The Quotient Rule Name Key

Find the critical points of the rational function as well as the equation of the tangent line at a given point when asked.

1) a) Find the critical points indicating a maximum or a minimum for the graph of $y = \dfrac{x-1}{x^2+3}$

$x^2+3 \neq 0$ so there are no POE's or VA's

$x-1=0$ so x-int at $(1,0)$

$x=0$ $y=-\frac{1}{3}$ so y-int at $\left(0,-\frac{1}{3}\right)$

Critical Points:

$$y' = \frac{(x^2+3)(1)-(x-1)(2x)}{(x^2+3)^2} = \frac{x^2+3-2x^2+2x}{(x^2+3)^2} = \frac{-x^2+2x+3}{(x^2+3)^2} = \frac{-(x^2-2x-3)}{(x^2+3)^2}$$

$$= \frac{-(x-3)(x+1)}{(x^2+3)^2} = 0 \text{ or undefined}$$

never because $x^2+3 \neq 0$

$-(x-3)(x+1)=0$

$x=-1,3$

y'	$-$	0	$+$	0	$-$
x		-1		3	

$\overbrace{x=-1 \quad y=-\frac{1}{2}}$ $\overbrace{x=3 \quad y=\frac{1}{6}}$

relative min at $\left(-1,-\frac{1}{2}\right)$

relative max at $\left(3,\frac{1}{6}\right)$

b) Find the equation of the tangent line at $x = -1$

$y' = \dfrac{-(x-3)(x+1)}{(x^2+3)^2}$ at $x=-1$ $y'=0$

$y=-\frac{1}{2}$

Tangent Line: $y+\frac{1}{2} = 0(x+1) \Rightarrow y=\frac{-1}{2}$

2) a) Find the critical points indicating a maximum or a minimum for the graph of $y = \dfrac{x^2+5}{x+2}$

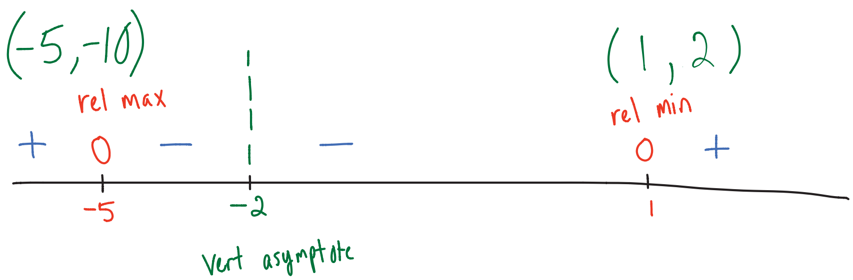
$f = x^2+5 \qquad g = x+2$

$f' = 2x \qquad g' = 1$

$$y' = \frac{2x(x+2) - 1(x^2+5)}{(x+2)^2} = \frac{2x^2+4x-x^2-5}{(x+2)^2} = \frac{x^2+4x-5}{(x+2)^2} = 0 \text{ or undef}$$

$$= 0 = \underline{x^2+4x-5} = (x+5)(x-1) \quad x = -5, 1$$

$$= \text{undef} \Rightarrow \underline{(x+2)}^2 = 0 \quad x = -2$$



b) Find the equation of the tangent line at $x = 0$

$m_T \Rightarrow$ plug $x = 0$ into y'

$$m_T = \frac{x^2+4x-5}{(x+2)^2} = \frac{0^2+4(0)-5}{(0+2)^2} = \frac{-5}{4}$$

$x = 0 \quad y = \dfrac{0^2+5}{0+2} = \dfrac{5}{2}$

Point-slope

$y = -\frac{5}{4}(x-0) + \frac{5}{2}$

or

$y = -\frac{5}{4}x + \frac{5}{2}$

3) Given $y = \dfrac{1}{x^4+x^2+1}$ find y'

$f = 1 \qquad g = x^4+x^2+1$

$f' = 0 \qquad g' = 4x^3+2x$

$$y' = \frac{0(x^4+x^2+1) - 1(4x^3+2x)}{(x^4+x^2+1)^2}$$

$$= \frac{-4x^3-2x}{(x^4+x^2+1)^2}$$