Math Analysis Final Exam Review

Chapter 1 Standards

1a	Use the Pythagorean Theorem to find missing sides in a right triangle
1b	Use the sine, cosine, and tangent functions to find missing sides in a right triangle
1c	Use the inverse sine, cosine, and tangent functions to find missing angles in a right triangle
1d	Apply Standards 1a, 1b, and 1c to solve mathematical models involving right triangles (*real-world problems*)
1e	Find missing sides and angles of an oblique triangle using the Law of Cosines
1f	Find missing sides of an oblique triangle using the Law of Sines
1g	Use the Laws of Cosines and Sines to solve mathematical models involving triangles (real world problems).

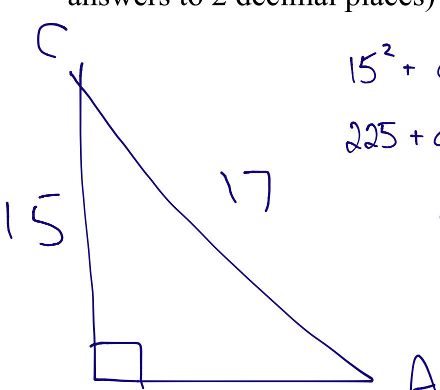
Chapter 2 Standards

2b	Find the quadrant and reference angles of a given angle in standard position.
2c	Given a point or the quadrant of the terminal side of an angle, find the six exact trigonometric values.
2d	Use exact values from the special triangles to simplify trigonometric expressions
2e	Convert between radians and degrees.
2f	Use a calculator to find approximate trigonometric values for a given angle and approximate angle values for a given trigonometric value.
2g	Find and draw a resultant vector from other component vectors.
2h	Find the direction angle of a resultant vector from other component vectors.

Chapter 3 Standards

3a	Graph a given equation of the sine or cosine function using the graphing calculator
3b	Identify and illustrate graphically the traits of a sinusoidal function
3c	Solving for values of x and y in a sinusoidal function

1) Find the missing sides and angles of the given right triangle: $\angle B = 90°$, $a = 15$, $b = 17$ (give degree answers to 2 decimal places) (Stds 1a through 1c)



$15^2 + c^2 = 17^2$

$225 + c^2 = 289$

$c^2 = 64$

$c = 8$

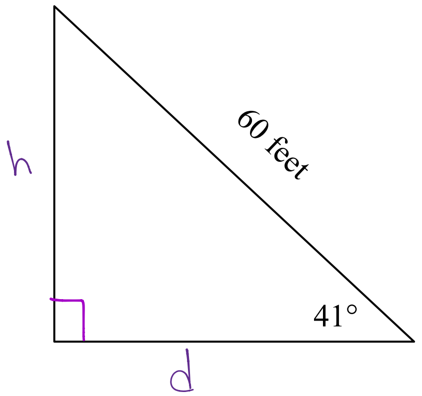
$\sin A = \frac{opp}{hyp} = \frac{15}{17}$

$\angle A = \sin^{-1}\left(\frac{15}{17}\right) \approx 61.928°$

$\angle C = 90 - \angle A \approx 28.072°$

C

2) Johnny is trying to raise the flag in front of campus but Carly keeps giving him the wrong instructions. Finally in frustration, he steps back allowing 60 feet of string to unwind completely and finds that the angle of elevation from his eyes to the top of the pole to be 41°. How high is the flagpole? How far is Johnny standing from the flagpole? (Std 1d)



Right triangle allows use to use SOHCAHTOA

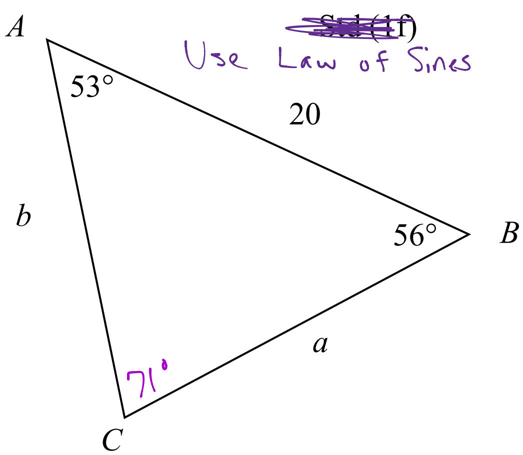
Flagpole height = h

$\frac{h}{60} = \sin 41° \Rightarrow h = 60\sin 41° \approx 39.364$ ft

distance = d $\quad \frac{d}{60} = \cos 41° \Rightarrow d = 60\cos 41° \approx 45.283$ ft

3) ~~Std (1f)~~

Use Law of Sines



$C = 71°$ $\quad a \approx 16.893$ $\quad b \approx 17.536$

$$\frac{a}{\sin 53} = \frac{20}{\sin 71}$$

$$a \sin 71 = 20 \sin 53$$

$$a = \frac{20 \sin 53}{\sin 71} \approx 16.893$$

$$\frac{a}{\sin 53} = \frac{b}{\sin 56} = \frac{20}{\sin 71}$$

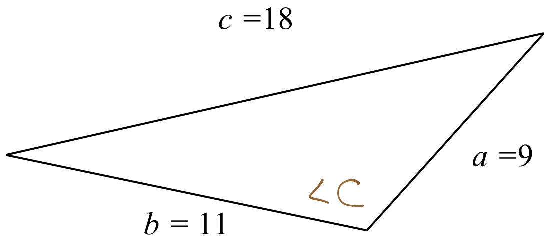
$$\frac{b}{\sin 56} = \frac{20}{\sin 71}$$

$$b \sin 71 = 20 \sin 56$$

$$b = \frac{20 \sin 56}{\sin 71} \approx 17.536$$

4) ~~Std(1e)~~ Use the Law of Cosines

$\angle A \approx 23.192^\circ$ $\angle B \approx 28.772^\circ$ $\angle C \approx 128.036^\circ$

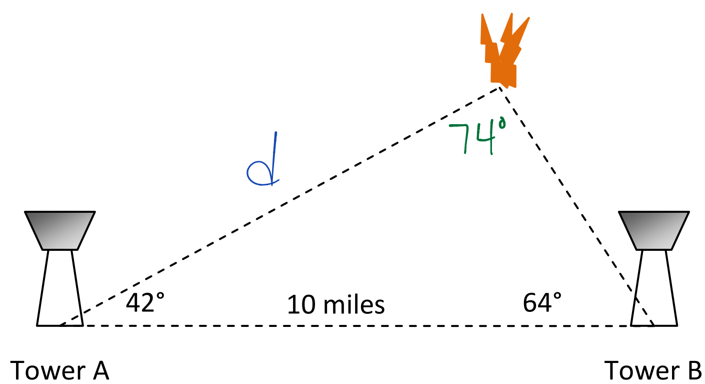


$$\angle C = \cos^{-1}\left(\frac{11^2 + 9^2 - 18^2}{2(11)(9)}\right) \approx 128.036^\circ$$

$$\angle A = \cos^{-1}\left(\frac{11^2 + 18^2 - 9^2}{2(11)(18)}\right) \approx 23.192^\circ$$

$$\angle B = 180 - \angle A - \angle C$$
$$\approx 28.772^\circ$$

5) Fire towers A and B are located 10 miles apart. They use the direction of the other tower as 0°. Rangers at fire tower A spots a fire at 42°, and rangers at fire tower B spot the same fire at 64°. How far from tower A is the fire to the nearest tenth of a mile? (~~Std 1g~~) LAW OF SINES



$$\frac{d}{\sin 64} = \frac{10}{\sin 74}$$

$$d \sin 74 = 10 \sin 64$$

$$d = \frac{10 \sin 64}{\sin 74} \approx 9.4 \text{ miles}$$

6) Convert the given angle measures to degrees (Std 2e)

a) $\frac{2\pi}{9}$

$\frac{2\cancel{\pi}}{\cancel{9}} \cdot \frac{\cancel{180}^{20}}{\cancel{\pi}} = 40^\circ$

b) $\frac{3\pi}{8}$

$\frac{3\cancel{\pi}}{2\cancel{8}} \cdot \frac{\cancel{180}^{45}}{\cancel{\pi}} = \frac{135}{2} = 67.5^\circ$

c) $\frac{25\cancel{\pi}}{\cancel{12}} \cdot \frac{\cancel{180}^{15}}{\cancel{\pi}} = 375^\circ$

7) Find the other five trig functions of θ given that $\sin\theta = \frac{\sqrt{3}}{2}$ and θ is in Quadrant II. (2c)

Unit Circle: $\sin\theta = \frac{y}{r} = \frac{\sqrt{3}}{2}$ $\quad x^2 + (\sqrt{3})^2 = 2^2 \Rightarrow x = -1$

$\cos\theta = -\frac{1}{2}$ $\quad \csc\theta = \frac{2}{\sqrt{3}}$ $\quad \cot\theta = -\frac{1}{\sqrt{3}}$

$\tan\theta = -\sqrt{3}$ $\quad \sec\theta = -2$

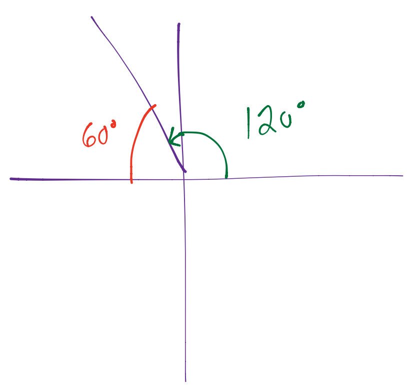
	0°	30°	45°	60°	90°
θ^{rad}	0^{rad}	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

8) Using the special angles shown in the table above, find the reference angle and the value of θ in #7. Express your answer in radians. (2b)

reference angle is 60°

QII

Answer: 120°



9) Simplify $\sec^2\frac{\pi}{3}+\cot^2\frac{\pi}{6}$ (2d)

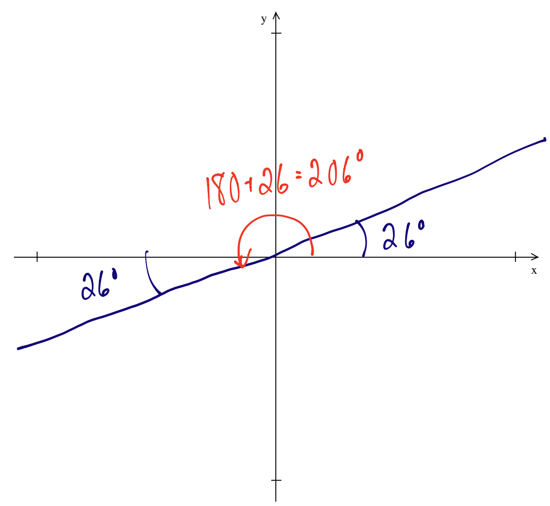
$$\underbrace{\frac{1}{\cos^2\left(\frac{\pi}{3}\right)}}+\underbrace{\frac{1}{\tan^2\left(\frac{\pi}{6}\right)}} = \frac{1}{\left(\frac{1}{2}\right)^2}+\frac{1}{\left(1/\sqrt{3}\right)^2} = 4+3 = 7$$

10) Find the two values of of θ given $0 \le \theta \le 360°$ that $\tan\theta = 0.4877325886$. Sketch the two angles on the axes below (Std 2f)

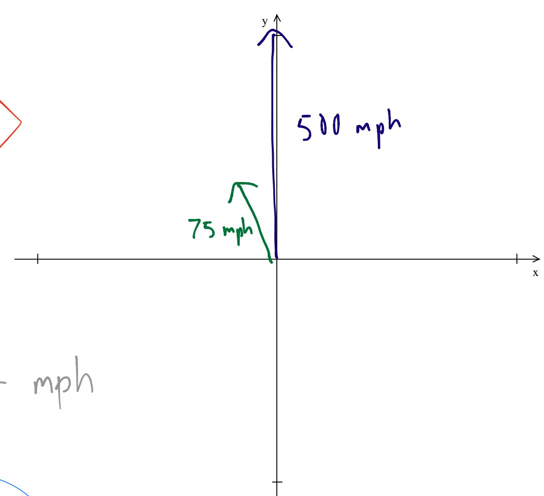
$$\tan^{-1}(0.4877325886) = 26°$$

Since $\tan\theta > 0$ we are in QI and QIII

$\theta = 26°$ and $206°$



11) A plane headed north at 500 mph enters a cross wind blowing 75 mph with a direction of 119°. Find the new speed and direction of the plane. (Std 2g & 2h)



$$\langle 500\cos 90 + 75\cos 119,\ 500\sin 90 + 75\sin 119\rangle$$

$$\langle -36.361,\ 565.596\rangle$$

$$\text{Speed} = \sqrt{(-36.361)^2+(565.596)^2} \approx 566.764 \text{ mph}$$

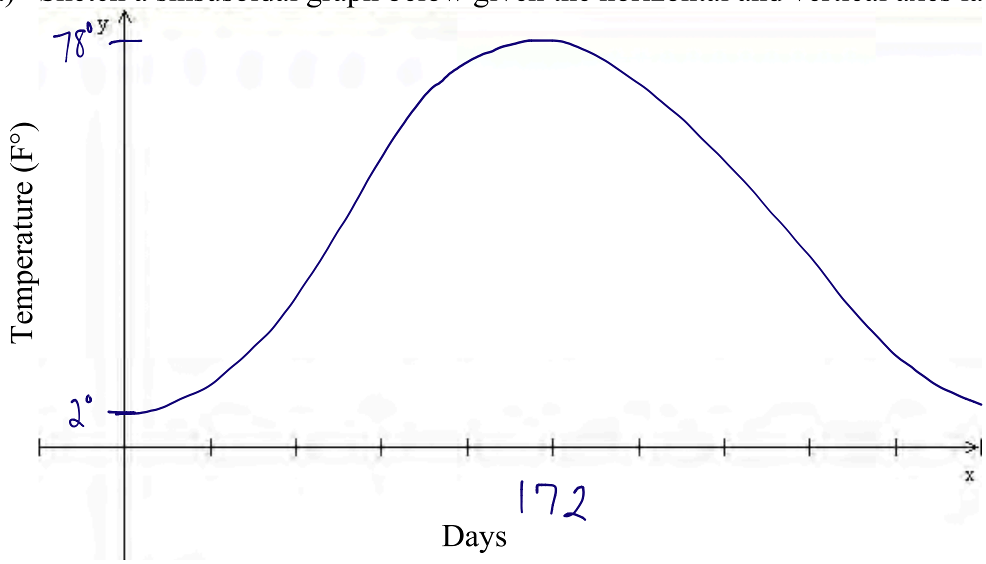
$$\text{Direction: } \theta = \pm\cos^{-1}\left(\frac{-36.361}{566.764}\right) \approx 93.678°$$

12) Christian and James, with a year's supply of food and a generator mainly to charge their iPads so they can use the internet to back up their philosophical arguments, head to a precise location north of the Equator to find a sinusoidal equation for the change in temperature throughout one year at that location. They find that on 172 the temperature hits its maximum of 78° and then reaches its minimum temperature of 2° on Day 355. (Std 3a-3d)

max

min

a) Sketch a sinusoidal graph below given the horizontal and vertical axes labels



$$\text{amplitude} = \frac{78-2}{2} = 38$$

$$\text{v-shift} = 78 - 38 = 40$$

$$\text{period} = 2(355-172)$$

$$= 2(183) = 366$$

$$b = \frac{2\pi}{366} = \frac{\pi}{183}$$

$$\text{h-shift}: 172$$

b) Write a sinusoidal equation of this graph

$$y = 40 + 38\cos\frac{\pi}{183}(t-172)$$

c) Using this model, predict which two days of this cycle that the temperature will be 52°

$$52 = 40 + 38\cos\frac{\pi}{183}(t-172)$$

$$12 = 38\cos\left[\frac{\pi}{183}(t-172)\right]$$

$$\frac{6}{19} = \cos\left[\frac{\pi}{183}(t-172)\right]$$

$$\cos^{-1}\left(\frac{6}{19}\right) = \frac{\pi}{183}(t-172)$$

$$\frac{183}{\pi}\cos^{-1}\left(\frac{6}{19}\right) = t-172$$

$$\frac{183}{\pi}\cos^{-1}\left(\frac{6}{19}\right) + 172$$

$$t = \frac{183}{\pi}\{\pm 1.250 \pm 2\pi n\} + 172$$

$$t = \pm 72.785 \pm 366n + 172$$

$$t = 99.215,\ 244.785 \text{ days}$$