Extreme Values of Functions II Name: Solutions (See last page for notes on how to approach these problems)

Find the critical values of the functions given and indicate which are maxima or minima and determine if it is relative or absolute. Sketch the graph of the function on the axes provided. <u>*Show the work*</u> that leads to your answers. You may use your calculators <u>*only*</u> to check your answers.

1) $y = x^3 - 6x^2 + 9x - 4$

y-intercept: $(0,-4)$

x-intercept: $(1,0)\ (4,0)$

How?

$$\begin{array}{c|cccc} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & 0 \end{array} \Rightarrow (x-1)(x^2-5x+4)=0$$

$$(x-1)(x-1)(x-4)=0$$

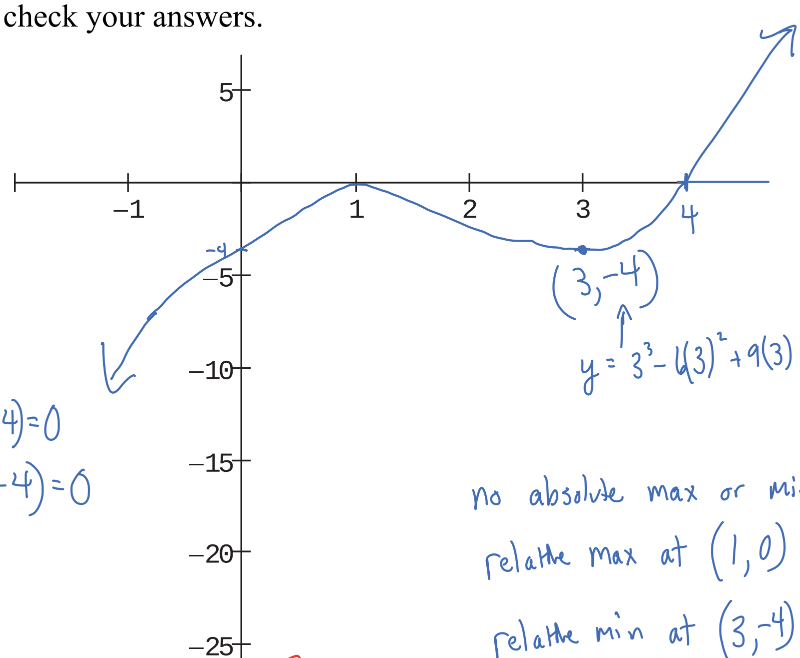
Critical values $(1,0)$ and $(3,-4)$

How?

$$\frac{dy}{dx} = 3x^2-12x+9=0$$

$$=3(x^2-4x+3)=0=3(x-1)(x-3) \Longrightarrow$$

dy/dx	$+$	0	$-$	0	$+$
	↗	max	↘	min	↗
x		1		3	



no absolute max or min

relative max at $(1,0)$

relative min at $(3,-4)$

2) $y = x^4 - 4x$

y-int $(0,0)$

x-int: $(\sqrt[3]{4},0)\ (0,0)$ How? See below

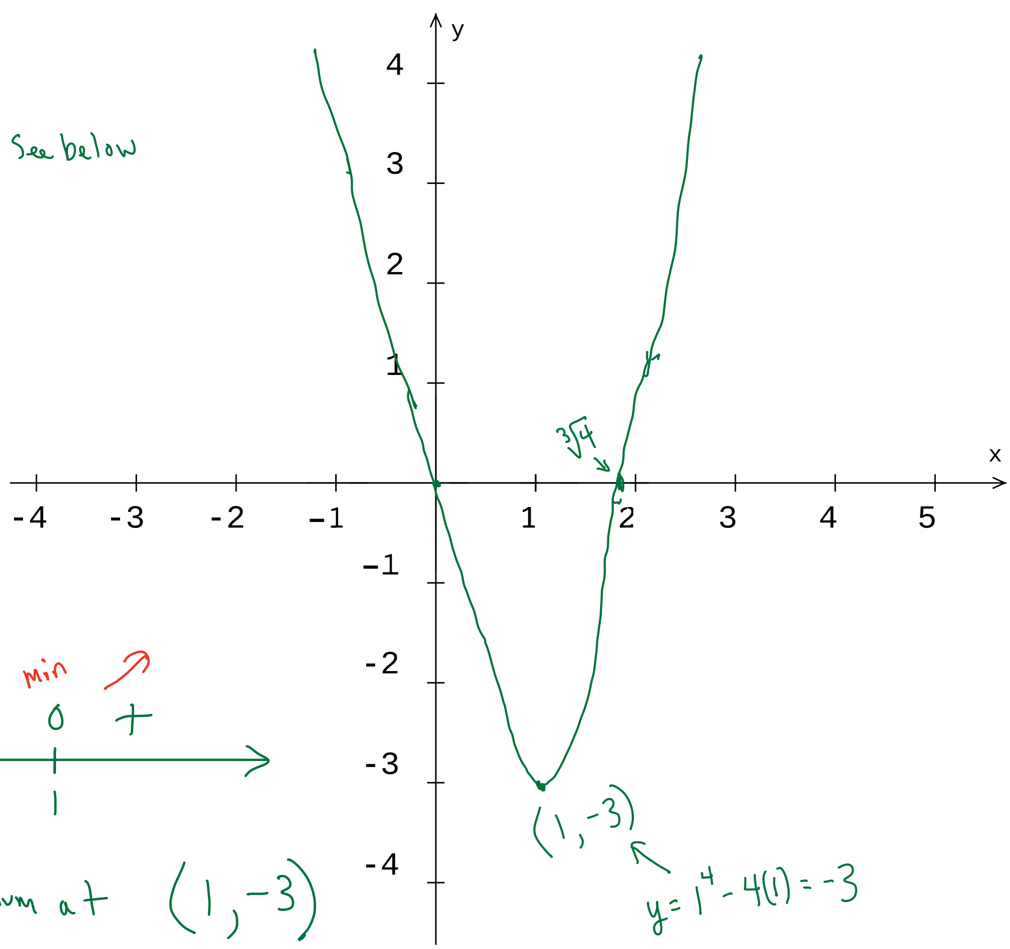
$$x^4-4x = x(x^3-4)=0 \Rightarrow x=0, \sqrt[3]{4}$$

Critical Values

$$\frac{dy}{dx} = 4x^3-4=0=4(x^3-1)$$

$$=4\underbrace{(x-1)}_{x=1}\underbrace{(x^2+x+1)}_{\text{always above the x-axis and therefore always }+}=0$$

dy/dx	$-$	0	$+$
	↘	min	↗
x		1	



absolute minimum at $(1,-3)$

3) $y = x^3 - 6x^2 + 12x - 7$

y-int: $(0,-7)$

x-int: $(1,0)$

$\underbrace{\text{x-int}}_{\text{How?}}$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 12 & -7 \\ & \downarrow & 1 & -5 & 7 \\ \hline & 1 & -5 & 7 & 0 \end{array}$$

$(x-1)\underbrace{(x^2-5x+7)}=0$

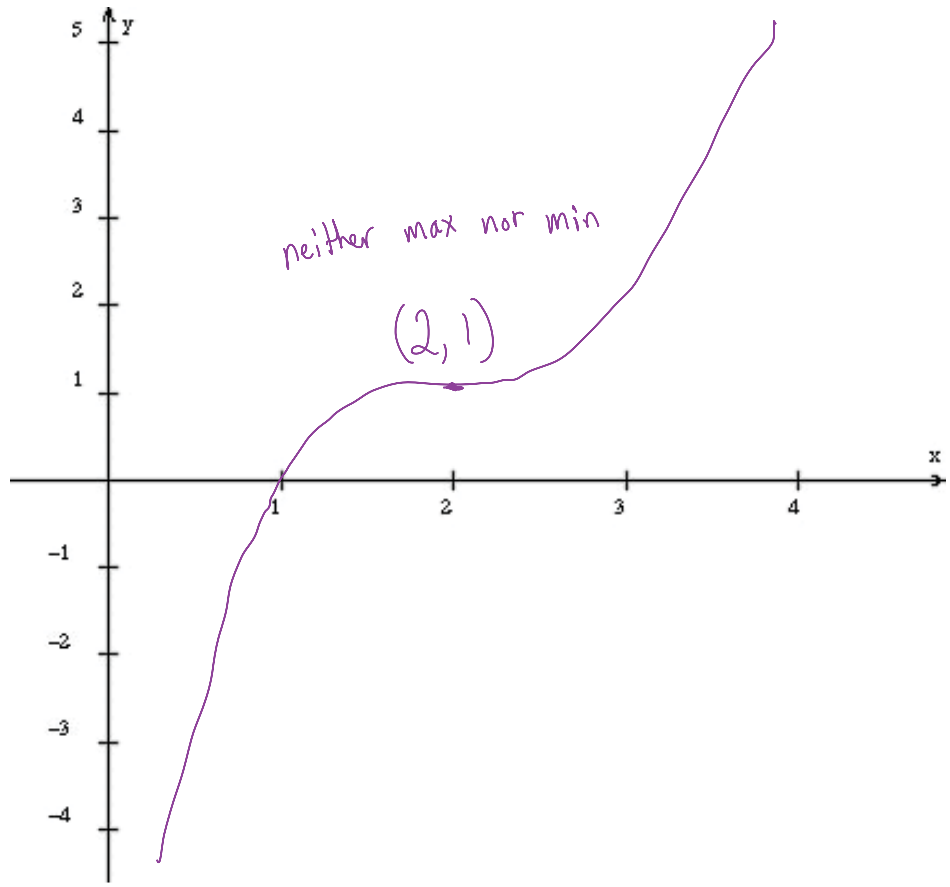
$x = \dfrac{5 \pm \sqrt{25-28}}{2}$ ← no solution

Critical values

$\frac{dy}{dx} = 3x^2 - 12x + 12 = 0$

$3(x^2-4x+4) = 0 = 3(x-2)^2$

$\frac{dy}{dx}$	+	0	+	
x		2		



4) $y = 2x^3 - 9x^2 - 60x + 32$

y-intercept: $(0, 32)$

x-intercept: $(-4,0)\left(\frac{1}{2},0\right)(8,0)$

$\underbrace{\text{x-intercept}}_{\text{How?}}$

$$\begin{array}{c|cccc} -4 & 2 & -9 & -60 & 32 \\ & & -8 & 68 & -32 \\ \hline & 2 & -17 & 8 & 0 \end{array}$$

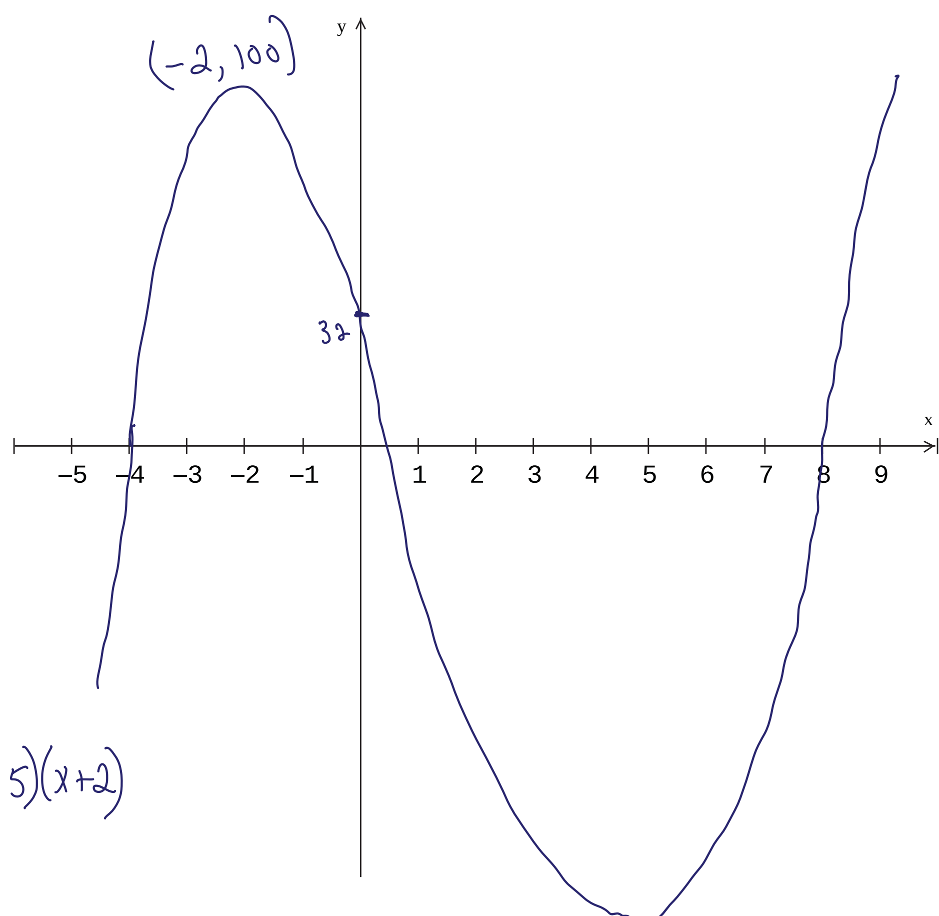
$(x+4)(2x^2-17x+8) = 0$

$(x+4)(2x-1)(x-8) = 0$

Critical values

$\frac{dy}{dx} = 6x^2 - 18x - 60 = 6(x^2-3x-10) = 6(x-5)(x+2)$

	↗	Max	↘	Min	↗
$\frac{dy}{dx}$	+	0	−	0	+
x		−2		5	



relative max $(-2, 100)$

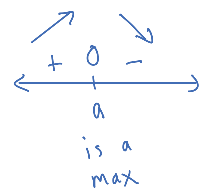
relative min $(5, -243)$

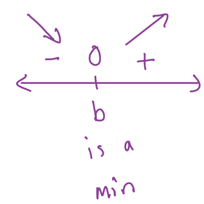
y-intercept: plug in 0 for x

x-intercept: plug in 0 for y and factor to solve for x

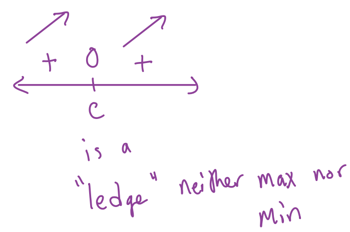
This happens <u>before</u> taking the derivative

Critical values: 1) Derivative
2) Set $\frac{dy}{dx} = 0$ and solve for x
3) Sign pattern → + means increasing
− means decreasing





Extreme Value: Plug critical values into y (<u>not</u> $\frac{dy}{dx}$)



If the domain is limited like [a, b] then
plug a and b into y to see if either
could be the absolute max/min