Position, Velocity, & Acceleration **Name**_____

Given the equation for the position of a particle at time *t*, indicate

a) when the particle changes directions $x'(t)=0$
b) where the particle changes directions $x(t)$ when $x'(t)=0$ (plug your answers to part (a) into $x(t)$)
c) what is its acceleration when it changes directions plug your answer to part (a) into $x''(t)$

1) $x(t)=t^2-9t-14$

a) $x'(t)=2t-9=0$

$2t=9$

$t=\frac{9}{2}$

b) $x\left(\frac{9}{2}\right)=\left(\frac{9}{2}\right)^2-9\left(\frac{9}{2}\right)-14=-34.25$

c) $x''(t)=2$

$x''\left(\frac{9}{2}\right)=2$

2) $y(t)=t^3-9t^2+15t+4$

moving downward

$1<t<5$

a) $y'(t)=3t^2-18t+15=0$

$=3(t^2-6t+5)=0$

$=3(t-1)(t-5)=0$

$t=1,5$

b) $x(1)=11$

$x(5)=-21$

c) $y''(t)=6t-18$

$y''(1)=-12$

$y''(5)=12$

$\langle 3t^4-22t^3+30t^2+48t+1, t\rangle$

3) A particle moves along the path given by the parametric equations $x(t)=3t^4-22t^3+30t^2+48t+1$ and $y(t)=t$ over the interval $0\le t\le 5$

a) Find the instant(s) when the particle is going straight up.

$x''(t)=12t^3-66t^2+60t+48=0=6(2t^3-11t^2+10t+8)$

$=6(t-2)(2t^2-7t-4)$

$(2t+1)(t-4)$

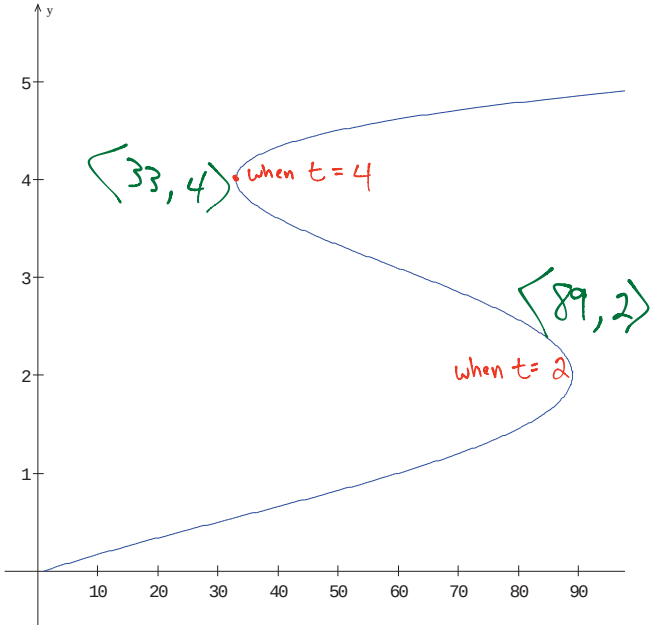
$t=-\frac{1}{2},2,4$

$t=2$ seconds and 4 seconds

b) Find the location of the particle at for the time(s) in part a)

$x(2)=89$ $y(2)=2$

$x(4)=33$ $y(4)=4$



4) Katrina, Michelle, and Lauren are sitting at the origin on the *x*-*y* plane bemoaning how long it has taken them to star in a Pre-Calc problem. When Lauren and Michelle begin arguing over who is the more deserving student, in a fit of rage, Michelle starts chasing Lauren. Hailey, seeing this coming, starts timing the chase at the moment Lauren starts running and continues to do so for 7 seconds. She also tracks the motion and determines the position vector for the path of the chase to be

$\langle x(t), y(t)\rangle = \langle t^4 - 15t^3 + 75t^2 - 125t - 2,\ t^2 - 6t\rangle$

a) At what point on the *x*-*y* plane are they sitting when the chase starts?

$\langle x(0), y(0)\rangle = \langle -2, 0\rangle$

b) In what directions (both *x* and *y*) do they initially start running? Justify your answer.

$\langle x'(0), y'(0)\rangle$

$x'(t) = 4t^3 - 45t^2 + 150t - 125$

$y'(t) = 2t - 6$

$\langle x'(0), y'(0)\rangle = \langle -125, -6\rangle$ so they are running down and to the left

c) In what directions are they running at $t = 2$ seconds? What is their speed at this time?

$\langle x'(2), y'(2)\rangle = \langle 27, 2\rangle$

Speed $= \sqrt{[x'(2)]^2 + (y'(2))^2} = \sqrt{27^2 + 2^2} = \sqrt{733}$

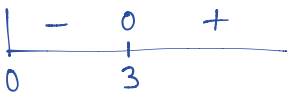
≈ 27.074

d) Do they ever change vertical directions? When and where?

$y'(t)$ changes signs

$2t - 6 = 0$

$t = 3$

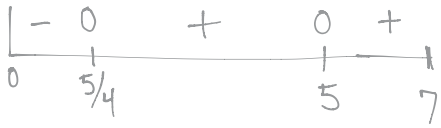


They turn from a downward direction to upward at $t = 3$ seconds when they are at the position $\langle x(3), y(3)\rangle = \langle -26, -9\rangle$

e) When is their horizontal motion to the left?

$x'(t) = 4t^3 - 45t^2 + 150t - 125 < 0$

Use synthetic div with $t = 5$



$x'(t) = (4t^2 - 25t + 25)(t - 5) = (4t - 5)(t - 5)(t - 5) = 0$

$0 < t < 1.25$ seconds

f) Does their horizontal motion ever stop and then start again without changing directions?

Yes at $t = 5$ (see sign pattern in part e))