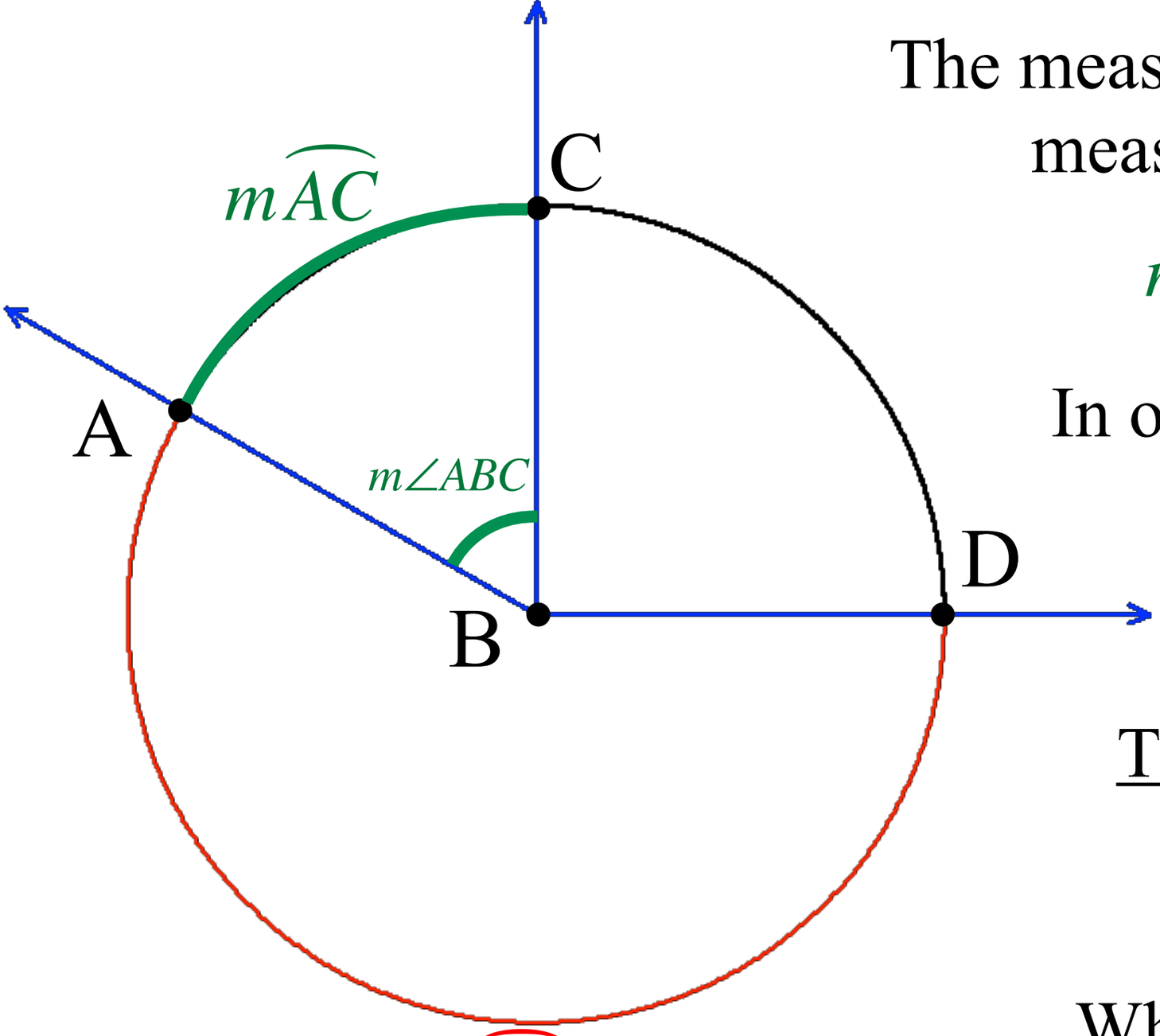
Arcs & Chords



Because $m\widehat{AD}$ is the arc here in red

The measure of an *Arc* is equal to the measure of its central angle

$$m\widehat{AC} = m\angle ABC$$

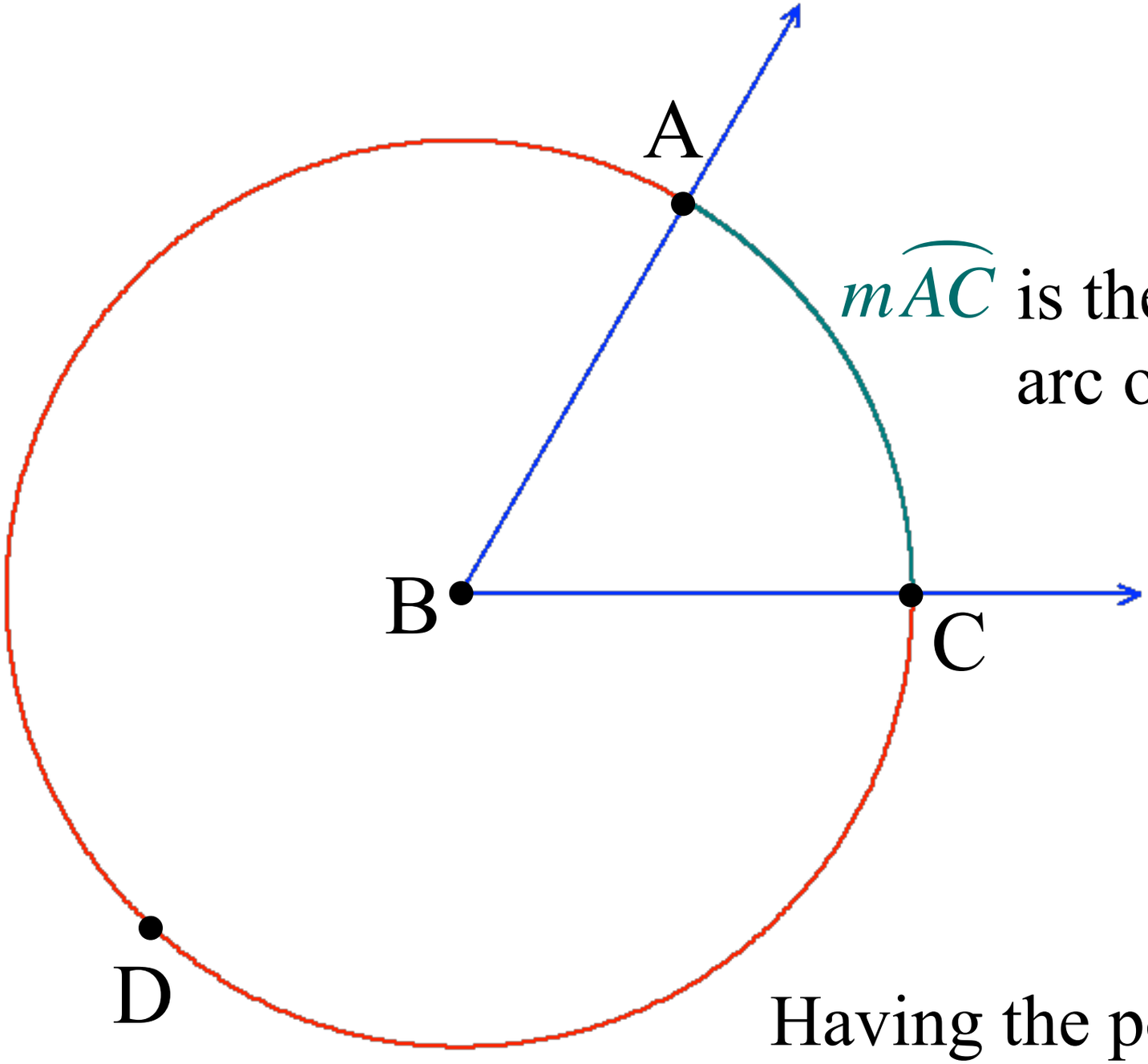
In other words, if $m\angle ABC = 60°$

then $m\widehat{AC} = 60°$

The Arc Addition Postulate

$$m\widehat{AC} + m\widehat{CD} = m\widehat{ACD}$$

Why didn't I just call it $m\widehat{AD}$ instead of $m\widehat{ACD}$?

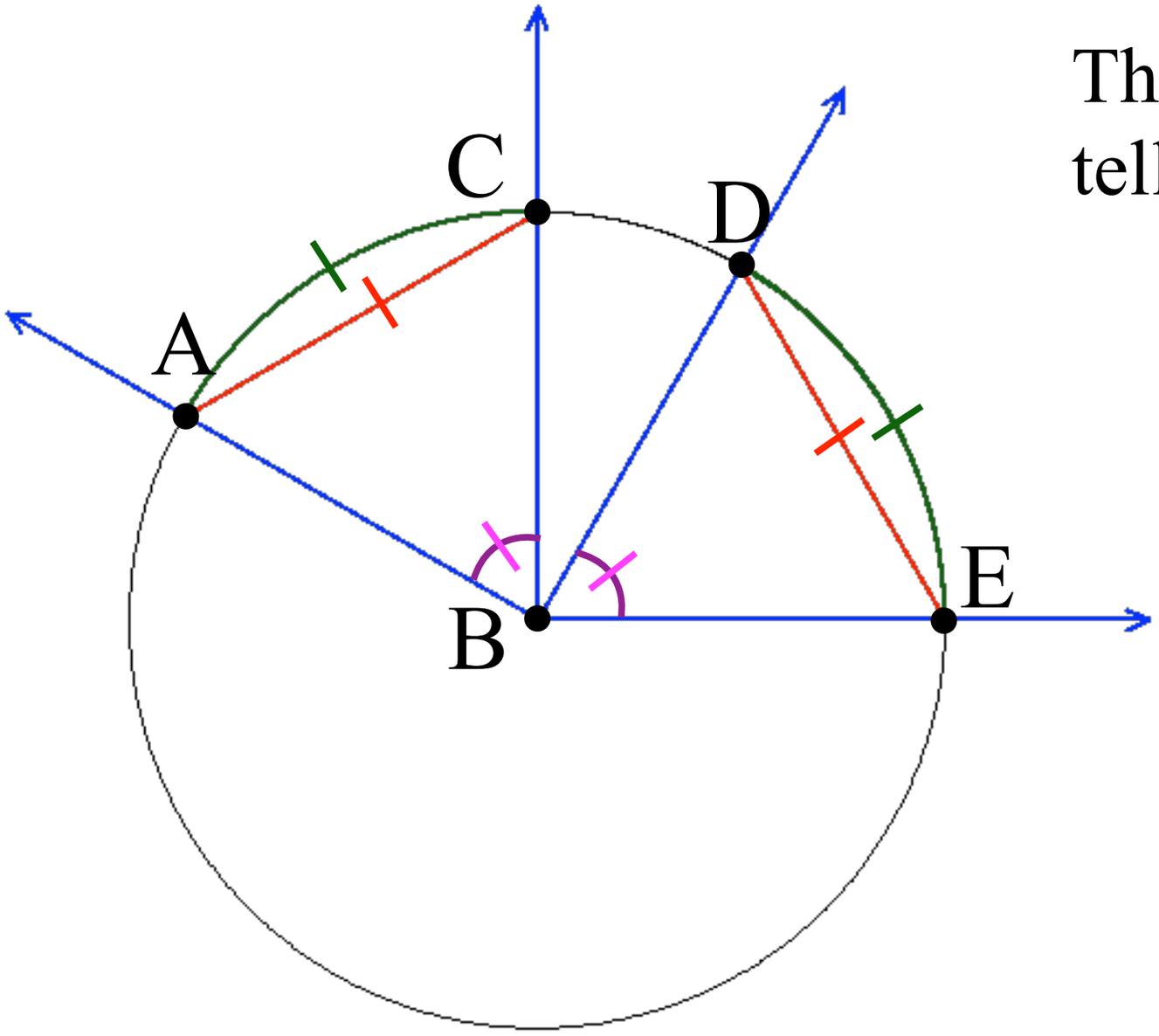


$m\overset{\frown}{AC}$ is the measure of the minor arc of central angle $m\angle ABC$

$$m\overset{\frown}{AC} = m\angle ABC$$

Having the point D helps us note the difference between the minor arc $m\overset{\frown}{AC}$ and the major arc $m\overset{\frown}{ADC}$

It is considered the *major arc* because it lies *outside* of the central angle $m\angle ABC$ while the *minor arc* lies *inside* the central angle



Theorem 12-2-2 (pg 803) simply tells us

Congruent Central Angles

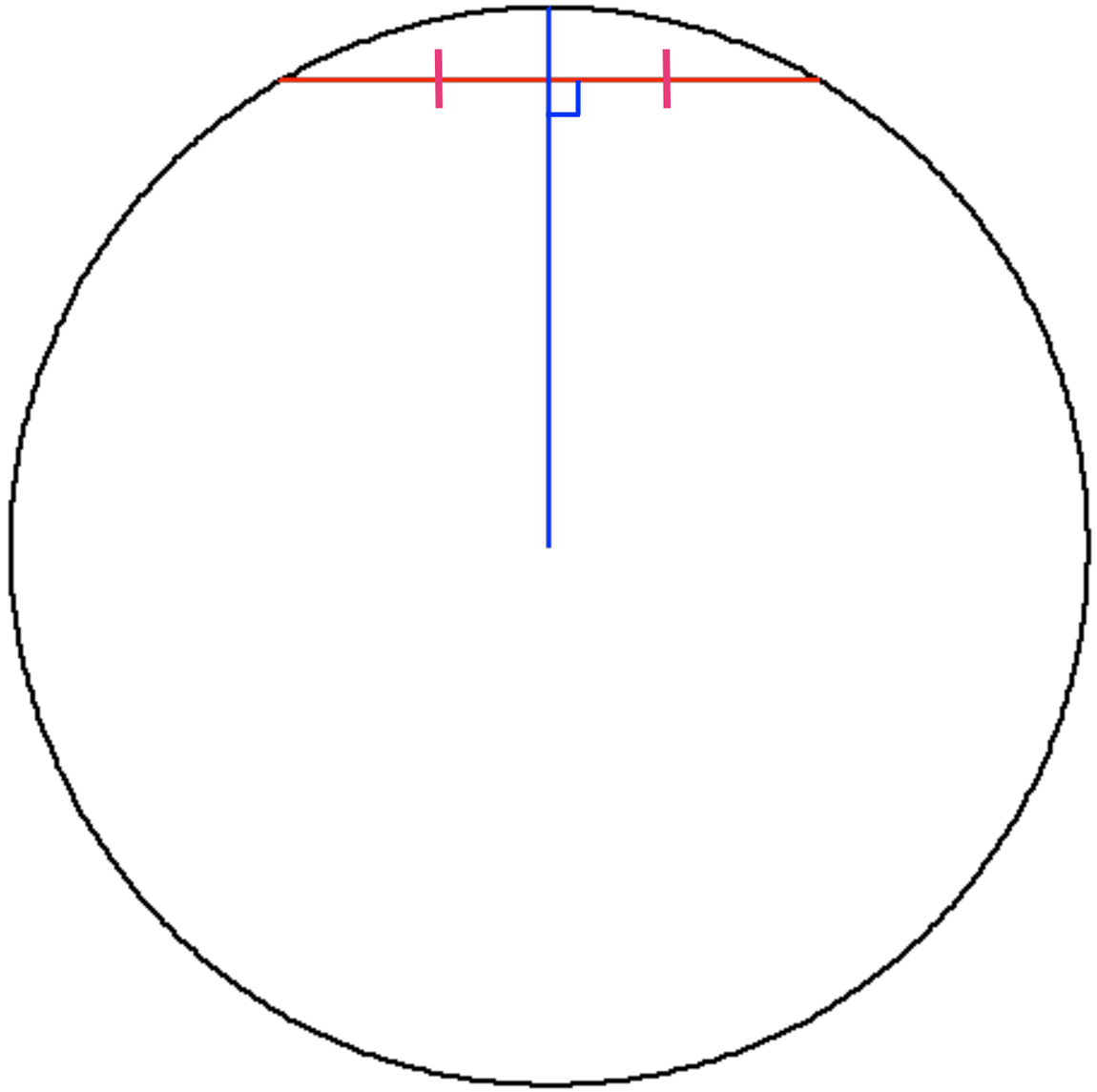
$$\angle ABC \approx \angle DBE$$

mean Congruent Chords

$$\overline{AC} \approx \overline{DE}$$

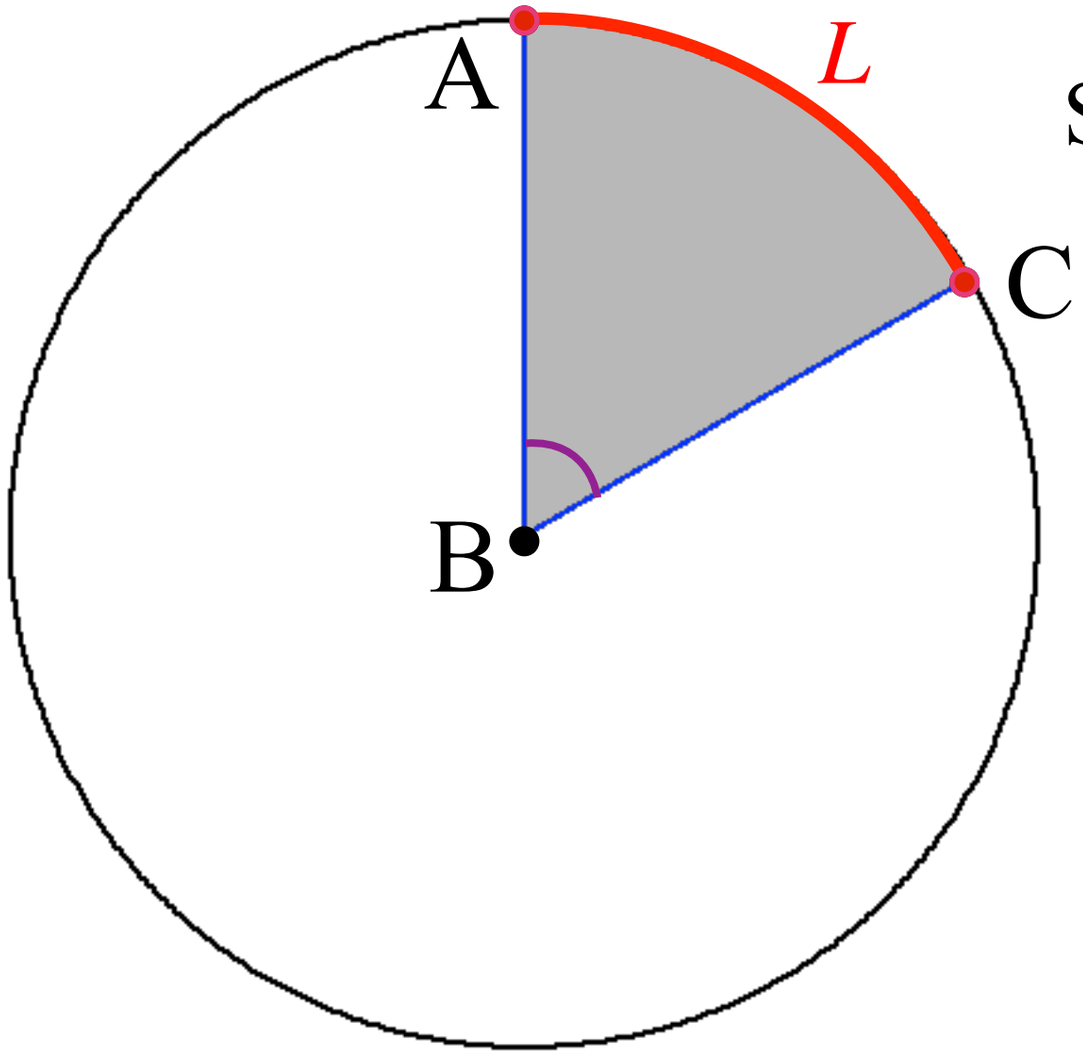
mean Congruent Arcs

$$\overset{\frown}{AC} \approx \overset{\frown}{DE}$$



A radius or diameter of a circle is perpendicular to the chord ***iff*** it bisects the chord

How would we find the area of this sector?



Since the area of the full circle is

$$A = \pi r^2$$

We just need to find what fraction of the circle this sector is

To do this, we need the measure of the central angle $m\angle ABC$

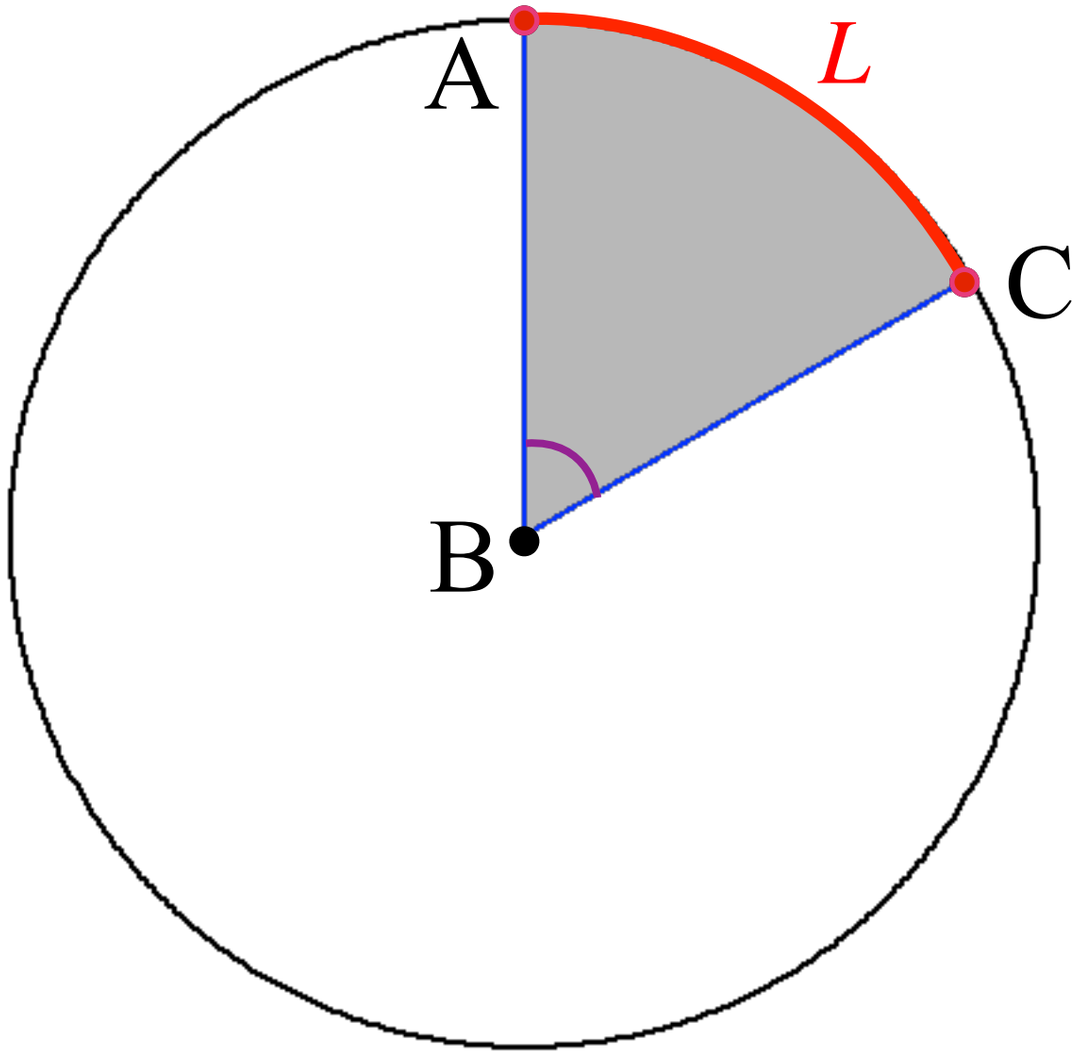
$$A_{sector} = \pi r^2 \frac{m\angle ABC}{360°}$$

Determines what portion of the circle is represented by the angle

In the same way we can find the distance along the arc $\overset{\frown}{AC}$ by multiplying the same fraction by circumference

$$L = 2\pi r \frac{m\angle ABC}{360°}$$

How would we find the area of this sector?



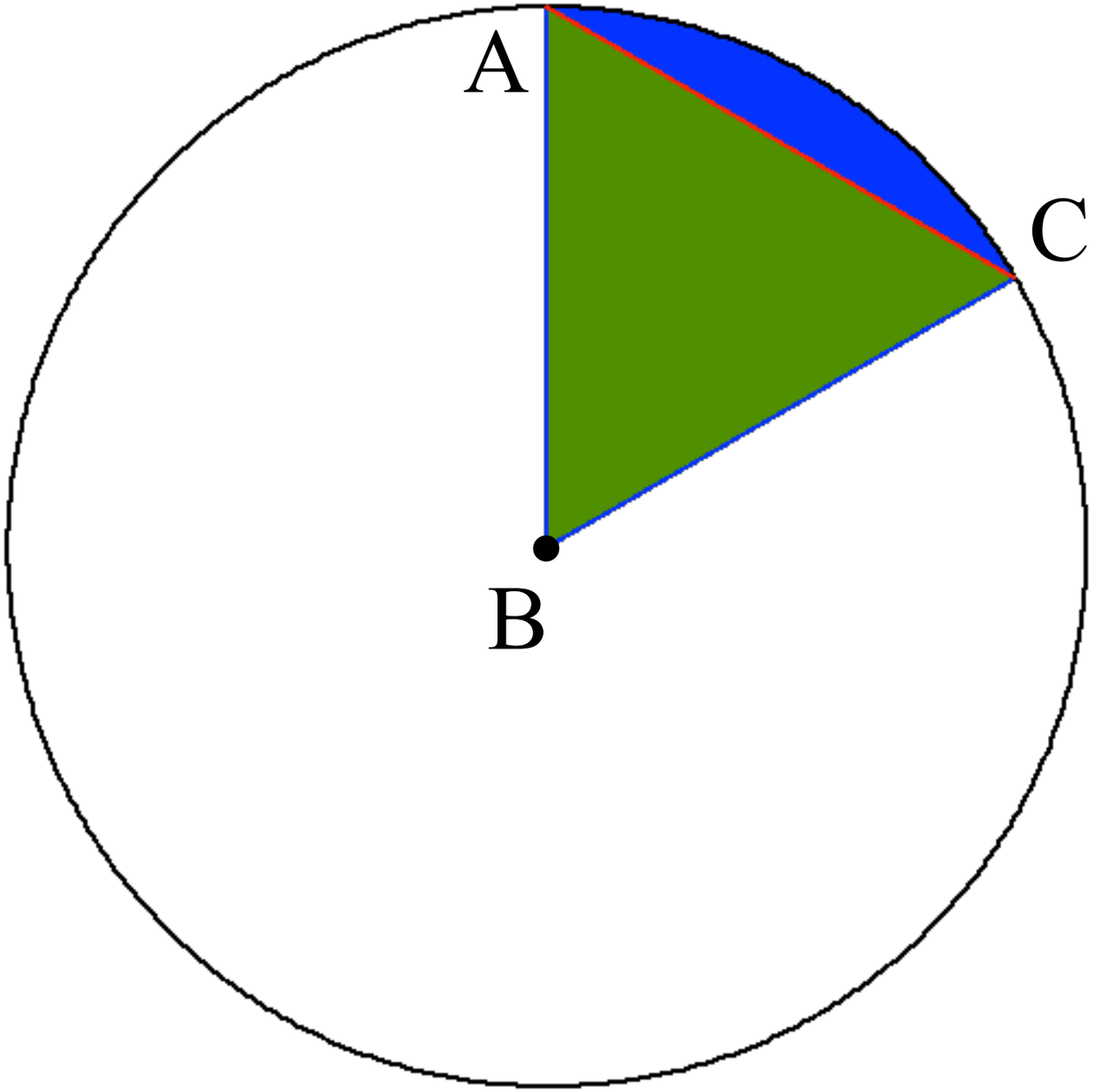
$m\angle ABC = 72°$

The radius of the circle is 4 cm

Find the area of the sector ABC and arc length $\overset{\frown}{AC}$

$$A_{sector} = \pi 4^2 \frac{72°}{360°} = \pi 4^2 \frac{1}{5} = \frac{16\pi}{5} \text{ cm}^2$$

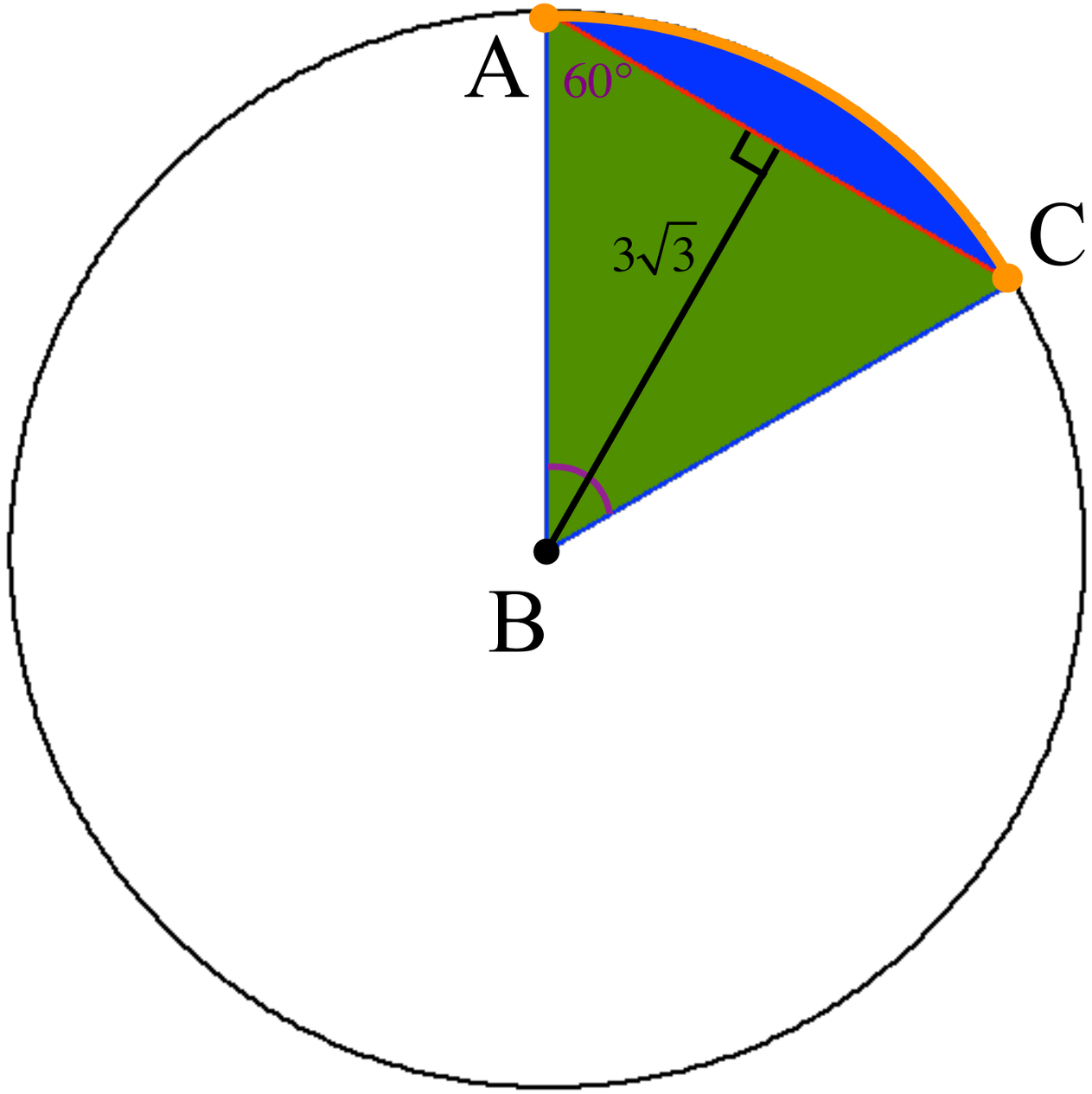
$$L = 2\pi 4 \frac{72°}{360°} = \frac{8\pi}{5} cm$$



We can also now find the area of both the triangle and the segment between it and the arc

$$A_{segment} = A_{sector} - A_{triangle}$$

Note that the triangle will always be at least isosceles so as long as we have the central angle we can find everything else



We can also now find the area of both the triangle and the segment between it and the arc

$$A_{segment} = A_{sector} - A_{triangle}$$

$$m\angle ABC = 60°$$

The radius of the circle is 6 cm

Find the area of the segment ACB and arc length $\overset{\frown}{AC}$

$$A_{sector} = \pi 6^2 \frac{60°}{360°} = 6\pi \longrightarrow A_{segment} = 6\pi - 9\sqrt{3}\ cm^2$$

$$A_{triangle} = \frac{1}{2}bh = \frac{1}{2}6\left(3\sqrt{3}\right) = 9\sqrt{3}$$

Since the central angle is 60º the triangle is equilateral

Arc Length

$$L = 2\pi 6\frac{60°}{360°} = 2\pi\ cm$$