

A projectile is launched from the ground with initial velocity 56 ft/sec. The equation for its height at time t (in seconds) is given by

$$h = -16t^2 + 56t$$

Find its maximum height

Since this is a parabolic equation, the projectile's maximum height is also the vertex

$$h = -16(t^2 - \frac{7}{2}t)$$

Factor -16 as a first step towards completing the square

$$-16\left(\frac{49}{16}\right) + h = -16\left(t^2 - \frac{7}{2}t + \frac{49}{16}\right)$$

Add $\left(\frac{b}{2}\right)^2 = \frac{b^2}{4} = \frac{(-7/2)^2}{4} = \frac{49}{16}$ to both sides factoring in -16 on the left

$$-49 + h = -16\left(t - \frac{7}{4}\right)^2$$

Write the perfect square

$$h = -16\left(t - \frac{7}{4}\right)^2 + 49$$

Isolate h

The vertex is $(\frac{7}{4}, 49)$ or $(1.75, 49)$
at $t = 1.75$ seconds
↑ ↑
the projectile reaches a maximum height of 49 feet

How long is the projectile in the air? What is its velocity when it hits the ground?

Since it is parabolic and therefore symmetric, the projectile is in the air for $2(1.75) = 3.5$ seconds

And it hits the ground at -56 ft/sec