

## Differential Equations Review

Name \_\_\_\_\_

- 1) Given the differential equation  $\frac{dy}{dx} = x - 2xy$  and  $y(0) = 1$
- a) Approximate  $y(1)$  using Euler's method with  $h = .25$

Answers:  $y(.25) = 1$ ,  $y(.5) = .9375$ ,  $y(.75) = .82813$ ,  $y(1) = .70508$

b) Solve for  $y$  using integrating factors

c) Solve for  $y$  using separation of variables

Answers(b & c):  $y = \frac{1}{2}(1 + e^{-x^2})$

- 2) Bacteria grows in an environment at a rate given by the differential equation  $\frac{dP}{dt} = 0.03P(0.2 - P)$  where  $P$  is the population of bacteria in the given environment.
- a) Solve for  $P$  in terms of  $t$  given that  $P(0) = 0.009$

Answer:  $P(t) = \frac{0.2}{1 + \frac{191}{9}e^{-0.006t}}$

b) What is the carrying capacity of the environment?

Answer:  $M = 0.2$

c) At what time will the population growth rate be a maximum?

Answer: 509.17 hours

3) Mr. Murphy is dangling 30 feet from the top of the veranda on a rope that weighs 20 lbs. Since Philip and Dominick have long since checked out (they were the ones who put Mr. Murphy there in the first place), Brian and Bennett are left to haul Mr. Murphy up by themselves. If Mr. Murphy weighs 180 lbs., how much work is done by Brian and Bennett in lifting Mr. Murphy?

Answer: 5700 ft-lbs.

4) After Mr. Murphy has been rescued, Alex and Michael attach Mr. Murphy to a metal spring and hang him over the side of the veranda again to test the elasticity of the spring for their physics class. Using Hooke's law, if it takes Mr. Murphy's 81.65 kg to stretch the spring from its natural length of 1 meter to 2 meters

a) Find the spring constant  $k$ .

Answer:  $k = 800.15$

b) How much work will it take to stretch the spring out another half meter?

Answer: 500.1 joules

1.  $\frac{dy}{dx} = 3xy$

2.  $\frac{dy}{dx} = y - x^2$

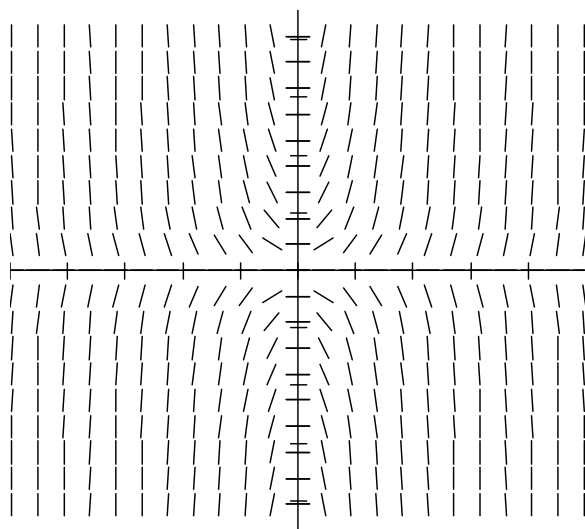
3.  $\frac{dy}{dx} = -\frac{y}{x}$

4.  $\frac{dy}{dx} = \frac{x}{y}$

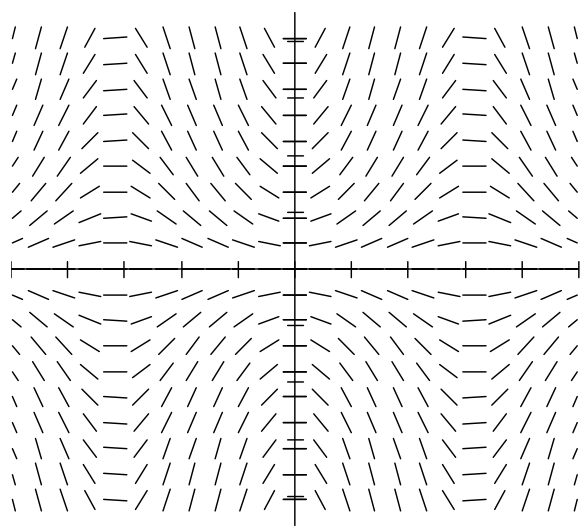
5.  $\frac{dy}{dx} = y \sin x$

6.  $\frac{dy}{dx} = \frac{y}{x}$

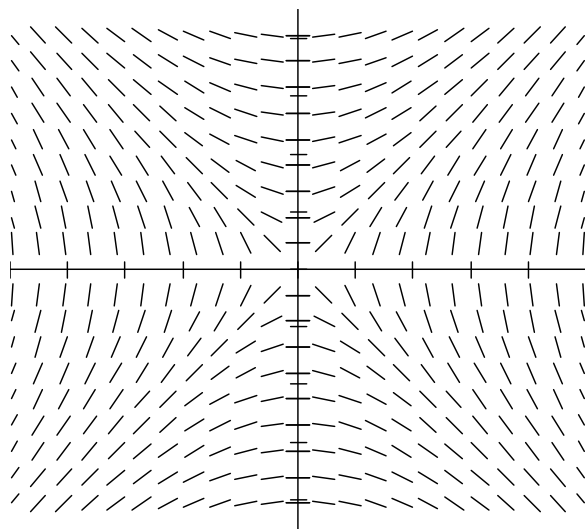
**A**



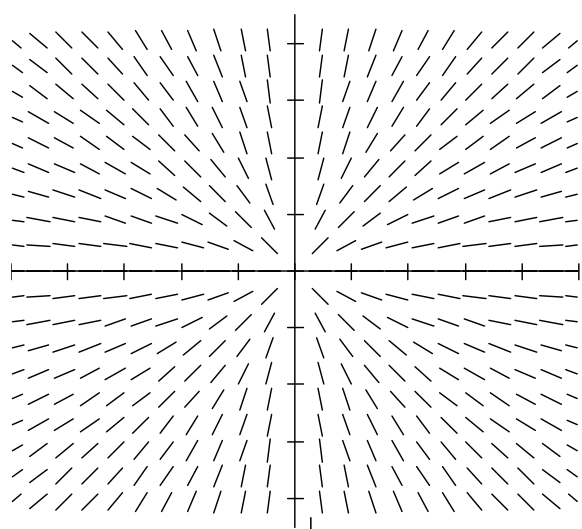
**B**



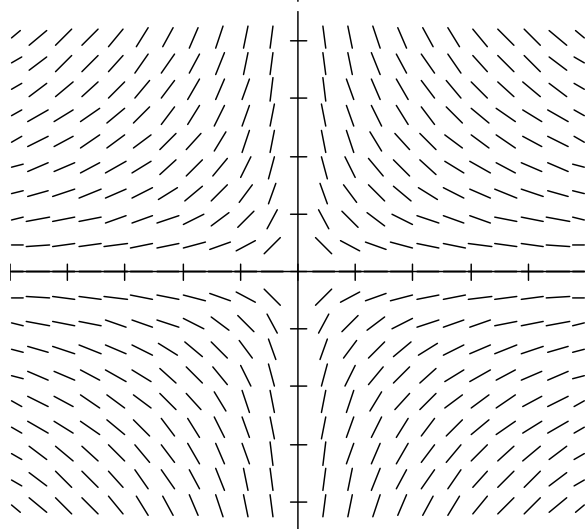
**C**



**D**



**E**



**F**

