Chapter 1 Standards

1a	Factor polynomials in order to find zeros
1b	Factor polynomials in order to simplify rational expressions
1c	Find equations and intercepts of lines from points, slopes, and parallel or perpendicular lines.
1d	Find equations, zeros, vertex, and range of a parabola.
1e	Apply quadratic functions to projectile motion problems
1f	Find complete graphs, zeros, and extreme points on the calculator.
1g	Apply polynomial functions to solve real-world application problems involving optimization.
1h	Factor polynomials to find zeros algebraically using synthetic substitution.
1i	Find the equation of a polynomial from its zeros.
1j	Use factoring to determine the sign patterns of a polynomial

Chapter 2 Standards

2a	Evaluate limits involving the indeterminate form 0/0

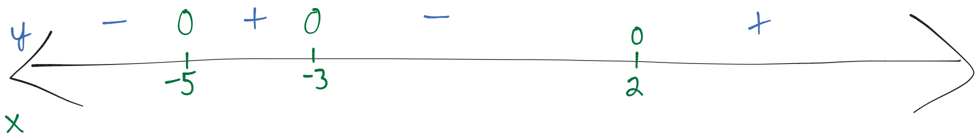
Pre-Calculus
Midterm Review

Find the values of x for which:

1) $x^3 + 6x^2 - x - 30 \geq 0$

$(x-2)(x+5)(x+3) \geq 0$

$$\begin{array}{r|rrrr} 2 & 1 & 6 & -1 & -30 \\ & & 2 & 16 & 30 \\ \hline & 1 & 8 & 15 & 0 \end{array}$$



$x \in [-5, -3] \cup [2, \infty)$

$-5 \leq x \leq -3$ or $x \geq 2$

2) Factor $x^3 - 64$ using synthetic substitution

$$\begin{array}{r|rrrr} 4 & 1 & 0 & 0 & -64 \\ & & 4 & 16 & 64 \\ \hline & 1 & 4 & 16 & 0 \end{array}$$

$(x-4)(x^2+4x+16)$

Simplify

$a = 2x \quad b = 1$

$(a-b)(a^2+ab+b^2)$

3) $\dfrac{x^2+4xy+3y^2}{3x^4+3x^2y^2} \div \dfrac{8x^3-1}{x^3-yx^2+xy^2-y^3} \text{ i } \dfrac{12x^3+6x^2+3x}{x^2-y^2}$

$$\frac{(x+3y)(x+y)}{3x^2(x^2+y^2)} \cdot \frac{x^2(x-y)+y^2(x-y)}{(2x-1)(4x^2+2x+1)} \cdot \frac{3x(4x^2+2x+1)}{(x-y)(x+y)}$$

$$\frac{(x+3y)\cancel{(x+y)}}{\cancel{3}x^{\cancel{2}}\cancel{(x^2+y^2)}} \cdot \frac{\cancel{(x^2+y^2)}\cancel{(x-y)}}{(2x-1)\cancel{(4x^2+2x+1)}} \cdot \frac{\cancel{3x}\cancel{(4x^2+2x+1)}}{\cancel{(x-y)}\cancel{(x+y)}}$$

$$\frac{x+3y}{x(2x-1)}$$

4) $\dfrac{x^3-5x^2+2x-10}{x^2-25} \div \dfrac{x^3+2x}{x^2}$

$$\frac{(x^2+2)(x-5)}{(x-5)(x+5)} \cdot \frac{x^2}{x(x^2+2)}$$

$$\frac{\cancel{(x^2+2)}\cancel{(x-5)}}{\cancel{(x-5)}(x+5)} \cdot \frac{x^{\cancel{2}}}{\cancel{x}\cancel{(x^2+2)}} = \frac{x}{x+5}$$

5) Find the equation of the line with an $\underbrace{x\text{-intercept of } 2}_{(2,0)}$ and perpendicular to one through the points (–4, –1) and (–2, –7)

$$M = \frac{-1-(-7)}{-4-(-2)} = \frac{6}{-2} = -3 \Rightarrow m_\perp = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x-2)$$

$$y = \frac{1}{3}x - \frac{2}{3}$$

6) Gavin and Chris are able to sell the idea of Mr Murphy being launched as a human cannonball for the Bruce Mahoney Rally provided that Mr. Murphy doesn't hit the ceiling of the gym which is 40 feet. Anthony and Jay volunteer to figure out if the cannon they are using will not cause this to happen. He finds that the equation for an object of equal weight being launched from the cannon at time t to be $h = 64t - 16t^2$ with $t = 0$ being the instant that Mr. Murphy is launched from the cannon.

a) What will Mr. Murphy's initial velocity be?

64 ft/sec

b) How long would Mr. Murphy be in the air if he doesn't hit the ceiling?

$$64t - 16t^2 = 0$$

$$-16t(t-4) = 0$$

$t = 4$ seconds

c) Will Mr. Murphy hit the ceiling of the gym if launched at this velocity? $\underbrace{\text{How do you know?}}_{}$

if max height ≥ 40 he hits the ceiling

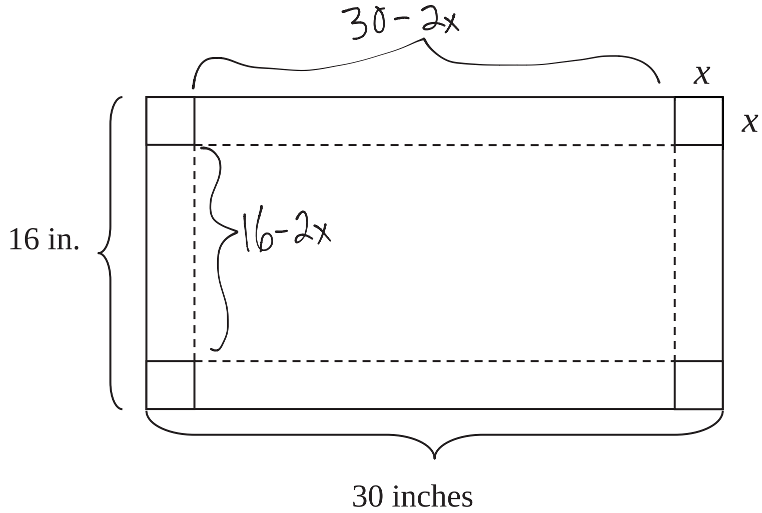
$$h = 64t - 16t^2$$

$$h = -16(t^2 - 4t + 4) - 4(-16)$$

$$h = -16(\underbrace{t-2}_{\text{time}})^2 + \underbrace{64}_{\text{max height}}$$

Since he would reach 64 ft then he will hit the ceiling

7) Lindsey still thinks that it is impossible to figure out the maximum volume possible of open box made from a 16 inch by 30 inch piece of cardboard by cutting out square of equal size from the four corners and bending up the sides. Jack and Maggie know that it can be done (Bray doesn't see the point so he just desists all together). What dimensions should he use to obtain a box with the largest possible volume?



$V = x(16-2x)(30-2x)$

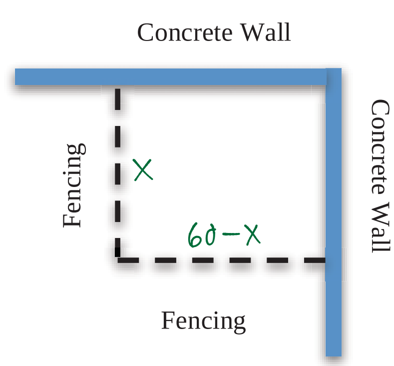
$0 < x < 8$

$x_{max} = 3\frac{1}{3} = \frac{10}{3}$

$16 - 2x = 9\frac{1}{3} = \frac{28}{3}$

$30 - 2x = 23\frac{1}{3}$

8) Emily, Laura, Kyle, and Maggie need to put up 2 sides of fencing at the intersection of two concrete fences. Maggie and Kyle are flustered because in most of their other math problems, there were three sides to their fence. Laura and Emily see through this ploy by Mr Murphy and know that they can easily solve this problem. If they have 60 feet of fencing to with which to work, find the dimensions of the largest rectangular area possible. State the maximum area.



$A = x(60-x)$

$A = 60x - x^2$

$A = 900\ ft^2$ at $x = 30\ ft$

30×30 ft square

9) Find $\lim_{x\to 5} \frac{\sqrt{x-2}-\sqrt{3}}{5-x}$ $\cdot \frac{\sqrt{x-2}+\sqrt{3}}{\sqrt{x-2}+\sqrt{3}} = \lim_{x\to 5} \frac{x-2-3}{(5-x)(\sqrt{x-2}+\sqrt{3})}$

$= \lim_{x\to 5} \frac{\cancel{x-5}\ -1}{\cancel{(5-x)}(\sqrt{x-2}+\sqrt{3})} =$

$- \frac{1}{\sqrt{3}+\sqrt{3}} = -\frac{1}{2\sqrt{3}}$

10) Find $\lim_{x\to -2} \frac{\frac{1}{x(x+3)}-\frac{1}{x}}{x+2}$ $= \frac{0}{0}$

$\lim_{x\to -2} \frac{\frac{1}{x(x+3)} - \frac{1}{x}\frac{(x+3)}{(x+3)}}{x+2} = \lim_{x\to -2} \frac{\frac{1-x-3}{x(x+3)}}{x+2} = \lim_{x\to -2} \frac{\frac{-x-2}{x(x+3)}}{x+2} = \lim_{x\to -2} \frac{\frac{-\cancel{(x+2)}}{x(x+3)}}{\cancel{x+2}} = \lim_{x\to -2} -\frac{1}{x(x+3)}$

$= -\frac{1}{-2(-2+3)} = -\frac{1}{2}$

Another approach to factoring this:

Take the original

multiply top and bottom by the common denominator of the top two fractions

$\lim_{x\to -2} \frac{\frac{1}{x(x+3)} - \frac{1}{x}}{x+2} \cdot \frac{x(x+3)}{x(x+3)} = \lim_{x\to -2} \frac{\frac{x(x+3)}{x(x+3)} - \frac{x(x+3)}{x}}{(x+2)x(x+3)} = \lim_{x\to -2} \frac{1-(x+3)}{x(x+2)(x+3)}$

$= \lim_{x\to -2} \frac{-x-2}{x(x+2)(x+3)} = \lim_{x\to -2} \frac{-\cancel{(x+2)}}{x\cancel{(x+2)}(x+3)} = \lim_{x\to -2} \frac{-1}{x(x+3)} = -\frac{1}{2}$