A.M.D.G.

PRECALCULUS ACCELERATED
Spring Practice Midterm – **CALCULATOR ALLOWED**

NAME: ______________________________
Date: __________________ Period: _____

Directions: Complete each of the following NEATLY IN PENCIL in the space provided. Show all work; round at **THREE** decimal places. Good luck!

Multiple Choice (3 pts. each)

1. The slope of the line tangent to the graph of $f(x) = -x^2 + 4\sqrt{x}$ at the point where $x = 4$ is

(a) –8
(b) –10
(c) –9
(d) –5
(e) –7

$f(x) = -x^2 + 4x^{\frac{1}{2}}$

$f'(x) = -2x + \frac{1}{2}(4)x^{-\frac{1}{2}} = -2x + 2x^{-\frac{1}{2}} = -2x + \frac{2}{\sqrt{x}}$

$f'(4) = -2(4) + \frac{2}{\sqrt{4}} = -8 + \frac{2}{2} = -8 + 1 = -7$

2. Suppose you can take out a 30-year loan for a $550,000 house, at a fixed APR of 5.25% compounded monthly. What are your monthly payments?

$S = P\left(1+\frac{r}{n}\right)^{nt}$, $S = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$, $A = P\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

(a) $114,245.95
(b) $630.87
(c) $3037.12
(d) $181.09
(e) $871.81

$550000 = P\frac{1-\left(1+\frac{0.0525}{12}\right)^{-360}}{\frac{0.0525}{12}}$

$\frac{\frac{0.0525}{12}\,550{,}000}{\left(1-\left(1+\frac{0.0525}{12}\right)^{-360}\right)} \approx \$3037.12$

3. If $\log_4 x + 3\log_4 x = 9$, then $x =$

(a) 1.86
(b) 2.25
(c) 9
(d) 22.6
(e) 256

$\log_4 x + \log_4 x^3 = 9$

$\log_4 x^4 = 9$

$\cancel{4^{\log_4}}\, x^4 = 4^9$

$x^4 = 4^9$

$x = \sqrt[4]{4^9}$

4. Given $y = x^2 \ln x$

a. $y' = 2$ b. $y' = 2x \cdot \frac{1}{x}$ c. $y' = 2x\ln x - x$ d. $y' = 2x\ln x + x$ e. $y' = \frac{2x}{\ln x}$

$f = x^2$ $g = \ln x$

$f' = 2x$ $g' = \frac{1}{x}$

$fg' + gf' = x^2\frac{1}{x} + 2x\ln x = x + 2x\ln x$

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$f = x - x^2 \quad g = e^{-x}$

$f' = 1 - 2x \quad g' = -e^{-x}$

5. Given $y = (x - x^2)e^{-x}$

a. $y' = (1 - x - x^2)e^{-x}$ b. $y' = (x^2 - 3x + 1)e^{-x}$ c. $y' = (1-2x)(-xe^{-x-1})$ d. $y' = \dfrac{1-2x}{e^{-x}}$ e. $y' = -\dfrac{1-2x}{e^{-x}}$

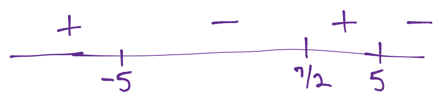
$-(x - x^2)\underline{e^{-x}} + (1-2x)\underline{e^{-x}} = e^{-x}\left[-(x-x^2) + 1 - 2x\right] = e^{-x}(-x + x^2 + 1 - 2x) = e^{-x}(x^2 - 3x + 1)$

Free Response

1. Find the domain, zeros, extreme points, and intervals of decreasing for $y = \sqrt{-2x^3 + 7x^2 + 50x - 175}$

Domain: $-2x^3 + 7x^2 + 50x - 175 = 0 = -(x+5)(x-5)(2x-7)$

Domain



$x \in (-\infty, -5] \cup [\tfrac{7}{2}, 5]$

Zeros: $x = \pm 5, \frac{7}{2}$

VA's: none

Extreme Points:

Intervals of Decreasing:

$y = (-2x^3 + 7x^2 + 50x - 175)^{1/2}$

$y' = \frac{1}{2}(-2x^3 + 7x^2 + 50x - 175)^{-1/2}(-6x^2 + 14x + 50)$

$= \dfrac{-6x^2 + 14x + 50}{2\underbrace{\sqrt{-2x^3 + 7x^2 + 50x - 175}}} = 0$ or undefined

We already know that this is zero when $x = \pm 5, \frac{7}{2}$ so the curve is vertical at these points

$-6x^2 + 14x + 50 = 0$ where $x \approx \underbrace{-1.947}_{\text{not in the domain}}, 4.280$

max at $(4.280, 3.229)$

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PRECALCULUS ACCELERATED
Spring Practice Midterm – **NO CALCULATOR ALLOWED**

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Date: __________________ Period: _____

Directions: Complete each of the following NEATLY IN PENCIL in the space provided. Show all work; round at **THREE** decimal places. Good luck!

Multiple Choice (3 pts. each)

4. The table at right gives the values of the differentiable functions *f* and *g* and their derivatives at $x=1$. If $h(x)=\underbrace{\left(2f(x)+3\right)\left(1+g(x)\right)}_{\text{requires the product rule}}$, then $h'(1)=$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	–2	–3	4

$u = 2f(x)+3$ $\quad v = 1+g(x)$

$u' = 2f'(x)$ $\quad v' = g'(x)$

(a) –28
(b) –16
(c) 40
(d) 44
(e) 47

$h'(x) = (2f(x)+3)g'(x) + 2f'(x)(1+g(x))$

$h'(1) = (2f(1)+3)g'(1) + 2f'(1)(1+g(1))$

$h'(1) = (2(3)+3)\cdot 4 + 2(-2)(1+(-3))$

$h'(1) = 36 + 8 = 44$

6. Which of the following is true about the function *f* if $f(x)=\sqrt{\dfrac{x^2+x-2}{2x^2+x-3}}$?

I. *f* has a zero at $x=1$. Can't have an x-int that is also a POE

II. The graph of *f* has a POE at $x=1$.

III. The graph of *f* has a horizontal asymptote at $y=\dfrac{1}{2}$. horiz asymptote at $y=\sqrt{\dfrac{x^2}{2x^2}} = \sqrt{\dfrac{1}{2}} \neq \dfrac{1}{2}$

(a) II only
(b) I and II only
(c) I and III only
(d) II and III only
(e) I, II and III

Free Response (10 pts. each)

4. List all traits and sketch $y=\sqrt{\dfrac{2x-3}{\underbrace{x^2+4}_{x^2+4\geq 4>0}}}$ $\rightarrow 2x-3\geq 0 \Rightarrow x\geq \frac{3}{2}$

$-$ 0 $+$; $\frac{3}{2}$

Domain: $x\geq 3/2$

Zeros: $(3/2, 0)$

y-int: none because plugging in zero for x gives us $y=\sqrt{-\frac{3}{4}}$

VAs: none because $x^2+4\neq 0$

EB: Horizontal asymptote at $y=0$

POEs: no

Extreme Points:

Range:

$$y'=\frac{1}{2}\left(\frac{2x-3}{x^2+4}\right)^{-1/2}\left(\frac{2x^2+8-4x^2+6x}{(x^2+4)^2}\right)$$

$$=\frac{1}{2}\left(\frac{x^2+4}{2x-3}\right)^{1/2}\frac{-2x^2+6x+8}{(x^2+4)^2}$$

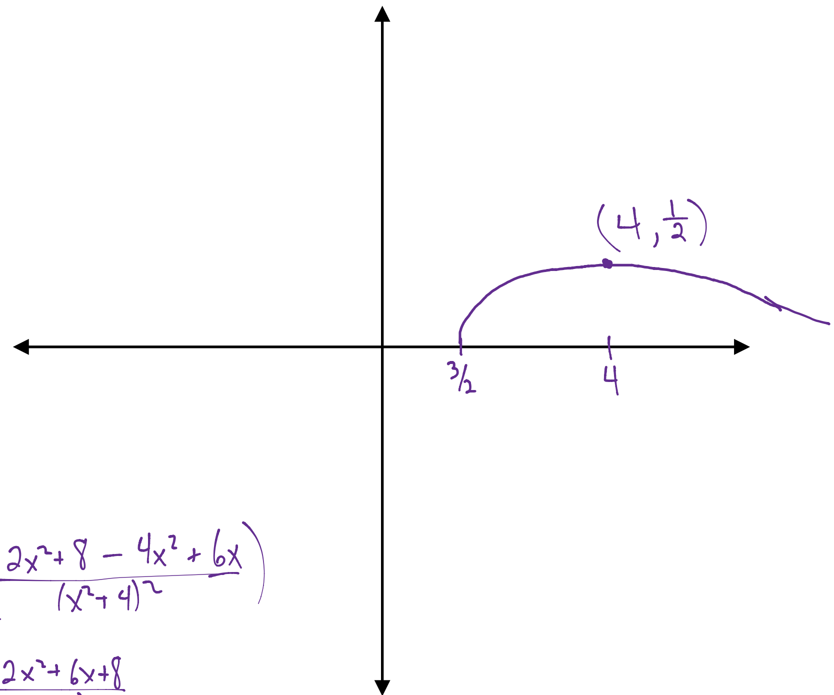
$$=\frac{-(x^2-3x-4)}{(x^2+4)^{3/2}} = 0 \text{ or undefined}$$

$\rightarrow -(x-4)(x+1)=0$

$x^2+4\neq 0$

vertical at $x=3/2$

not in domain; vertical graph; $+$; 0 ; -1 ; $3/2$; 4



5. List all traits **and** sketch $y=\ln x$.

Domain: $x>0$

Zeros: $\ln x=0$ when $x=1$

y-int: none because $\ln 0$ does not exist

VAs: at $x=0$ (y-axis)

EB: always increasing so up to the right

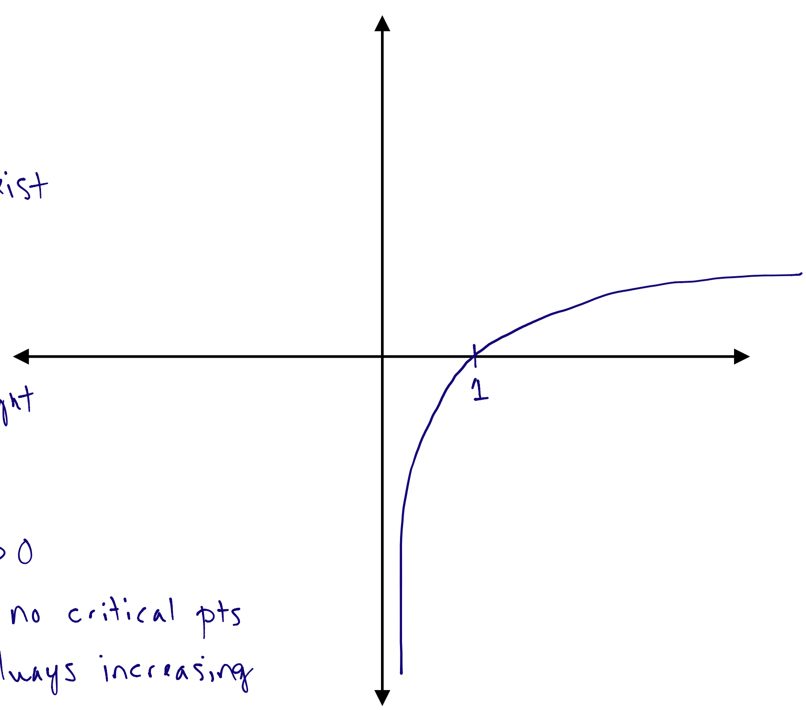
POEs: none

Extreme Points: $y'=\frac{1}{x}>0$ for all $x>0$

So there are no critical pts and $\ln x$ is always increasing

Range:

all reals



6. List all traits **and** sketch $y = a^x$.

Domain: all reals (you can plug in any value for x)

Zeros: none because $a^x \neq 0$

y-int: (0,1) because $a^0 = 1$

VAs: none

EB: up to the right

POEs: none

Extreme Points: $y' = a^x \ln a > 0$ for all x

so it is always increasing with no critical pts

Range:

$a^x > 0$ for all x so horiz asymptote at $y = 0$