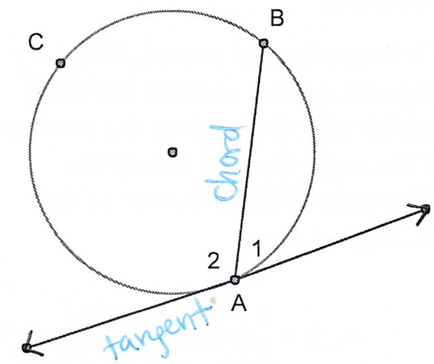
12-5: Angle Relationships in Circles

Theorem:

If a tangent and a chord (or secant) ***intersect*** at a point ***on*** a circle (point of tangency), then the measure of the angle formed is ***half*** the measure of its intercepted arc.

$$m\angle 1=\frac{1}{2}\left(m\overset{\frown}{AB}\right) \qquad m\angle 2=\frac{1}{2}\left(m\overset{\frown}{ACB}\right)$$

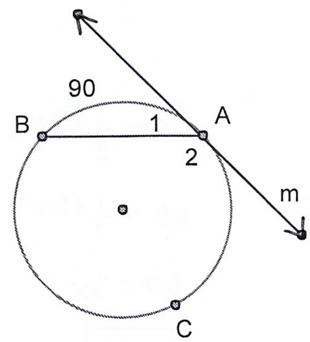


EX 1) Line m is tangent to the circle. Find the following:

a) $m\angle 1=$ 45°

b) $m\overset{\frown}{ACB}=$ 270°

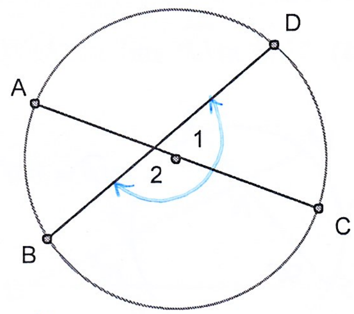
c) $m\angle 2=$ 135°



Theorem:

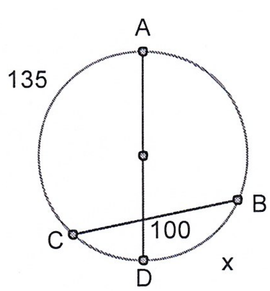
If two chords (or secants) ***intersect inside*** a circle, then the measure of each angle formed is ***half*** the ***sum*** of the measures of the arcs that are intercepted by the angle and its vertical angle.

$$m\angle 1=\frac{1}{2}\left(m\overset{\frown}{CD}+m\overset{\frown}{AB}\right) \qquad m\angle 2=\frac{1}{2}\left(m\overset{\frown}{BC}+m\overset{\frown}{AD}\right)$$



* Recall: $m\angle 1+m\angle 2=180°$ by L.P.T.

EX 2) Find the value of x.



$100=\frac{1}{2}(x+135)$

$200=x+135$

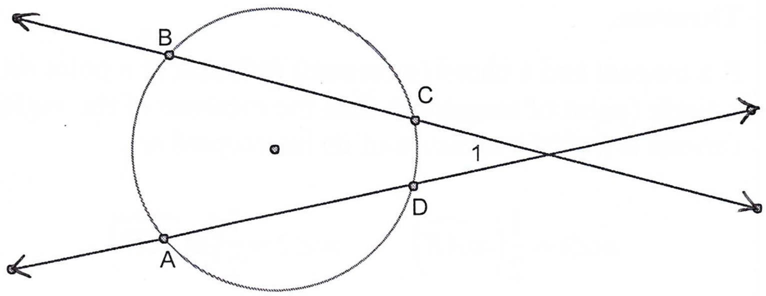
$\boxed{x=65}$

Theorem:

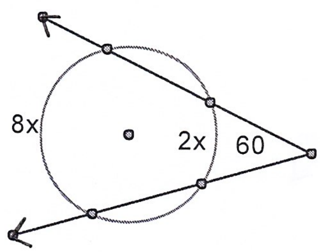
If a tangent and a secant, two tangents, or two secants ***intersect outside*** of a circle, then the measure of the angle formed is ***half*** the ***difference*** of the measure of the intercepted arcs.

$$m\angle 1 = \frac{1}{2}\left(m\overset{\frown}{AB}^{*} - m\overset{\frown}{CD}\right)$$

$= \frac{1}{2}$ (large arc − small arc)



EX 3) Solve for x.



$60 = \frac{1}{2}(8x - 2x)$

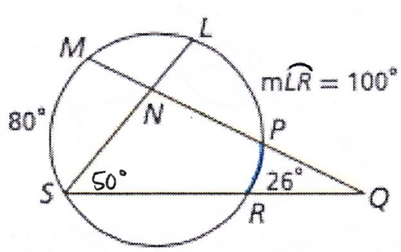
$60 = \frac{1}{2}(6x)$

$60 = 3x$

$\boxed{x = 20}$

<u>**Challenge:**</u>

EX 4) Find $m\overset{\frown}{LP}$ and $m\angle MNL$.



$\overset{\frown}{PR}$: $26 = \frac{1}{2}(80 - \overset{\frown}{PR})$

$52 = 80 - PR$

$\overset{\frown}{PR} = 28$

$\overset{\frown}{LR} = \overset{\frown}{LP} + \overset{\frown}{PR}$

$100 = \overset{\frown}{LP} + 28$

$\boxed{\overset{\frown}{LP} = 72°}$

$m\angle S = \frac{1}{2}\overset{\frown}{LR}$

$m\angle S = 50°$ $\therefore$ $m\angle SNQ = 104°$ ($\triangle$ sum)

$\boxed{m\angle MNL = 104°}$ (vertical $\angle$s)