Math Analysis Worksheet 2
4.1 Identities

Prove the following identities.

1. $\sin^2\theta + \tan^2\theta + \cos^2\theta = \sec^2\theta$

$\sin^2\theta + \cos^2\theta + \tan^2\theta = \sec^2\theta$

$1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta = \sec^2\theta$

2. $\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta$

$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$

$\frac{\sin^2 x}{\sin^2 x \cos^2 x}\cdot\frac{1}{1} + \frac{\cos^2 x}{\cos^2 x \sin^2 x}\cdot\frac{1}{1} = \frac{1}{\cos^2 x \sin^2 x}$

$\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\cos^2 x \sin^2 x} = \frac{1}{\cos^2 x \sin^2 x}$

$\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\cos^2 x \sin^2 x}$

3. $\cos x(\sec x - \cos x) = \sin^2 x$

$\cos x \sec x - \cos^2 x = \sin^2 x$

$1 - \cos^2 x = \sin^2 x$ ✓

$\sin^2 x = \sin^2 x$ ✓

4. $\tan x + \cot x = \sec x \csc x$

$$\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \sec x \csc x$$

$$\frac{\sin x}{\sin x}\frac{\sin x}{\cos x} + \frac{\cos x}{\cos x}\frac{\cos x}{\sin x} = \sec x \csc x$$

$$\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\cos x \sin x} = \sec x \csc x$$

$$\frac{\sin^2 x}{\sin x \cos x} + \frac{\cos^2 x}{\cos x \sin x} = \sec x \csc x$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} = \sec x \csc x$$

$$\frac{1}{\cos x \sin x} = \sec x \csc x$$

$$\sec x \csc x = \sec x \csc x \quad \checkmark$$

5. $\cos x\left(\sec x + \cos x \csc^2 x\right) = \csc^2 x$

$$\cos x\left(\frac{1}{\cos x} + \cos x \frac{1}{\sin^2 x}\right) = \csc^2 x$$

$$1 + \cos^2 x \frac{1}{\sin^2 x} = \csc^2 x$$

$$1 + \frac{\cos^2 x}{\sin^2 x} = \csc^2 x$$

$$1 + \cot^2 x = \csc^2 x \quad \checkmark$$

6. $\dfrac{(\cos x - \sin x)^2}{\sin x} = \csc x - 2\cos x$

$$(\cos x - \sin x)^2 = (\cos x - \sin x)(\cos x - \sin x) = \cos^2 x - 2\sin x \cos x + \sin^2 x$$

$$\frac{\cos^2 x - 2\sin x \cos x + \sin^2 x}{\sin x} = \csc x - 2\cos x$$

$$\cos^2 x - 2\sin x \cos x + \sin^2 x = (\csc x - 2\cos x)\sin x$$ Multiplied both sides by $\sin x$

$$\cos^2 x - 2\sin x \cos x + \sin^2 x = \underbrace{\sin x \csc x}_{1/\sin x} - 2\cos x \sin x$$

$$\underbrace{\cos^2 x + \sin^2 x}_{1} - 2\sin x \cos x = 1 - 2\cos x \sin x$$

$$1 - 2\sin x \cos x \quad \checkmark$$

7. $\dfrac{\cos x}{\sec x}+\dfrac{\sin x}{\csc x}=1$

$\dfrac{\cos x}{1/\cos x}+\dfrac{\sin x}{1/\sin x}=1$

$\cos x\cdot\dfrac{\cos x}{1}+\sin x\cdot\dfrac{\sin x}{1}=1$

$\cos^2 x+\sin^2 x=1$ ✓

8. $\dfrac{\cos x}{1-\sin x}+\dfrac{1-\sin x}{\cos x}=2\sec x$

$\dfrac{(1+\sin x)\cos x}{(1+\sin x)(1-\sin x)}+\dfrac{(1-\sin x)\cos x}{\cos x\cos x}=2\sec x$

$\dfrac{(1+\sin x)(\cos x)}{1-\sin^2 x}+\dfrac{(1-\sin x)\cos x}{\cos^2 x}=2\sec x$

$\dfrac{(1+\sin x)(\cos x)}{\cos^2 x}+\dfrac{(1-\sin x)\cos x}{\cos^2 x}=2\sec x$

$\dfrac{(1+\sin x)\cos x+(1-\sin x)\cos x}{\cos^2 x}=2\sec x$

$\dfrac{\cos x+\cancel{\cos x\sin x}+\cos x-\cancel{\cos x\sin x}}{\cos^2 x}=2\sec x$

$\dfrac{\cos x+\cos x}{\cos^2 x}=\dfrac{2\cos x}{\cos^2 x}=$

$\dfrac{2}{\cos x}=2\sec x$ ✓

9. $\dfrac{\cos x\sec x}{\tan x}=\cot x$

$\dfrac{\cancel{\cos x}\ \cancel{\frac{1}{\cos x}}}{\tan x}=\cot x$

$\dfrac{1}{\tan x}=\cot x$ ✓

... Carly's solution to #8

$$\frac{\cos x \cos x}{\cos x(1-\sin x)} + \frac{(1-\sin x)(1-\sin x)}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{\cos^2 x}{\cos x(1-\sin x)} + \frac{1-2\sin x+\sin^2 x}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{1+1-2\sin x}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{2-2\sin x}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{2(1-\sin x)}{\cos x(1-\sin x)} = 2\sec x$$

$$\frac{2}{\cos x} = 2\sec x$$

$$2\sec x = 2\sec x \quad \checkmark$$