Extreme Values of Functions Name Solutions

Find the critical values of the functions given and indicate which are maxima or minima and determine if it is relative or absolute. Sketch the graph of the function on the axes provided. *Show the work* that leads to your answers. Use your calculators *only* to check your answers.

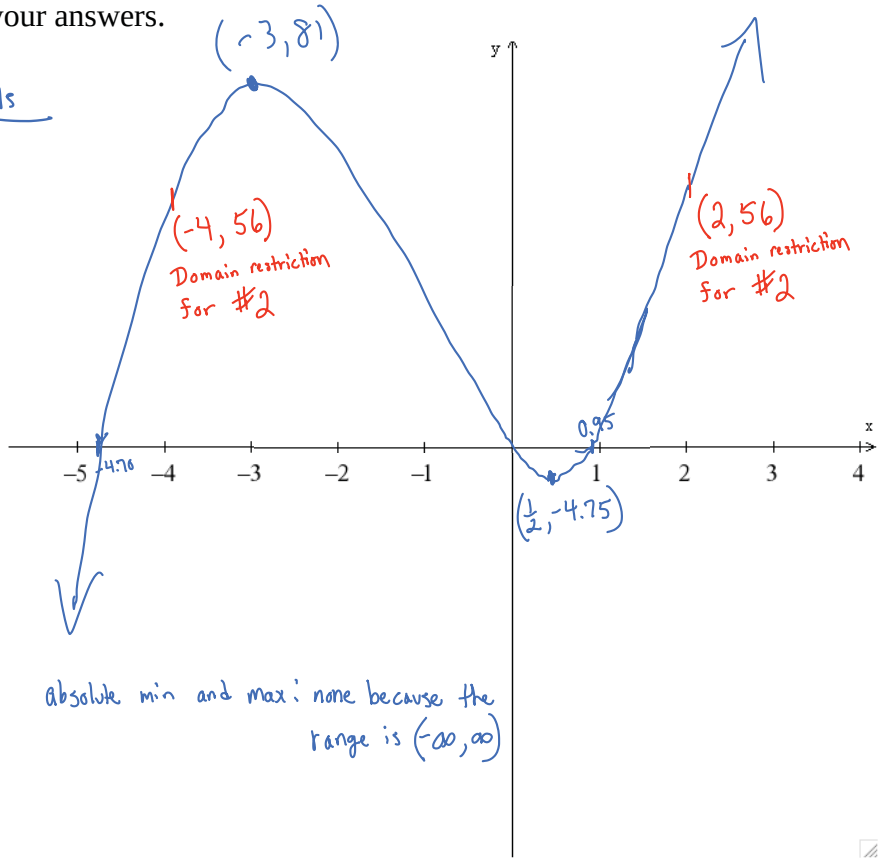
1) $y = 4x^3 + 15x^2 - 18x$

x and y-intercepts

y-int: $(0,0)$

x-int: $x(4x^2+15x-18)=0$

$(0,0)$ Quadratic formula $x = -4.70, 0.95$



Absolute min and max: none because the range is $(-\infty, \infty)$

Critical values

$\frac{dy}{dx} = 12x^2 + 30x - 18 = 0$

$6(2x^2+15x-6)=0$

$6(2x-1)(x+3)=0$

dy/dx: + 0 − 0 +

x: −3 (rel max), ½ (rel min)

plug −3 and ½ into y

$(-3,81)$ $\left(\frac{1}{2}, -4.75\right)$

2) Indicate changes in your answer for #1 if the domain were restricted to [–4, 2]

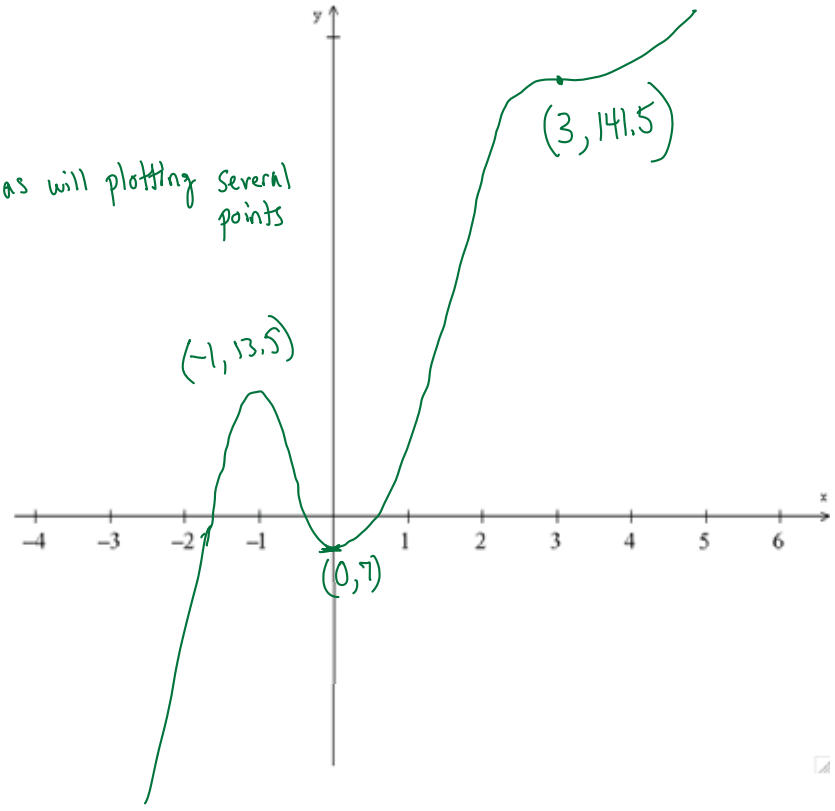
See red notations on the graph above:

1) $(-3,81)$ is now an absolute maximum

2) $(\frac{1}{2},-4.75)$ is now an absolute minimum

3) Checking the endpoints gives us $(-4,56)$ and $(2,56)$ with are neither max or min over the interval $x \in [-4,2]$

3) $y = 2x^5 - \frac{25x^4}{2} + 10x^3 + 45x^2 - 7$



x-ints require a calculator for this problem as will plotting several points

y int: $(0, -7)$

$\frac{dy}{dx} = 10x^4 - 50x^3 + 30x^2 + 90x = 0$

$\frac{dy}{dx} = 10x(x^3 - 5x^2 + 3x + 9) = 0$

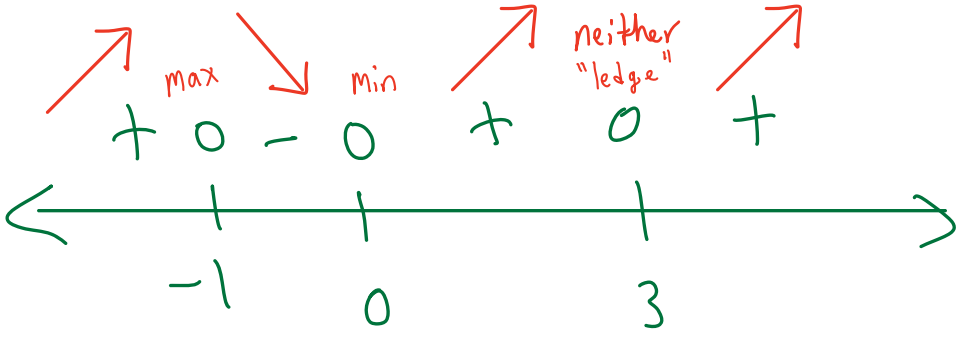
$3 \mid 1 \quad -5 \quad 3 \quad 9$

$\underline{\quad 3 \quad -6 \quad -9}$

$1 \quad -2 \quad -3 \quad 0$

$(x-3)(x^2 - 2x - 3) = 0$

$10x(x-3)^2(x+1) = 0$



4) Shannon, Alena, Cassie, and Lauren, jealous of Diego, Jeneiah, Adeeb, and Campbell for being in a worksheet problem try to design the most challenging particle motion problem they can create. The result is below:

The position of a particle moving along the *x*-axis is described by the equation

$x(t) = \frac{t^5}{5} - 2t^4 + 5t^3 + 2t^2 - 20t + 3$ over the interval $0 \le t \le 6$

(a) When and where does the particle change directions?

$v(t) = t^4 - 8t^3 + 15t^2 + 4t - 20 = 0$

2	1	−8	15	4	−20
		2	−12	6	20
	1	−6	3	10	0
−1		−1	7	−10	
	1	−7	10	0	

$(t+1)(t-2)(t-2)(t-5) = 0$

$(t-2)^2(t-5)$

− 0 − 0 +

1 2 5

Changes direction at $t=5$ not at $t=2$ which is just a stopping point

$x(5) = -47$

(b) At what time is the particle furthest to the left of the origin?

$t=5$

Since it stops moving to the left for good at $t=5$ then this is when it has moved furthest to the left