Work

$W = Fd$ ➔ Work = Force x distance ➔ $W = \sum F(x) \cdot \Delta x$ ➔ $\int_a^b F(x)\,dx$

Units for Work are either foot-lbs or Newton-meters(aka joules)

Hooke's Law: $f(x) = kx$ or the amount of force required to stretch a spring x meters is proportional to x with k as the spring constant.

Don't forget: $F = ma$ $m = D \cdot V$ (mass = Density x Volume)

1) Find the amount of work done in lifting a 1.2 kg book off the floor and onto a table that is 0.7 meters high.

$1.2\text{ kg} \cdot 9.8\text{ m/s}^2 = 11.76\text{ N}$

$W = F \cdot d = 11.76N(.7m) = 8.232$ Nm (joules)

2) How much work is done lifting a 20 lb weight 6 feet off the ground?

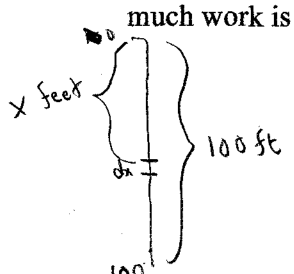
120 ft·lbs

3) When a particle is located a distance of x feet from the origin, a force of $3x^2$ pounds acts on it. Find the work done moving the particle 5 feet from the origin.

$F(x) = 3x^2$ distance = dx

$W = \int_0^5 3x^2\,dx = x^3\Big]_0^5 = 125$ ft·lbs

4) A 200 lb. cable measuring 100 feet in length is raised to the top of a building. How much work is done lifting the cable?



Weight = 2 lb/ft

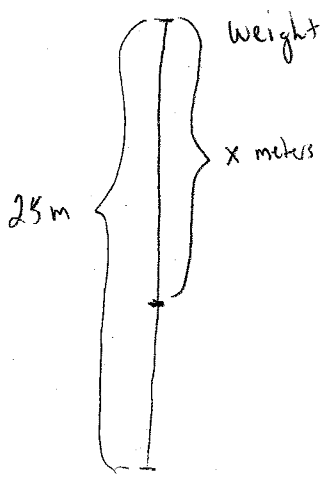
$F(x) = 2\cdot dx$

distance = X

$W = \int_0^{100} 2x\,dx$

$= x^2\Big]_0^{100} = 10{,}000$ ft·lbs

5) A heavy rope, 25 m long weighs 200 g/cm and hangs off the edge of a building. How much work is done pulling the rope to the top of the building?



weight = .2 kg/cm = 20 kg/m = 196 N/m

$F(x) = 196 \cdot dx$ distance = X

$W = \int_0^{25} 196x\,dx$

$= 98x^2\Big]_0^{25} = 61250$ Nm (joules)

6) A force of 20 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?

Hookes Law $\Rightarrow F(x) = Kx \Rightarrow 20\text{ N} = K(5\text{ cm}) = K(.05\text{ m})$

$400 = K = 400\text{ N/m}$

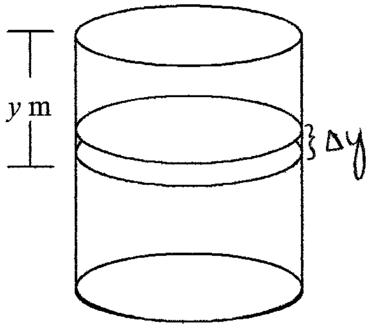
$F(x) = 400x \qquad W = \int_{.15}^{.18} 400x\,dx = 200x^2\Big]_{.15}^{.18} = \boxed{1.98 \text{ joules}}$

7) A force of 40 lbs. is required to hold a spring that has been stretched from its natural length of 6 inches to a length of 9 inches. How much work is done in stretching the spring from 9 to 12 inches?

$F(x) = Kx \Rightarrow 40 = K(\tfrac{1}{4}\text{ ft}) \Rightarrow k = 160 \text{ lbs/ft}$

$F(x) = 160x \qquad W = \int_{.75}^{1} 160x\,dx = 35 \text{ ft lbs}$

8) A cylindrical tank of height 15 meters and a radius of 5 m is filled with water. The water will be pumped through a pipe attached to the top of the tank. Using the fact that the density of water is 1000 kg/m^3, find the amount of work required to empty the tank.



Each individual slab has to go y meters

the volume is $\pi r^2 \Delta y = 25\pi\,\Delta y$

The mass is density $\times$ volume $= (1000\text{ kg/m}^3)(25\pi\,\Delta y)$

$= 25000\pi\,\Delta y$ = mass of one slab

weight $= 25000\pi\,\Delta y(9.8\text{ m/s}^2) = 245000\pi\,\Delta y$ N

Force required is $245000\pi\,\Delta y$ N

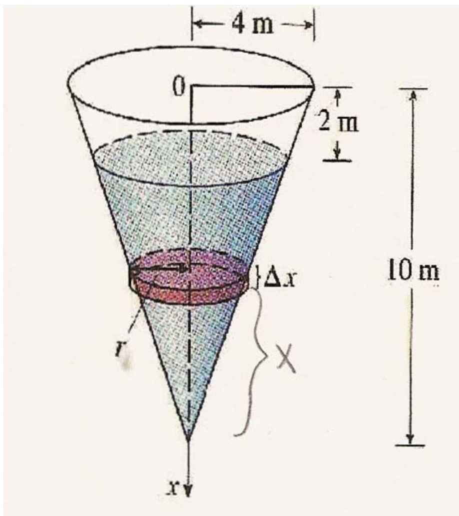
Work done moving one slab over a distance of y meters is

$W = F \cdot d = 245000\pi\, y\,\Delta y$

Work done moving all the water is $\sum 245000\pi\, y\,\Delta y$

$= \int_0^{15} 245000\pi\, y\,dy = 122500\pi y^2\Big]_0^{15} = 27{,}562{,}500\pi \text{ joules}$

9) An inverted conical tank has a height of 10 m and a radius of 4 m. It is filled with water to a height of 8 m. Find the work required to empty the tank by pumping all of the water to the top of the tank. (The density of water is 1000 kg/m^3)



m = density · volume

One slab has a volume of

$\pi r^2 \Delta x$ Assuming the cone's depth is X $\Rightarrow$ $\frac{r}{X} = \frac{4}{10}$

$r = \frac{2}{5}X$

$V = \pi\left(\frac{4}{25}X^2\right)\Delta X = \frac{4\pi}{25}X^2\,\Delta X$

$\text{Mass} = 1000\,\text{kg/m}^3 \cdot \frac{4\pi}{25}X^2\,\Delta X$

$= 160\pi X^2 \Delta X$

$\text{Force} = 160\pi X^2 \Delta X\,(9.8\,\text{m/s}^2)$

$= 1568\pi X^2 \Delta X$

The distance each slab has to be pushed is

$10 - X$

$W = \int_0^8 1568\pi X^2 (10 - X)\,dx$

$= 1568\pi\left(\frac{2048}{3}\right) = 1070421.333\pi = 3{,}362{,}827.797$ joules

10) A bucket weighing 5 kg is filled with 10 kg of water. It is attached to a rope that weighs 1 kg and is being lifted to a height of height of 20 meters. While the bucket is rising it is slowly leaking water so that it is empty at the moment it reaches the top. Find the work done

a) lifting the bucket alone.

$$W = 5 \cdot 9.8 \cdot 20 = \boxed{980 \text{ joules}}$$

b) lifting the rope alone

rope = .05 kg/m

$F = (.05)(9.8)x$ where x = # of meters of length

$$W = \int_0^{20} .49x \, dx = .245x^2 \Big]_0^{20} = 98 \text{ joules}$$

c) lifting the water.

$$\text{Weight} = 10 \text{ kg}\,(9.8)\underbrace{\left(\frac{20-x}{20}\right)}_{\text{proportion of water left}} N = 98\left(1 - \frac{x}{20}\right) \text{ Newtons}$$

$$W = 98\int_0^{20} 1 - \frac{x}{20} \, dx$$

$$= 98\left[x - \frac{x^2}{40}\right]_0^{20} = 98\left[20 - \frac{400}{40}\right]$$

$$= 98\left[20 - 10\right] = 980 \text{ joules}$$