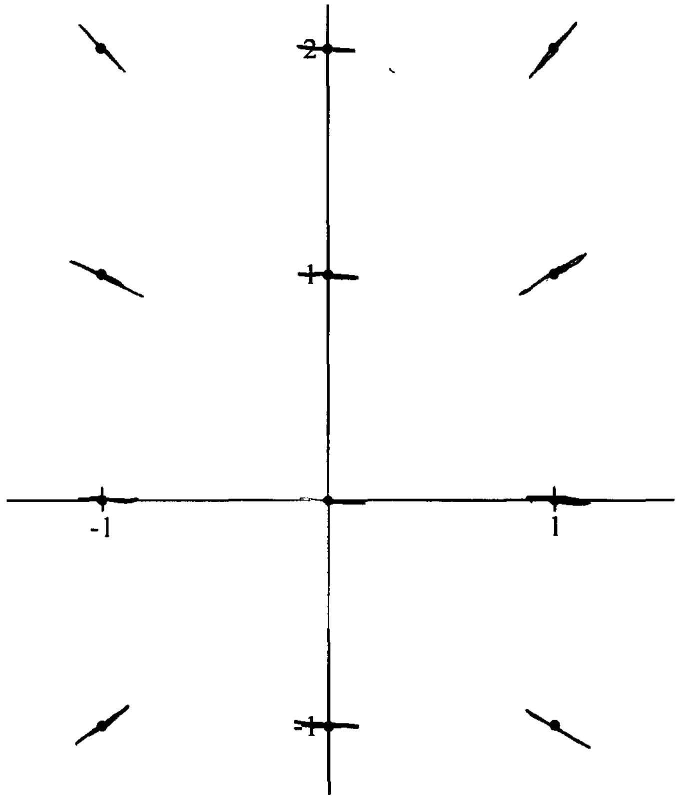
Slope Field Worksheet 3 Name Solutions

1) Consider the differential equation $\frac{dy}{dx} = \frac{xy}{2}$

a) On the axes below, sketch the slope field for the given differential equation at the twelve points indicated.



b) Describe all points in the xy-plane for which the slopes are positive.

$x>0,\ y>0$
or
$x<0\quad y<0$

Quadrants I and III

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

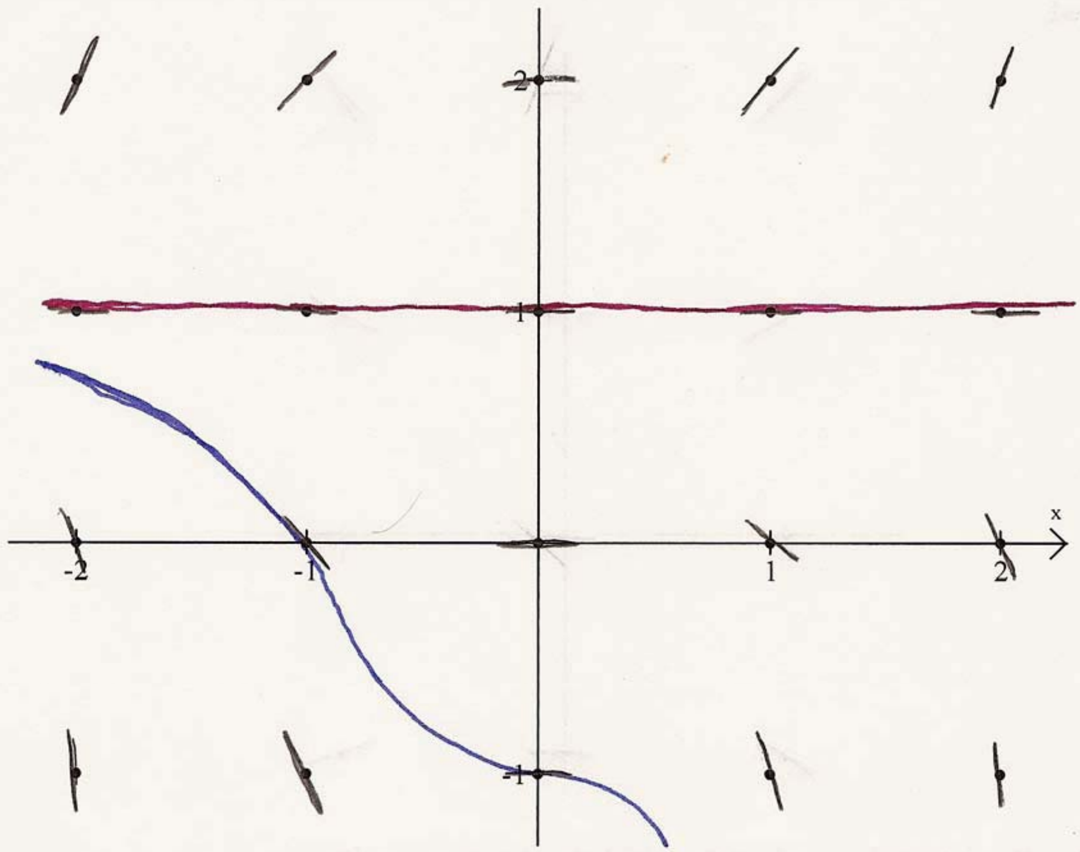
$$\frac{dy}{y} = \frac{1}{2}x\,dx \Rightarrow \int \frac{dy}{y} = \int \frac{1}{2}x\,dx \Rightarrow \ln|y| = \frac{x^2}{4} + C$$

$$\Rightarrow y = e^{\frac{x^2}{4}+C} \Rightarrow y = Ae^{\frac{x^2}{4}} \Rightarrow 3 = Ae^{\frac{0}{3}} \Rightarrow A = 3$$

$$y = 3e^{x^2/4}$$

2) Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$

a) On the axes below, sketch the slope field for the given differential equation at the twenty points indicated.



b) Sketch an approximate graph of y with an initial point of $(0, 1)$. Red

c) Sketch an approximate graph of y with an initial point of $(0, -1)$. Blue

Both sketches are approximate graphs

d) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

$$\frac{dy}{dx} = x^2(y-1) \Rightarrow \frac{dy}{y-1} = x^2\,dx \Rightarrow \int \frac{dy}{y-1} = \int x^2\,dx$$

$$\Rightarrow \ln|y-1| = \frac{x^3}{3} + C \Rightarrow y-1 = e^{\left(\frac{x^3}{3}+C\right)} \Rightarrow y-1 = Ae^{x^3/3}$$

$$y = Ae^{x^3/3} + 1 \Rightarrow 3 = Ae^0 + 1 \Rightarrow A = 2$$

$$y = 2e^{x^3/3} + 1$$