$\boxed{10\text{-}1}$ Homework: p. 558, #1–21

1. $\cos^2 x + \tan^2 x \cos^2 x = 1$

$\cos^2 x + \dfrac{\sin^2 x \cancel{\cos^2 x}}{\cancel{\cos^2 x}} =$

$\cos^2 x + \sin^2 x =$

1 ✓

2. $2\cos\theta = \dfrac{\cos\theta \tan\theta + \sin\theta}{\tan\theta}$

$= \left(\dfrac{\cancel{\cos\theta}\sin\theta}{\cancel{\cos\theta}} + \sin\theta\right) \div \left(\dfrac{\sin\theta}{\cos\theta}\right)$

$= 2\cancel{\sin\theta} \cdot \dfrac{\cos\theta}{\cancel{\sin\theta}}$

$= 2\cos\theta$ ✓

3. $1 + 2\tan^2\beta = \sec^4\beta - \tan^4\beta$

$= (\sec^2\beta - \tan^2\beta)(\sec^2\beta + \tan^2\beta)$

$= 1(\sec^2\beta + \tan^2\beta)$

$= \tan^2\beta + 1 + \tan^2\beta$

$= 1 + 2\tan^2\beta$ ✓

4. $4 + (\tan\sigma - \cot\sigma)^2 = \sec^2\sigma + \csc^2\sigma$

$4 + \tan^2\sigma - 2\tan\sigma\cot\sigma + \cot^2\sigma =$

$\cancel{4} + \sec^2\sigma \cancel{-1} - 2\cancel{\tan\sigma} \cdot \dfrac{1}{\cancel{\tan\sigma}} + \csc^2\sigma \cancel{-1} =$

$\sec^2\sigma + \csc^2\sigma$ ✓

5. $\dfrac{(\cos\theta)(\cos\theta) - (-\sin\theta)(\sin\theta)}{\cos^2\theta} = \sec^2\theta$

$\dfrac{\cos^2\theta - (-\sin^2\theta)}{\cos^2\theta} =$

$\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} =$

$\dfrac{1}{\cos^2\theta} = \sec^2\theta$ ✓

6. $\frac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x} = -\csc^2 x$

$\frac{-\sin^2 x-\cos^2 x}{\sin^2 x} =$

$\frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x} =$

$\frac{-1}{\sin^2 x} =$

$-\csc^2 x$ ✓

7. $\frac{-(-\sin\theta)}{\cos^2\theta} = \sec\theta\tan\theta$

$\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta} =$

$\tan\theta\sec\theta$ ✓

8. $\frac{-\cos\varphi}{\sin^2\varphi} = -\csc\varphi\cot\varphi$

$\frac{-\cos\varphi}{\sin\varphi} \cdot \frac{1}{\sin\varphi} =$

$-\cot\varphi\csc\varphi$ ✓

9. $\frac{(\cos a+\sin a)(\cos a+\sin a)-1}{2\cos a} = \sin a$

$\frac{\cos^2 a+2\sin a\cos a+\sin^2 a-1}{2\cos a} =$

$\frac{\cancel{1}+2\sin a\cos a-\cancel{1}}{2\cos a} =$

$\frac{2\sin a\cancel{\cos a}}{\cancel{2\cos a}} =$

$\sin a$ ✓

10. $\tan W(\cot W \cos W + \sin W) = \sec W$

$\tan W \cot W \cos W + \tan W \sin W =$

$\cancel{\tan W}\left(\frac{1}{\cancel{\tan W}}\right)\cos W + \left(\frac{\sin W}{\cos W}\right)\sin W =$

$\cos W + \frac{\sin^2 W}{\cos W} =$

$\frac{\cos^2 W + \sin^2 W}{\cos W} =$

$\frac{1}{\cos W} =$

$\sec W$ ✓

11. $\frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \cot x$

$\frac{\sec x \cos x - \sin^2 x}{\sin x \cos x} =$

$\frac{1 - \sin^2 x}{\sin x \cos x} =$

$\frac{\cos^{\cancel{2}} x}{\sin x \cancel{\cos x}} =$

$\frac{\cos x}{\sin x} = \cot x$ ✓

12. $\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 2\csc x$

$\frac{\sin^2 x + (1-\cos x)(1-\cos x)}{\sin x(1-\cos x)} =$

$\frac{\sin^2 x + 1 - 2\cos x + \cos^2 x}{\sin x(1-\cos x)} =$

$\frac{1+1-2\cos x}{\sin x(1-\cos x)} =$

$\frac{2\cancel{(1-\cos x)}}{\sin x \cancel{(1-\cos x)}} =$

$\frac{2}{\sin x} = 2\csc x$ ✓

13. $\cos^2\theta = \dfrac{\cot\theta\csc\theta\tan\theta - \sin\theta}{\csc\theta}$

$= \left[\left(\dfrac{\cancel{\cos\theta}}{\sin\theta}\right)\left(\dfrac{1}{\cancel{\sin\theta}}\right)\left(\dfrac{\cancel{\sin\theta}}{\cancel{\cos\theta}}\right) - \sin\theta\right] \div \dfrac{1}{\sin\theta}$

$= \left(\dfrac{1}{\sin\theta} - \sin\theta\right) \cdot \sin\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$ ✓

14. $\tan b + \cot b = \dfrac{\csc b}{\cos b}$

$\dfrac{\sin b}{\cos b} + \dfrac{\cos b}{\sin b} =$

$\dfrac{\sin^2 b + \cos^2 b}{\cos b \sin b} =$

$\dfrac{1}{\cos b} \cdot \dfrac{1}{\sin b} =$

$\dfrac{\csc b}{\cos b}$ ✓

15. $\dfrac{\cos x}{1-\cos x} = \cot x \csc x + \cot^2 x$

$= \left(\dfrac{\cos x}{\sin x}\right)\left(\dfrac{1}{\sin x}\right) + \left(\dfrac{\cos^2 x}{\sin^2 x}\right)$

$= \dfrac{\cos x + \cos^2 x}{\sin^2 x}$

$= \dfrac{\cos x(1+\cos x)}{1-\cos^2 x}$

$= \dfrac{\cos x(1+\cancel{\cos x})}{(1-\cos x)(1+\cancel{\cos x})}$

$= \dfrac{\cos x}{1-\cos x}$ ✓

#16-24 not for ACC

16. $\dfrac{\tan^2 W - \sec W - 5}{\tan^2 W + 3\sec W + 3} = \dfrac{\sec W - 3}{\sec W + 1}$

$\dfrac{(\sec^2 W - 1) - \sec W - 5}{(\sec^2 W - 1) + 3\sec W + 3} =$

$\dfrac{\sec^2 W - \sec W - 6}{\sec^2 W + 3\sec W + 2} =$

$\dfrac{(\sec W - 3)\cancel{(\sec W + 2)}}{\cancel{(\sec W + 2)}(\sec W + 1)}$

$\dfrac{\sec W - 3}{\sec W + 1} = \dfrac{\sec W - 3}{\sec W + 1}$

17. $\dfrac{2\sin^2 W + 5\cos W + 1}{6\sin^2 W + 5\cos W - 2} = \dfrac{\cos W - 3}{3\cos W - 4}$

$\dfrac{2(1 - \cos^2 W) + 5\cos W + 1}{6(1 - \cos^2 W) + 5\cos W - 2} =$

$\dfrac{2 - 2\cos^2 W + 5\cos W + 1}{6 - 6\cos^2 W + 5\cos W - 2} =$

$\dfrac{-2\cos^2 W + 5\cos W + 3}{-6\cos^2 W + 5\cos W + 4} =$

$\dfrac{+(2\cos^2 W - 5\cos W - 3)}{+(6\cos^2 W - 5\cos W - 4)} =$

$\dfrac{\cancel{(2\cos W + 1)}(\cos W - 3)}{\cancel{(2\cos W + 1)}(3\cos W - 4)} =$

$\dfrac{\cos W - 3}{3\cos W - 4} = \dfrac{\cos W - 3}{3\cos W - 4}$

18. $\dfrac{3\sec^2 t - 8\tan t + 1}{\sec^2 t - \tan t - 3} = \dfrac{3\tan t - 2}{\tan t + 1}$

$\dfrac{3(\tan^2 t + 1) - 8\tan t + 1}{(\tan^2 t + 1) - \tan t - 3} =$

$\dfrac{3\tan^2 t + 3 - 8\tan t + 1}{\tan^2 t - \tan t - 2} =$

$\dfrac{3\tan^2 t - 8\tan t + 4}{\tan^2 t - \tan t - 2} =$

$\dfrac{(3\tan t - 2)\cancel{(\tan t - 2)}}{\cancel{(\tan t - 2)}(\tan t + 1)} =$

$\dfrac{3\tan t - 2}{\tan t + 1} = \dfrac{3\tan t - 2}{\tan t + 1}$

19. $\dfrac{2\csc^2 y - 7\cot y - 6}{6\csc^2 y - 5\cot y - 10} = \dfrac{\cot y - 4}{3\cot y - 4}$

$\dfrac{2(1 + \cot^2 y) - 7\cot y - 6}{6(1 + \cot^2 y) - 5\cot y - 10} =$

$\dfrac{2 + 2\cot^2 y - 7\cot y - 6}{6 + 6\cot^2 y - 5\cot y - 10} =$

$\dfrac{2\cot^2 y - 7\cot y - 4}{6\cot^2 y - 5\cot y - 4} =$

$\dfrac{\cancel{(2\cot y + 1)}(\cot y - 4)}{\cancel{(2\cot y + 1)}(3\cot y - 4)} =$

$\dfrac{\cot y - 4}{3\cot y - 4} = \dfrac{\cot y - 4}{3\cot y - 4}$

20. $\dfrac{\cot B+1}{\cot B} = \dfrac{1+\tan^3 B}{-\tan B+\sec^2 B}$

$= \left(1+\dfrac{\sin^3 B}{\cos^3 B}\right) \div \left(\dfrac{-\sin B}{\cos B} + \dfrac{1}{\cos^2 B}\right)$

$= \left(\dfrac{\cos^3 B+\sin^3 B}{\cos^3 B}\right) \cdot \left(\dfrac{\cos^2 B}{-\sin B\cos B+1}\right)$

$= \dfrac{(\cos B+\sin B)(\cos^2 B-\sin B\cos B+\sin^2 B)}{\cos B(1-\sin B\cos B)}$

$= \dfrac{(\cos B+\sin B)(1-\cancel{\sin B}\cos B)}{\cos B(1-\cancel{\sin B\cos B})}$

$1+\dfrac{1}{\cot B} = \dfrac{\cos B+\sin B}{\cos B}$

$1+\tan B = 1+\tan B$ ✓

$\dfrac{(1+\tan B)(1-\cancel{\tan B}+\tan^2 B)}{-\cancel{\tan B}+\tan^2 B+1}$

$= 1+\tan B$ ✓

21. $\dfrac{\sin^2 B-\sin B+1}{1-\sin B} = \dfrac{1+\sin^3 B}{1-\sin^2 B}$

$= \dfrac{(1+\cancel{\sin B})(1-\sin B+\sin^2 B)}{(1+\cancel{\sin B})(1-\sin B)}$

$= \dfrac{\sin^2 B-\sin B+1}{1-\sin B}$ ✓

22. $\dfrac{\cot^2 x-4\csc x-11}{\cot^2 x-3} = \dfrac{\csc x-6}{\csc x-2}$

$\dfrac{\csc^2 x-1-4\csc x-11}{\csc^2 x-1-3} =$

$\dfrac{\csc^2 x-4\csc x-12}{\csc^2 x-4} =$

$\dfrac{(\csc x-6)(\cancel{\csc x+2})}{(\csc x-2)(\cancel{\csc x+2})} =$

$\dfrac{\csc x-6}{\csc x-2}$ ✓

23. $\dfrac{1+\sin x}{1-\sin x} = 2\sec^2 x + 2\sec x\tan x - 1$

$$= \frac{2}{\cos^2 x} + \left(\frac{2}{\cos x}\right)\left(\frac{\sin x}{\cos x}\right) - 1$$

$$= \frac{2 + 2\sin x - \cos^2 x}{\cos^2 x}$$

$$= \frac{2 + 2\sin x - 1 + \sin^2 x}{\cos^2 x}$$

$$= \frac{1 + 2\sin x + \sin^2 x}{1 - \sin^2 x}$$

$$= \frac{\cancel{(1+\sin x)}(1+\sin x)}{\cancel{(1+\sin x)}(1-\sin x)}$$

$$= \frac{1+\sin x}{1-\sin x} \checkmark$$

24. $\csc^6 x - \cot^6 x = 1 + 3\csc^2 x\cot^2 x$

$(\csc^3 x - \cot^3 x)(\csc^3 x + \cot^3 x) =$

$(\csc x - \cot x)(\csc^2 x + \csc x\cot x + \cot^2 x)(\csc x + \cot x)(\csc^2 x - \csc x\cot x + \cot^2 x)$

$(\csc^2 x - \cot^2 x)(\csc^4 x - \cancel{\csc^3 x\cot x} + \cancel{\csc^2 x\cot^2 x} + \cancel{\csc^3 x\cot x} - \cancel{\csc^2 x\cot^2 x} + \cancel{\csc x\cot^3 x}$

$+ \csc^2 x\cot^2 x - \cancel{\csc x\cot^3 x} + \cot^4 x =$

$1(\csc^4 x + \csc^2 x\cot^2 x + \cot^4 x) =$

$\csc^2 x\csc^2 x + \csc^2 x\cot^2 x + \cot^2 x\cot^2 x =$

$\csc^2 x(1+\cot^2 x) + \csc^2 x\cot^2 x + \cot^2 x(\csc^2 x - 1) =$

$\csc^2 x + \csc^2 x\cot^2 x + \csc^2 x\cot^2 x + \csc^2 x\cot^2 x - \cot^2 x =$

$\csc^2 x - \cot^2 x + 3\csc^2 x\cot^2 x =$

$1 + 3\cos^2 x\cot^2 x \checkmark$