Math Analysis Trig Identity Worksheet 3
4.1 Identities

Prove the following identities.

1. $\sin x(\cot x + \cos x \tan x) = \cos x + \sin^2 x$

$\sin x\left(\frac{\cos x}{\sin x} + \cos x \frac{\sin x}{\cos x}\right) = \cos x + \sin^2 x$

$\sin x \frac{\cos x}{\sin x} + \sin x \cos x \frac{\sin x}{\cos x} = \cos x + \sin^2 x$

$\cancel{\sin x} \frac{\cos x}{\cancel{\sin x}} + \sin x \cancel{\cos x} \frac{\sin x}{\cancel{\cos x}} = \cos x + \sin^2 x$

$\cos x + \sin^2 x = \cos x + \sin^2 x$ ✓

2. $\sec x - \cos x = \sin x \tan x$

$\frac{1}{\cos x} - \cos x = \sin x \frac{\sin x}{\cos x}$ WRITE IN TERMS OF SINE AND COSINE

$\frac{1}{\cos x} - \frac{\cos x}{\cos x} \cos x = \sin x \frac{\sin x}{\cos x}$ ← COMMON DENOMINATOR

$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

BRING THE TWO FRACTIONS TOGETHER

$\frac{1 - \cos^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

$\frac{\sin^2 x}{\cos x} = \frac{\sin^2 x}{\cos x}$

3. $\cot^2 x + \csc^2 x = 2\csc^2 x - 1$

$\csc^2 x - 1 + \csc^2 x = 2\csc^2 x - 1$

$2\csc^2 x - 1 = 2\csc^2 x - 1$ ✓

4. $\sin x \cos x \tan x = 1 - \cos^2 x$

$\sin x \cos x \frac{\sin x}{\cos x} = 1 - \cos^2 x$

$\sin x \cancel{\cos x} \frac{\sin x}{\cancel{\cos x}} = 1 - \cos^2 x$

$\sin^2 x = 1 - \cos^2 x$ ✓

5. $\sin y + \sin y \cot^2 y = \csc y$

$\sin y\left(1+\cot^2 y\right) = \csc y$

$\sin y\left(\csc^2 y\right) = \csc y$

$\sin y\left(\frac{1}{\sin^2 y}\right) = \csc y$

$\frac{1}{\sin y} = \csc y$ ✓

6. $\frac{1}{1-\cos x} + \frac{1}{1+\cos x} = 2\csc^2 x$

$\frac{1+\cos x}{1+\cos x}\cdot\frac{1}{1-\cos x} + \frac{1-\cos x}{1-\cos x}\cdot\frac{1}{1+\cos x} = 2\csc^2 x$ } common denominator on the left

$\frac{1+\cos x}{1-\cos^2 x} + \frac{1-\cos x}{1-\cos^2 x} = 2\csc^2 x$

$\frac{1+\cos x + 1-\cos x}{1-\cos^2 x} = 2\csc^2 x$ combine the fractions

$\frac{1+\cancel{\cos x}+1-\cancel{\cos x}}{1-\cos^2 x} = 2\csc^2 x$

$\frac{2}{1-\cos^2 x} = 2\csc^2 x$

$\frac{2}{\sin^2 x} = 2\csc^2 x$

$2\csc^2 x = 2\csc^2 x$ ✓