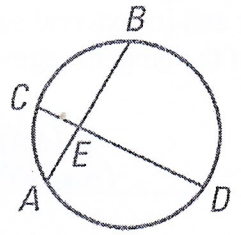
12-6: Segment Relationships in Circles

When two chords intersect inside a circle, each chord is divided into two segments called **segments of a chord**.

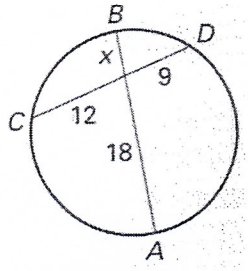
Theorem:

If two chords ***intersect inside*** a circle, then the product of the segment lengths of one chord is equal to the product of the segment lengths of the other chord.

$$EA \cdot EB = EC \cdot ED$$



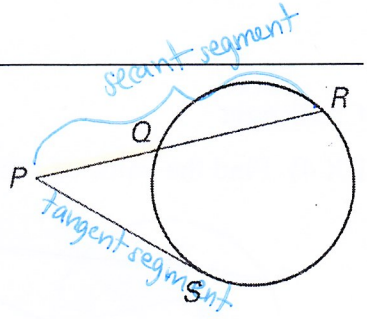
EX 1) Find the value of x.



$18x = 12 \cdot 9$

$\boxed{x = 6}$

In the figure, $\overline{PS}$ is a **tangent segment** because it is tangent to the circle at an endpoint (S). $\overline{PR}$ is a **secant segment** because one of the two intersection points with the circle is an endpoint (R). $\overline{PQ}$ is the **external segment** of $\overline{PR}$.

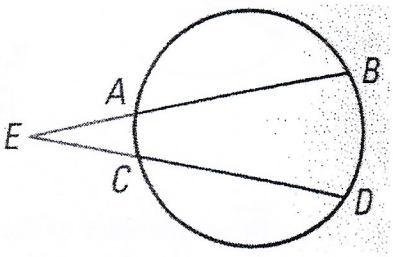


Theorem:

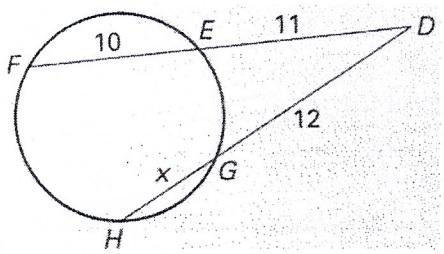
If two secant segments share the same endpoint outside a circle, then the product of the lengths of one secant segment and its external segment equals the product of the lengths of the other secant segment and its external segment.

$$EA \cdot EB = EC \cdot ED$$

external · whole = external · whole



EX 2) Find the value of x.



$11(10+11) = 12(12+x)$

$231 = 144 + 12x$

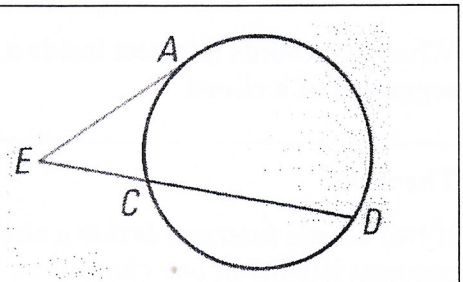
$\boxed{x = 7.25}$

Theorem:

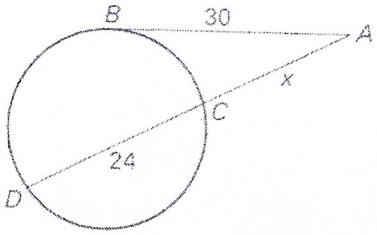
If a secant segment and a tangent segment share an endpoint outside a circle, then the product of the lengths of the secant segment and its external segment equals the square of the length of the tangent segment.

$$(EA)^2 = EC \cdot ED$$

(tangent)^2 = external · whole



EX 3) Find the value of x.



$30^2 = x(x+24)$

$x^2 + 24x - 900 = 0$

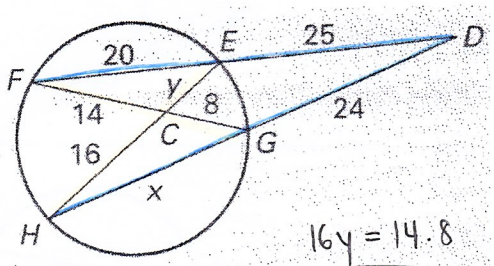
$x = \dfrac{-24 \pm \sqrt{24^2 - 4(1)(-900)}}{2(1)}$

$= \dfrac{-24 \pm \sqrt{4176}}{2}$

$x = \dfrac{-24 \pm 12\sqrt{29}}{2} = -12 \pm 6\sqrt{29} \therefore$ $\boxed{x = -12 + 6\sqrt{29} \approx 20.311}$

Challenge:

EX 4) Find the value of x and y.



$25(20+25) = 24(24+x)$

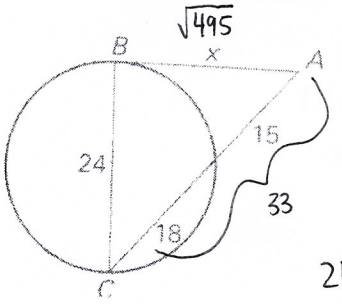
$1125 = 576 + 24x$

$\boxed{x = 22.875}$

$16y = 14 \cdot 8$

$\boxed{y = 7}$

EX 5) Is $\overline{BC}$ a diameter of the circle? (Hint: What do you recall about a radius intersecting a tangent at the point of tangency?) a radius + tangent are ⊥ at pt. of tangency



$x^2 = 15(15+18)$

$x = \pm\sqrt{495} = 3\sqrt{55}$

$24^2 \square \sqrt{495}^2 + 33^2$

$576 \square 495 + 1089$

$576 \neq 1584 \therefore$ $\overline{BC}$ is <u>not</u> a diameter b/c $\triangle CBA$ is not rt.