)^n$

both are convergent geometric series so the series converges

ln n < n for all n > 2

9) $\sum_{n=1}^{\infty} \frac{\ln n}{n^3} < \sum_{n=1}^{\infty} \frac{n}{n^3} = \sum_{n=1}^{\infty} \frac{1}{n^2}$

by direct comparison the series converges

10) $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

Ratio test:

$\lim_{n\to\infty} \left| \frac{(n+1)\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n \ln n} \right|$

$= \lim_{n\to\infty} \left| \frac{(n+1)\ln(n+1)}{2n \ln n} \right|$ since $\frac{n+1}{n} \to 1$ and $\frac{\ln(n+1)}{\ln n} \to 1$

The limit is $\frac{1}{2} < 1$ ∴ Series Converges

11) $\sum_{n=1}^{\infty} e^{-n} n^3 = \sum_{n=1}^{\infty} \frac{n^3}{e^n}$

Ratio Test:

$\lim_{n\to\infty} \left| \frac{(n+1)^3}{e^{n+1}} \cdot \frac{e^n}{n^3} \right|$

$\lim_{n\to\infty} \left| \frac{(n+1)^3}{n^3 e} \right| = \frac{1}{e} < 1$

∴ Series Converges

12) $\sum_{n=1}^{\infty} \frac{1}{(2n)!}$ Ratio Test

$\lim_{n\to\infty} \left| \frac{1}{[2(n+1)]!} \cdot \frac{(2n)!}{1} \right|$

$\lim_{n\to\infty} \left| \frac{\cancel{(2n)!}}{(2n+2)(2n+1)\cancel{(2n)!}} \right|$

$\lim_{n\to\infty} \frac{1}{(2n+2)(2n+1)} = 0 < 1$

∴ Series converges

13) $\sum_{n=1}^{\infty} \frac{(2n)!}{2^n}$

Ratio test: $\lim_{n\to\infty} \left| \frac{(2n+2)!}{2^{n+1}} \cdot \frac{2^n}{(2n)!} \right|$

$\lim_{n\to\infty} \left| \frac{(2n+2)(2n+1)}{2} \right| = \infty > 1$

∴ series diverges

14) $\sum_{n=1}^{\infty} \frac{5^n}{n!}$ Ratio Test

$\lim_{n\to\infty} \left| \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} \right|$

$\lim_{n\to\infty} \left| \frac{5}{n+1} \right| = 0 < 1$

Series Converges

15) $\sum_{n=1}^{\infty} \left(\frac{n}{3n+1}\right)^n$ Root Test

$\lim_{n\to\infty} \left| \sqrt[n]{\left(\frac{n}{3n+1}\right)^n} \right|$

$\lim_{n\to\infty} \frac{n}{3n+1} = \frac{1}{3} < 1$

Series Converges