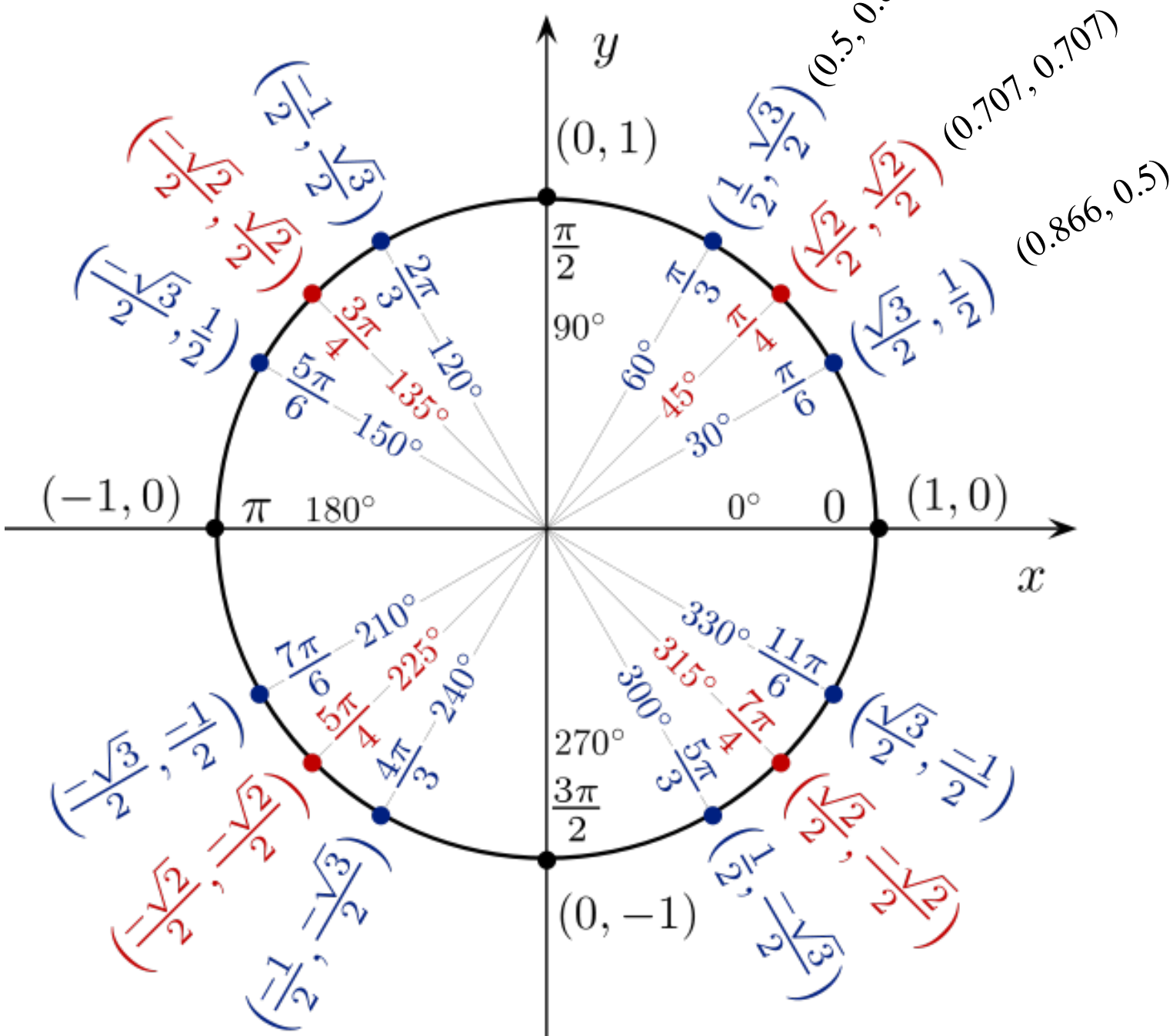
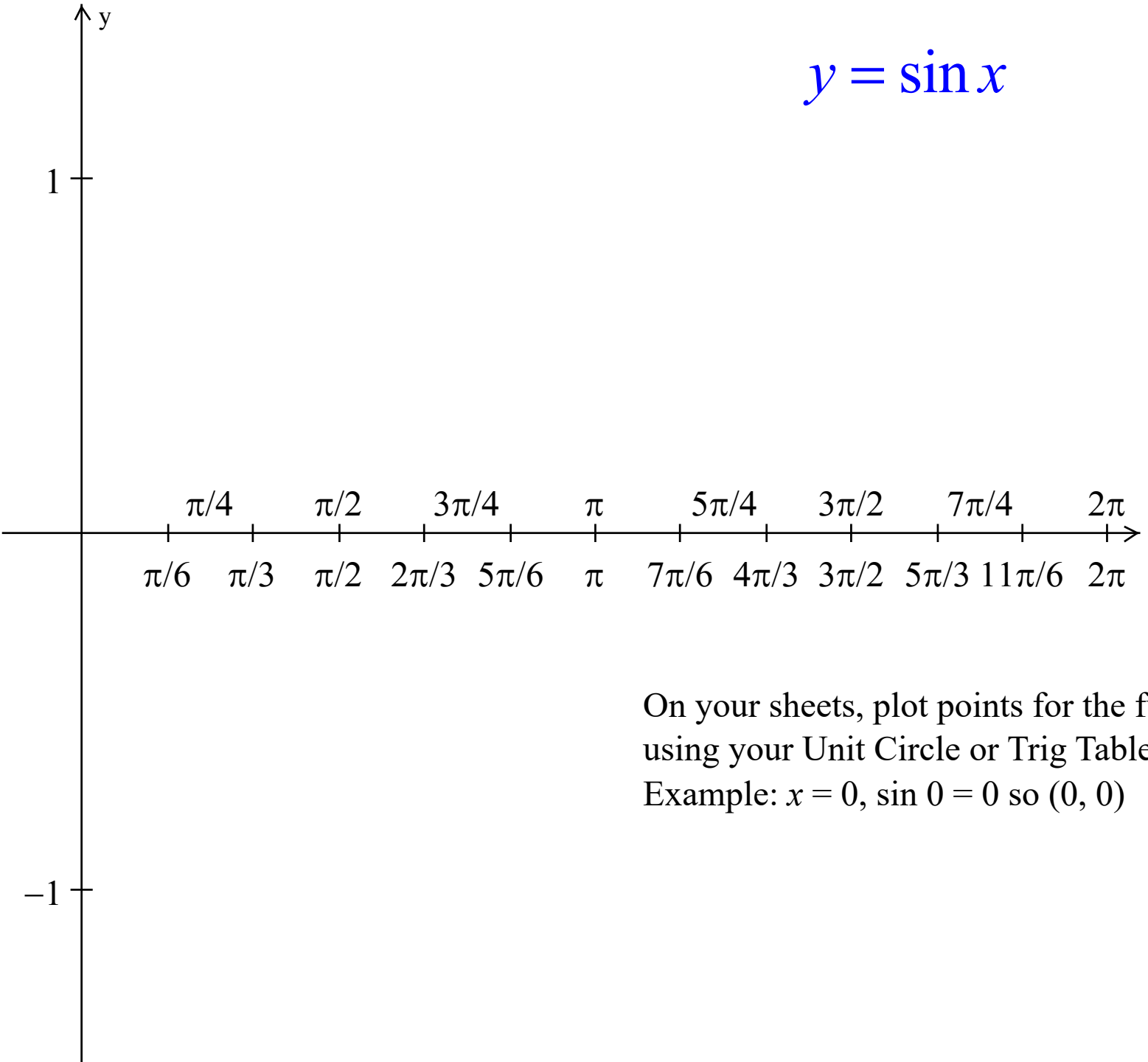
Graphing Sine and Cosine


y
x
(0, 1)
(1, 0)
(−1, 0)
(0, −1)
π/2
90°
π/3
60°
π/4
45°
π/6
30°
0°
0
π
180°
2π/3
120°
3π/4
135°
5π/6
150°
7π/6
210°
5π/4
225°
4π/3
240°
3π/2
270°
5π/3
300°
7π/4
315°
11π/6
330°
(1/2, √3/2) (0.5, 0.866)
(√2/2, √2/2) (0.707, 0.707)
(√3/2, 1/2) (0.866, 0.5)
(−1/2, √3/2)
(−√2/2, √2/2)
(−√3/2, 1/2)
(−√3/2, −1/2)
(−√2/2, −√2/2)
(−1/2, −√3/2)
(1/2, −√3/2)
(√2/2, −√2/2)
(√3/2, −1/2)



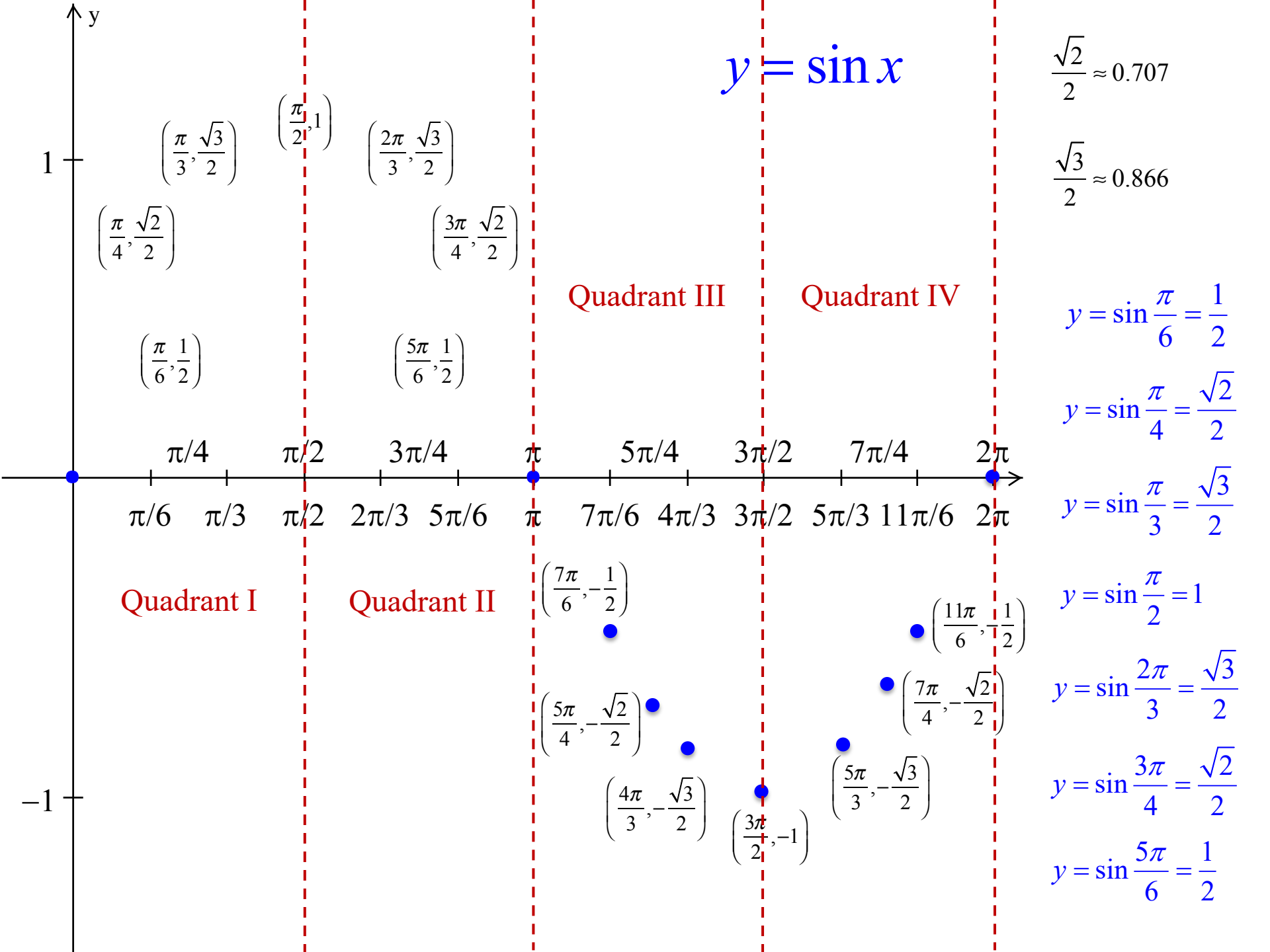
$y = \sin x$

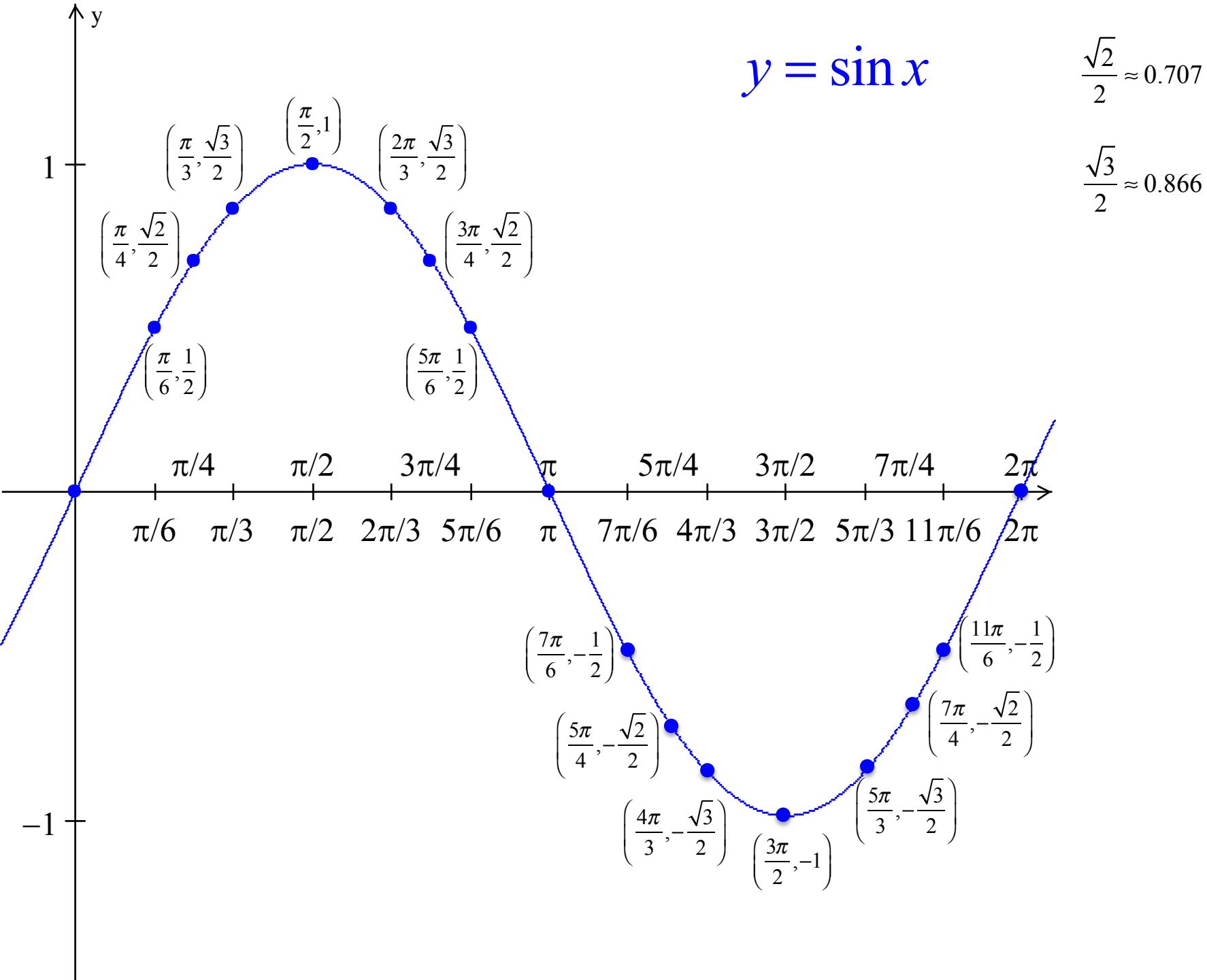
$\frac{\sqrt{2}}{2} \approx 0.707$

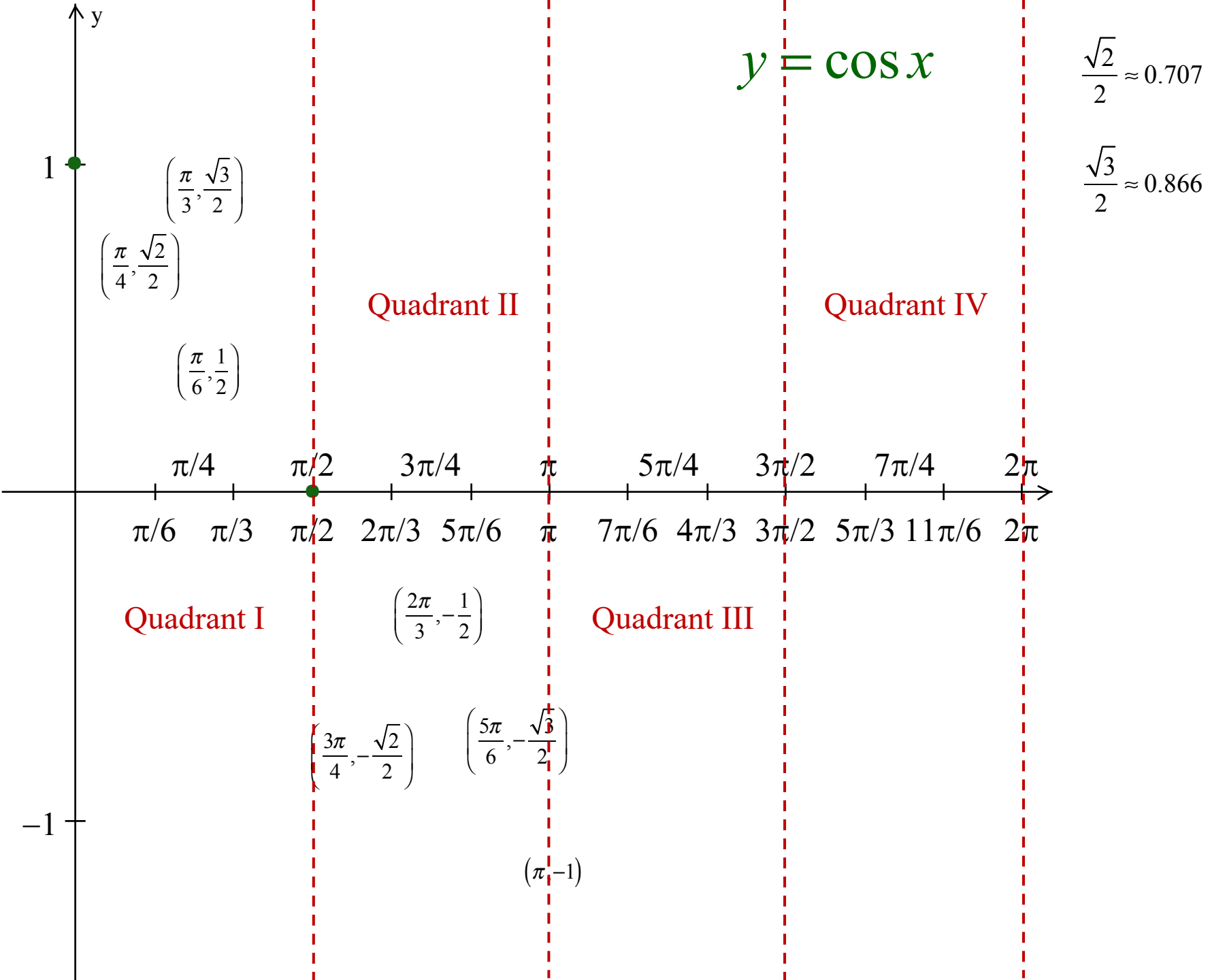
$\frac{\sqrt{3}}{2} \approx 0.866$

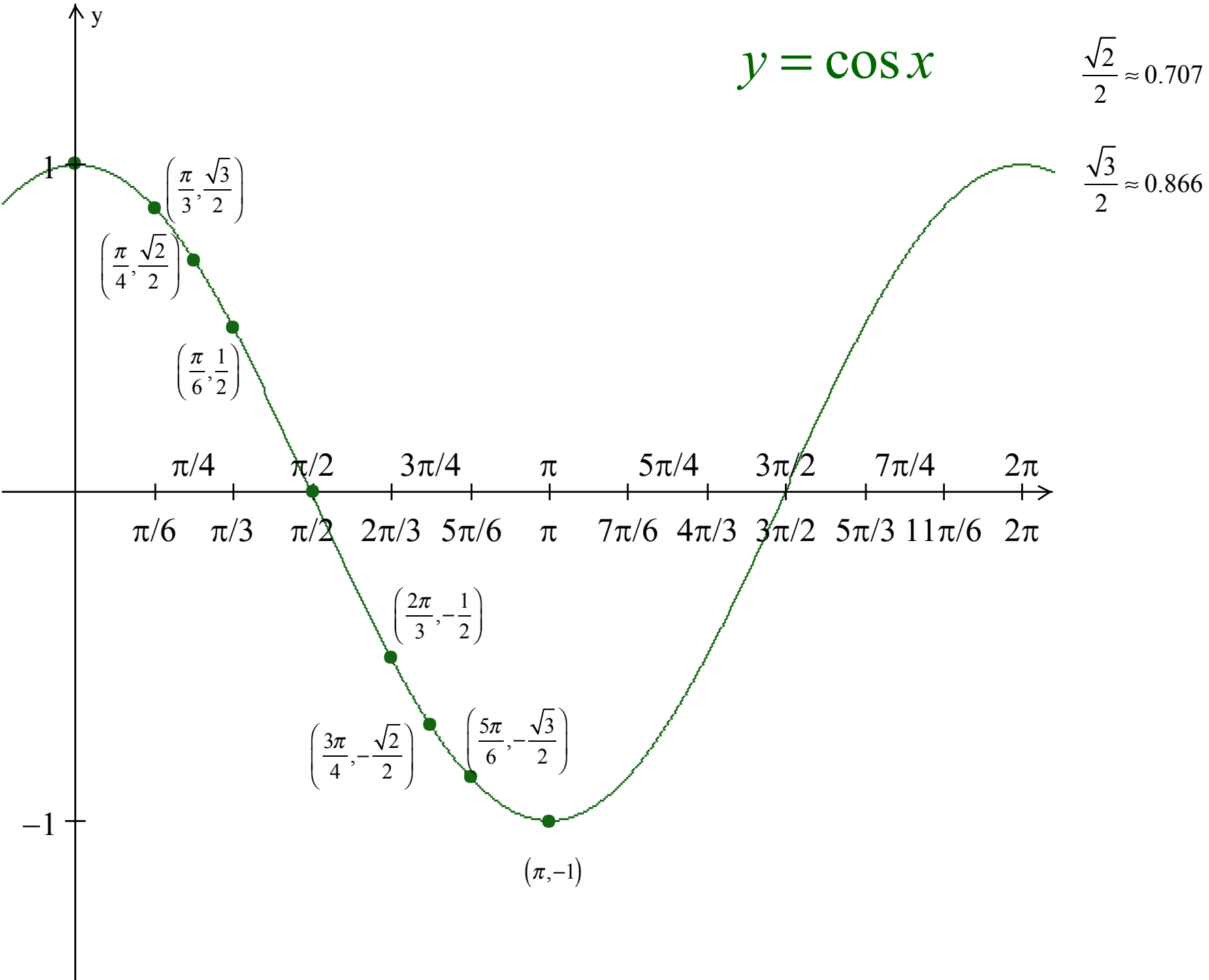
On your sheets, plot points for the function using your Unit Circle or Trig Table

Example: $x = 0$, $\sin 0 = 0$ so $(0, 0)$


y
$y = \sin x$
$\frac{\sqrt{2}}{2} \approx 0.707$
$\frac{\sqrt{3}}{2} \approx 0.866$
1
−1
$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\pi}{2}, 1\right)$
$\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{3\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
Quadrant III
Quadrant IV
π/4 π/2 3π/4 π 5π/4 3π/2 7π/4 2π
π/6 π/3 π/2 2π/3 5π/6 π 7π/6 4π/3 3π/2 5π/3 11π/6 2π
Quadrant I
Quadrant II
$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$
$\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{7\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{3\pi}{2}, -1\right)$
$y = \sin\frac{\pi}{6} = \frac{1}{2}$
$y = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$
$y = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
$y = \sin\frac{\pi}{2} = 1$
$y = \sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$
$y = \sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$
$y = \sin\frac{5\pi}{6} = \frac{1}{2}$


y
$y = \sin x$
$\frac{\sqrt{2}}{2} \approx 0.707$
$\frac{\sqrt{3}}{2} \approx 0.866$
1
−1
$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\pi}{2}, 1\right)$
$\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{3\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{5\pi}{6}, \frac{1}{2}\right)$
π/4 π/2 3π/4 π 5π/4 3π/2 7π/4 2π
π/6 π/3 π/2 2π/3 5π/6 π 7π/6 4π/3 3π/2 5π/3 11π/6 2π
$\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$
$\left(\frac{5\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{4\pi}{3}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{3\pi}{2}, -1\right)$
$\left(\frac{5\pi}{3}, -\frac{\sqrt{3}}{2}\right)$
$\left(\frac{7\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{11\pi}{6}, -\frac{1}{2}\right)$


y
$y = \cos x$
$\frac{\sqrt{2}}{2} \approx 0.707$
$\frac{\sqrt{3}}{2} \approx 0.866$
1
−1
$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
Quadrant II
Quadrant IV
Quadrant I
Quadrant III
π/4 π/2 3π/4 π 5π/4 3π/2 7π/4 2π
π/6 π/3 π/2 2π/3 5π/6 π 7π/6 4π/3 3π/2 5π/3 11π/6 2π
$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
$\left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right)$
$(\pi, -1)$


y
$y = \cos x$
$\frac{\sqrt{2}}{2} \approx 0.707$
$\frac{\sqrt{3}}{2} \approx 0.866$
1
−1
$\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$
$\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$
$\left(\frac{\pi}{6}, \frac{1}{2}\right)$
π/4 π/2 3π/4 π 5π/4 3π/2 7π/4 2π
π/6 π/3 π/2 2π/3 5π/6 π 7π/6 4π/3 3π/2 5π/3 11π/6 2π
$\left(\frac{2\pi}{3}, -\frac{1}{2}\right)$
$\left(\frac{3\pi}{4}, -\frac{\sqrt{2}}{2}\right)$
$\left(\frac{5\pi}{6}, -\frac{\sqrt{3}}{2}\right)$
$(\pi, -1)$

General Equation: $y = k + A \bullet f\left[B\left(x - h\right)\right]$

where $|A|$ = amplitude – vertical distance from extreme value to the sinusoidal axis

$|B|$ = the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|}$ = period**

$\frac{1}{|period|}$ = frequency

k = vertical shift

h = horizontal shift (a.k.a. phase shift)

and f = the sinusoidal function

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$$y = k + A\sin[B(x - h)]$$

$$y = k + A\cos[B(x - h)]$$

Same as sine function only shifted to the right by h

or the left if h is negative

General Equation: $y = k + A \bullet f\left[B\left(x - h\right)\right]$

where $|A|$ = amplitude – vertical distance from extreme value to the sinusoidal axis

$|B|$ = the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|}$ = period**

$\frac{1}{|period|}$ = frequency

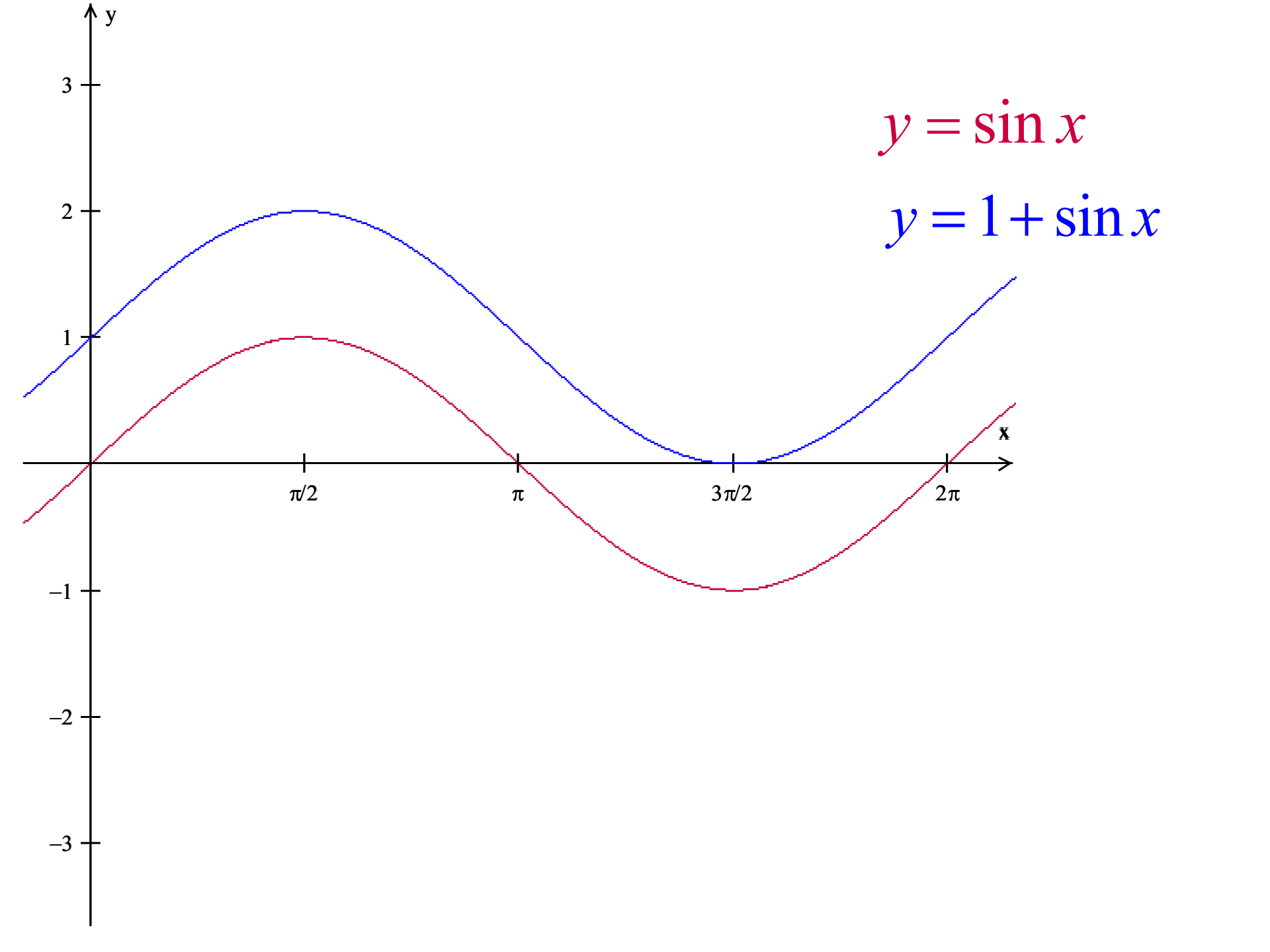
k = vertical shift

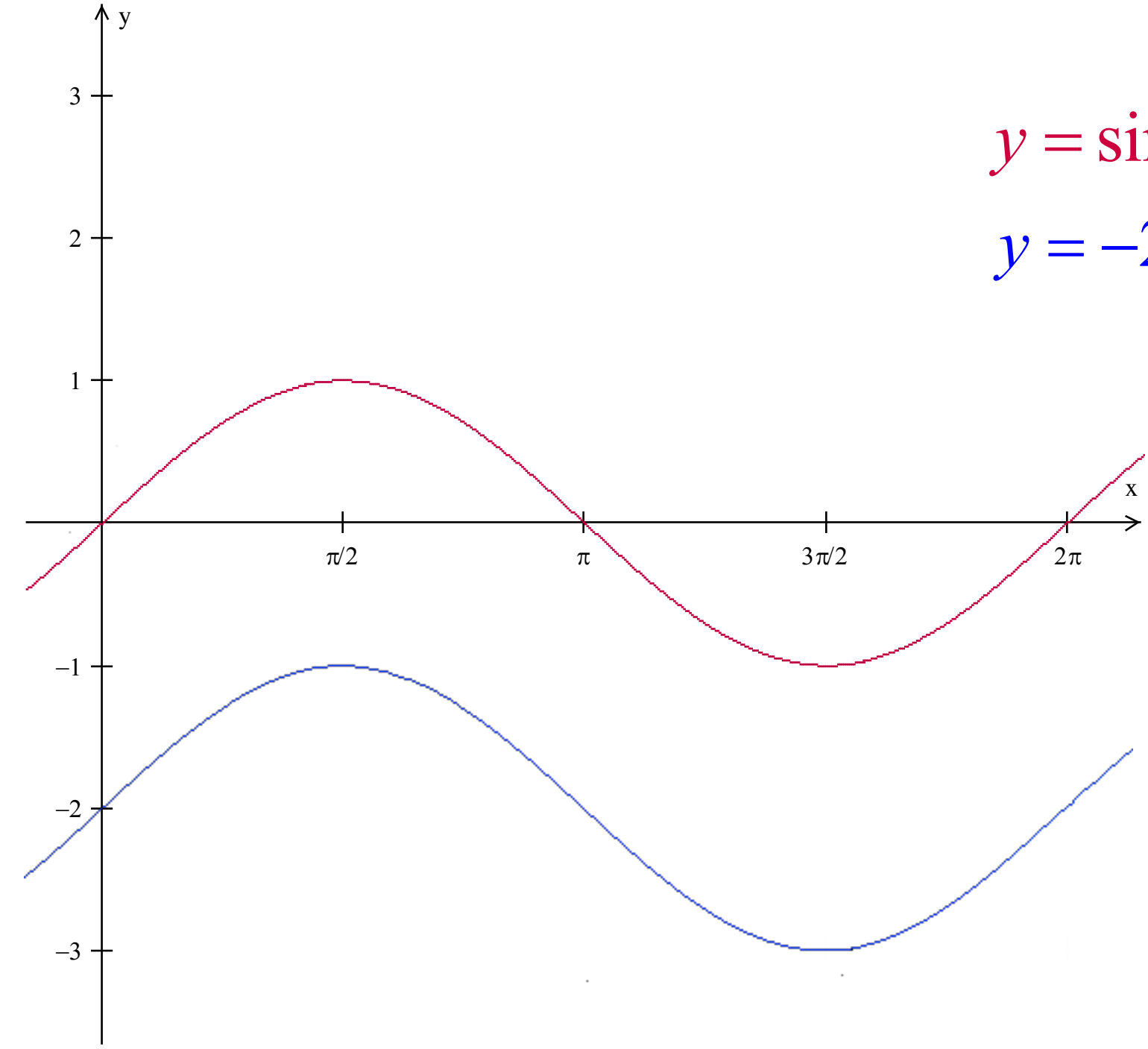
h = horizontal shift (a.k.a. phase shift)

and f = the sinusoidal function

$$y = k + A\sin[B(x - h)]$$

Moves the function up or down


y
3
2
1
–1
–2
–3
x
π/2
π
3π/2
2π
$y = \sin x$
$y = 1 + \sin x$



$y = \sin x$

$y = -2 + \sin x$

General Equation: $y = k + A \bullet f\left[B\left(x - h\right)\right]$

where $|A|$ = amplitude – vertical distance from extreme value to the sinusoidal axis

$|B|$ = the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|}$ = period**

$\frac{1}{|period|}$ = frequency

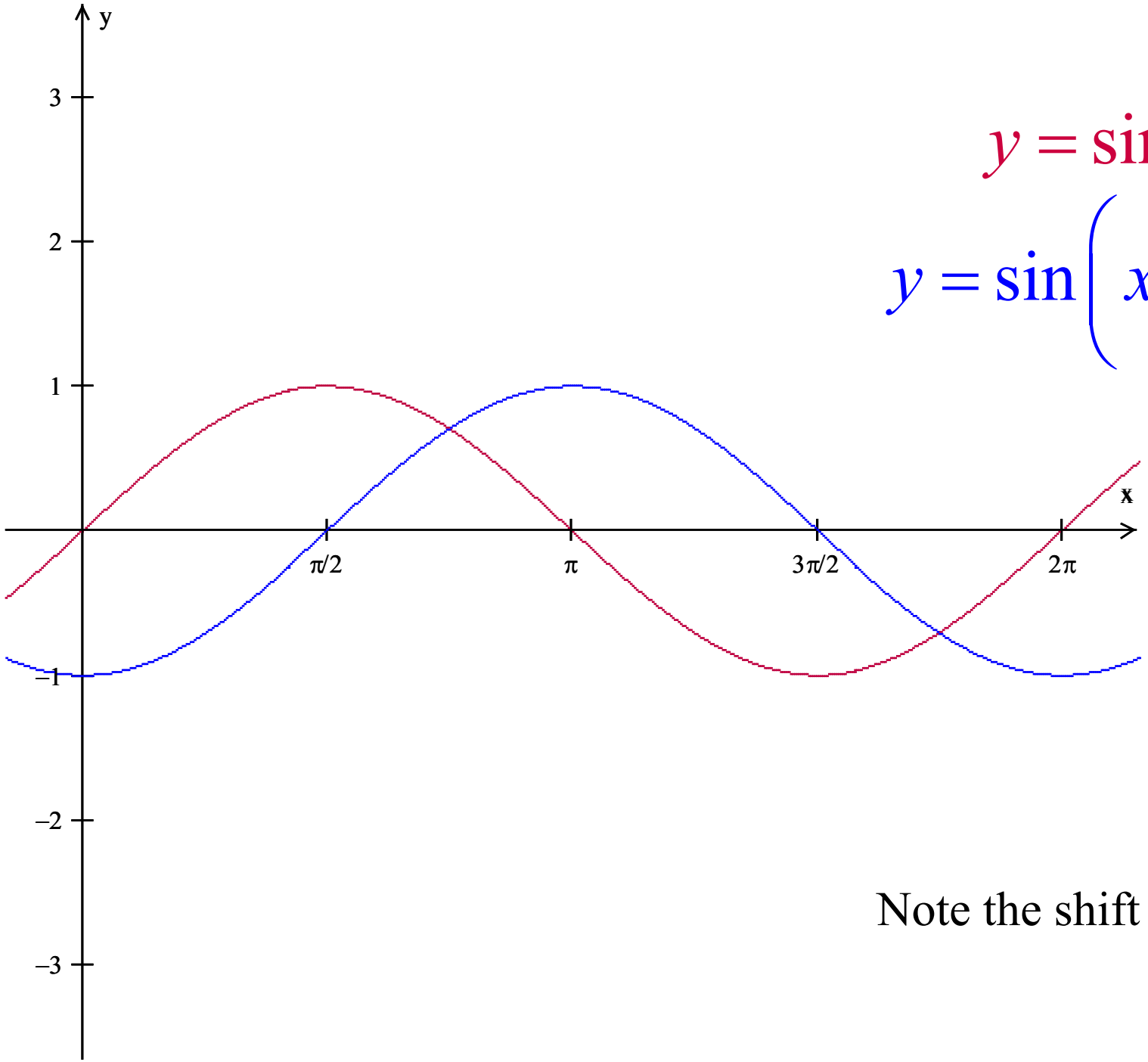
k = vertical shift

h = horizontal shift (a.k.a. phase shift)

and f = the sinusoidal function

$$y = k + A\sin[B(x - h)]$$

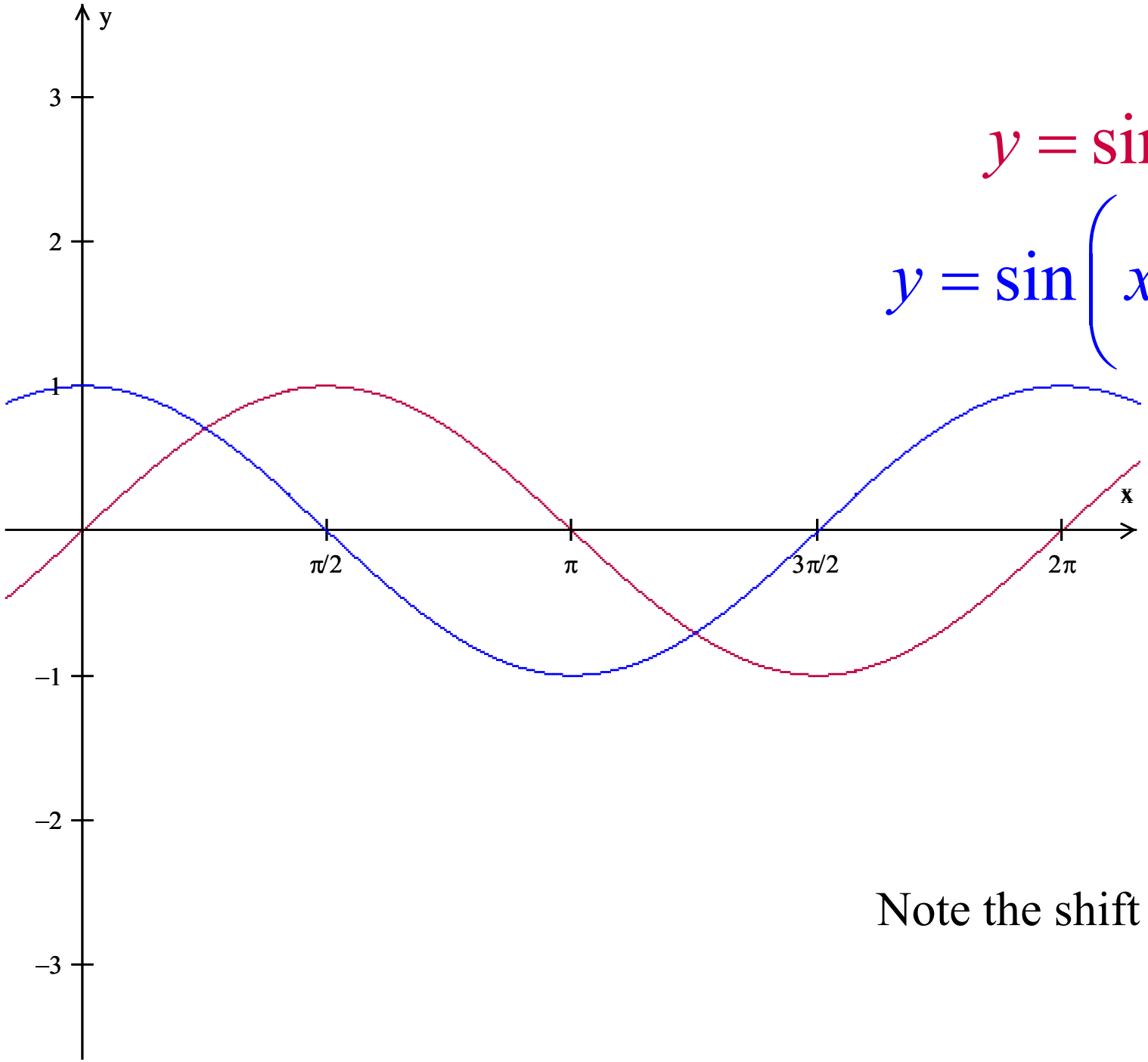
Moves the function left or right



$$y = \sin x$$

$$y = \sin\left(x - \frac{\pi}{2}\right)$$

Note the shift to the right



$$y = \sin x$$

$$y = \sin\left(x + \frac{\pi}{2}\right)$$

Note the shift to the left

General Equation: $y = k + A \cdot f\left[B(x-h)\right]$

where $|A|$ = amplitude – vertical distance from extreme value to the sinusoidal axis

$|B|$ = the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|}$ = period**

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$\frac{1}{|period|}$ = frequency

k = vertical shift

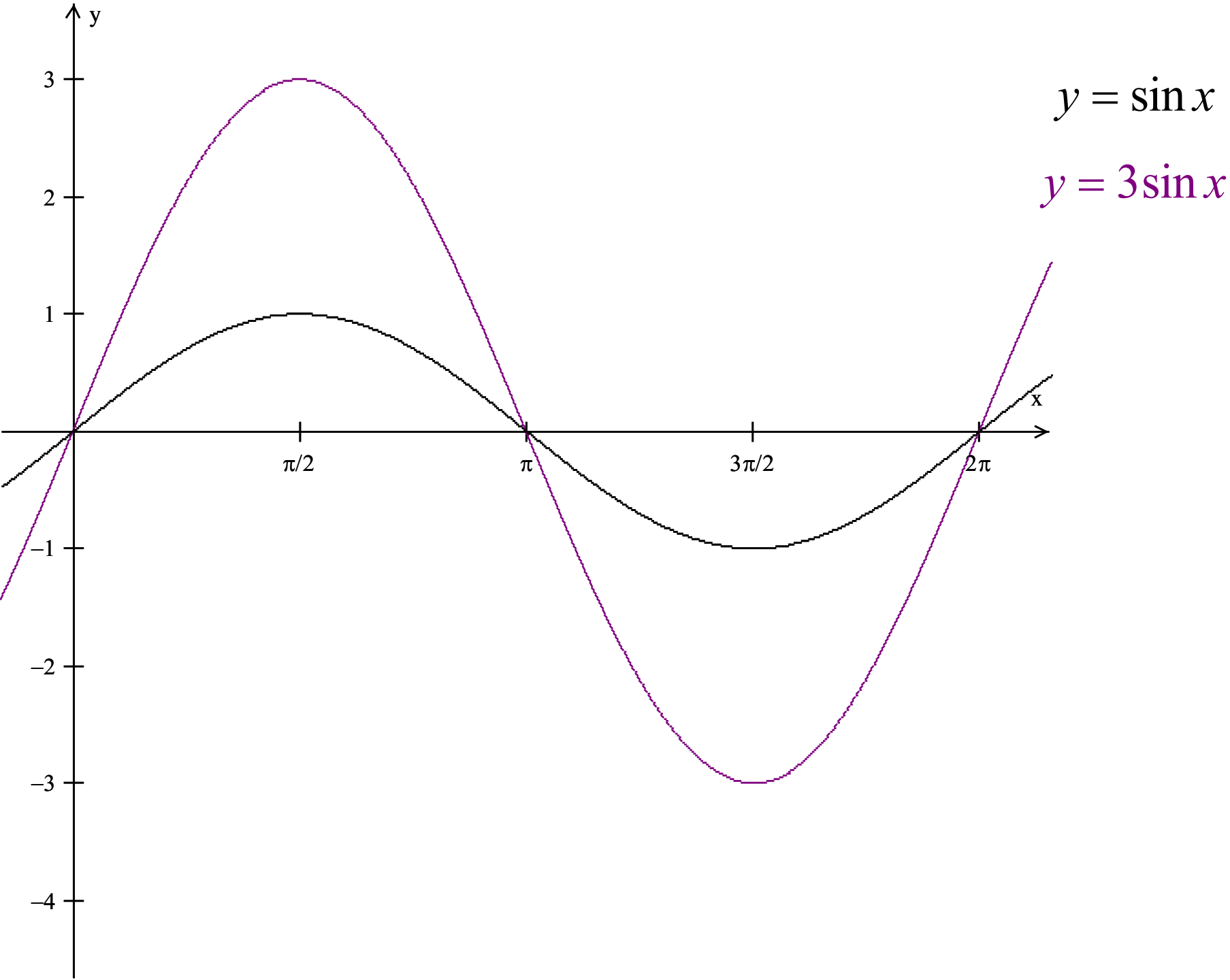
h = horizontal shift (a.k.a. phase shift)

and f = the sinusoidal function

$$y = k + A\sin[B(x-h)]$$

Stretches the function vertically

like this…


y
3
2
1
−1
−2
−3
−4
x
π/2
π
3π/2
2π
$y = \sin x$
$y = 3\sin x$

General Equation: $y = k + A \cdot f\left[B(x-h)\right]$

where $|A|$ = amplitude – vertical distance from extreme value to the sinusoidal axis

$|B|$ = the number of cycles between 0 and 2π*

$\frac{2\pi}{|B|}$ = period**

$\frac{1}{|period|}$ = frequency

k = vertical shift

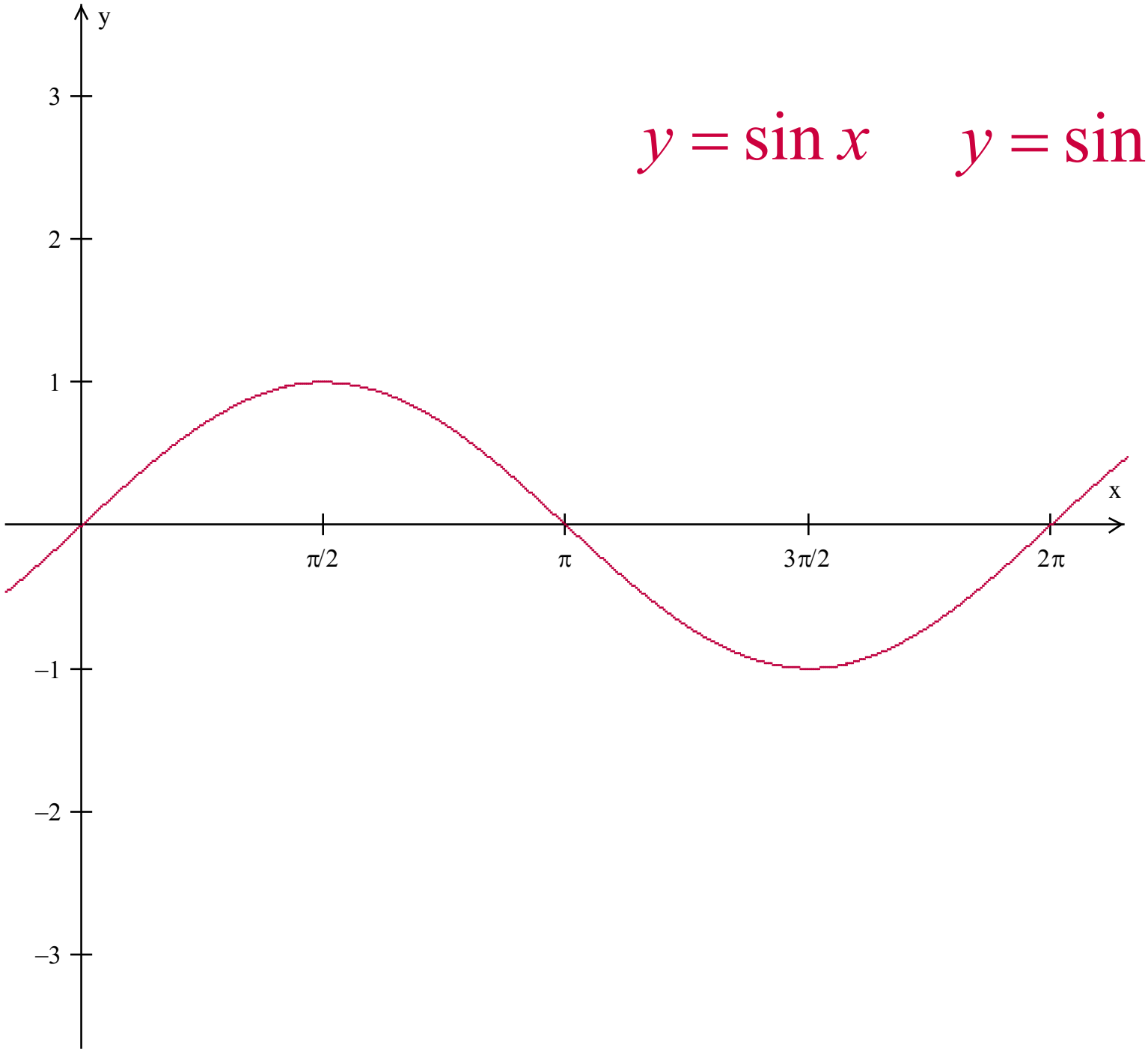
h = horizontal shift (a.k.a. phase shift)

and f = the sinusoidal function

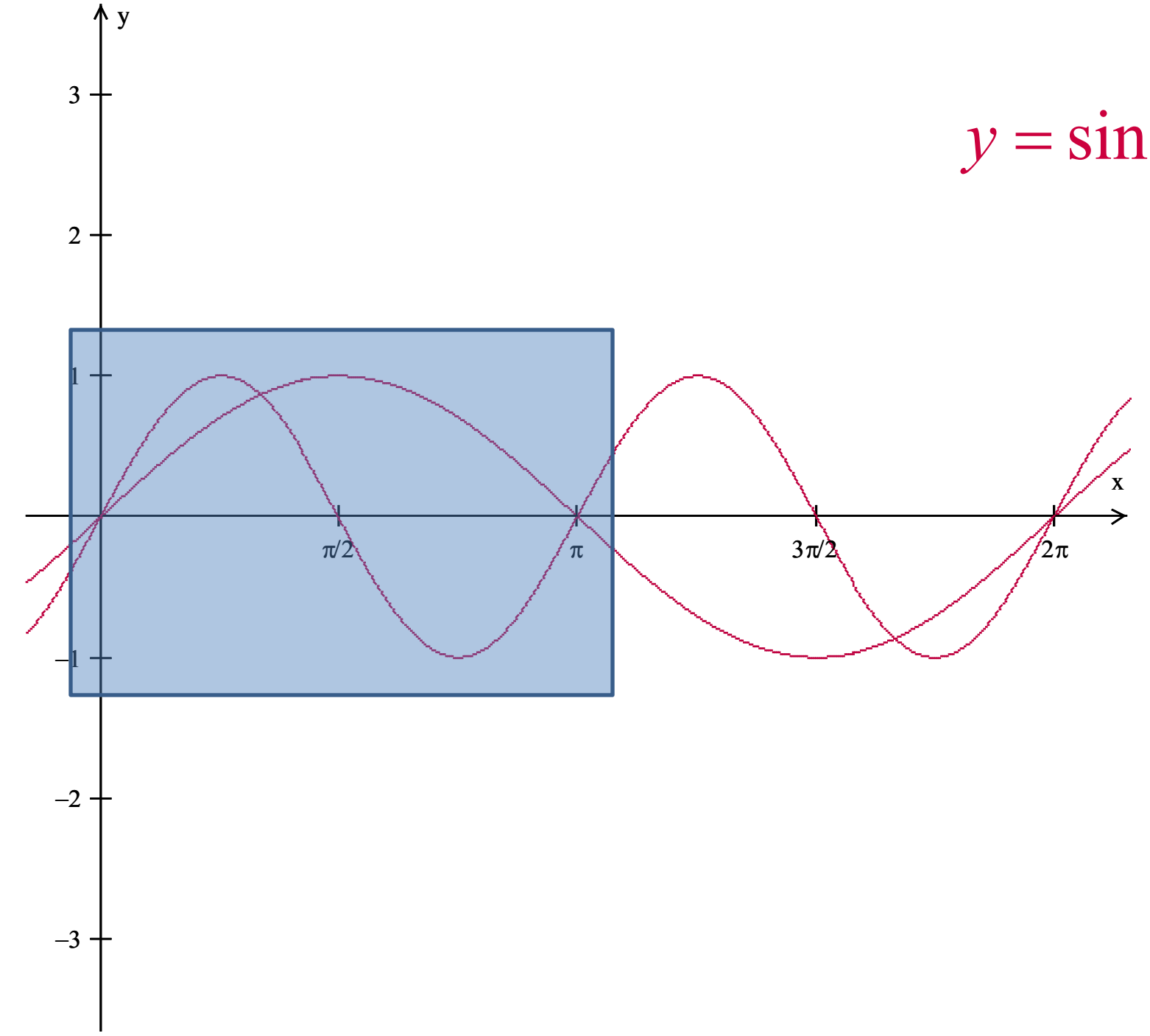
$$y = k + A\sin[B(x-h)]$$

Stretches or shrinks the function horizontally

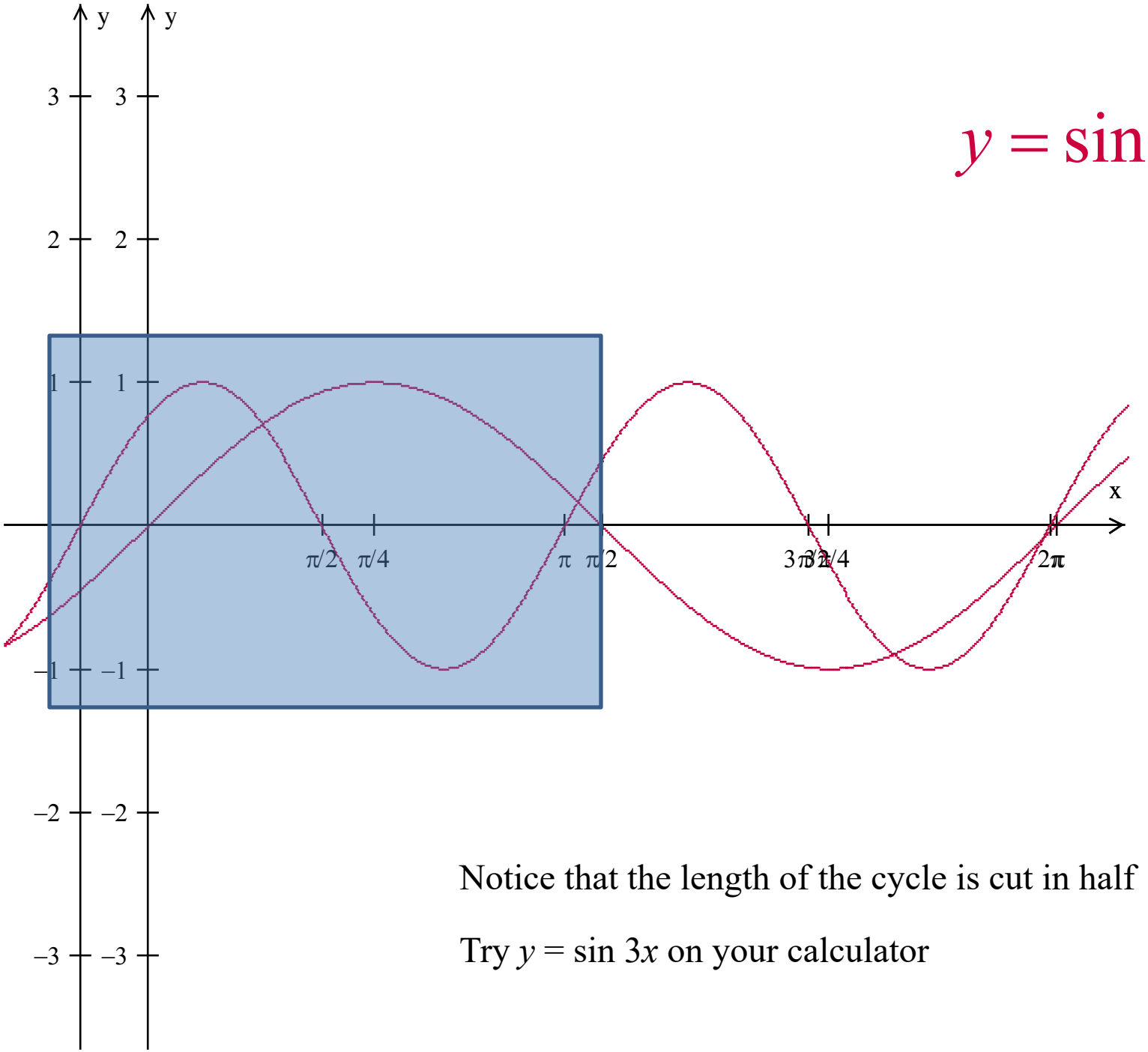
like pushing in or stretching out a spring



$y = \sin x$ $y = \sin 2x$?



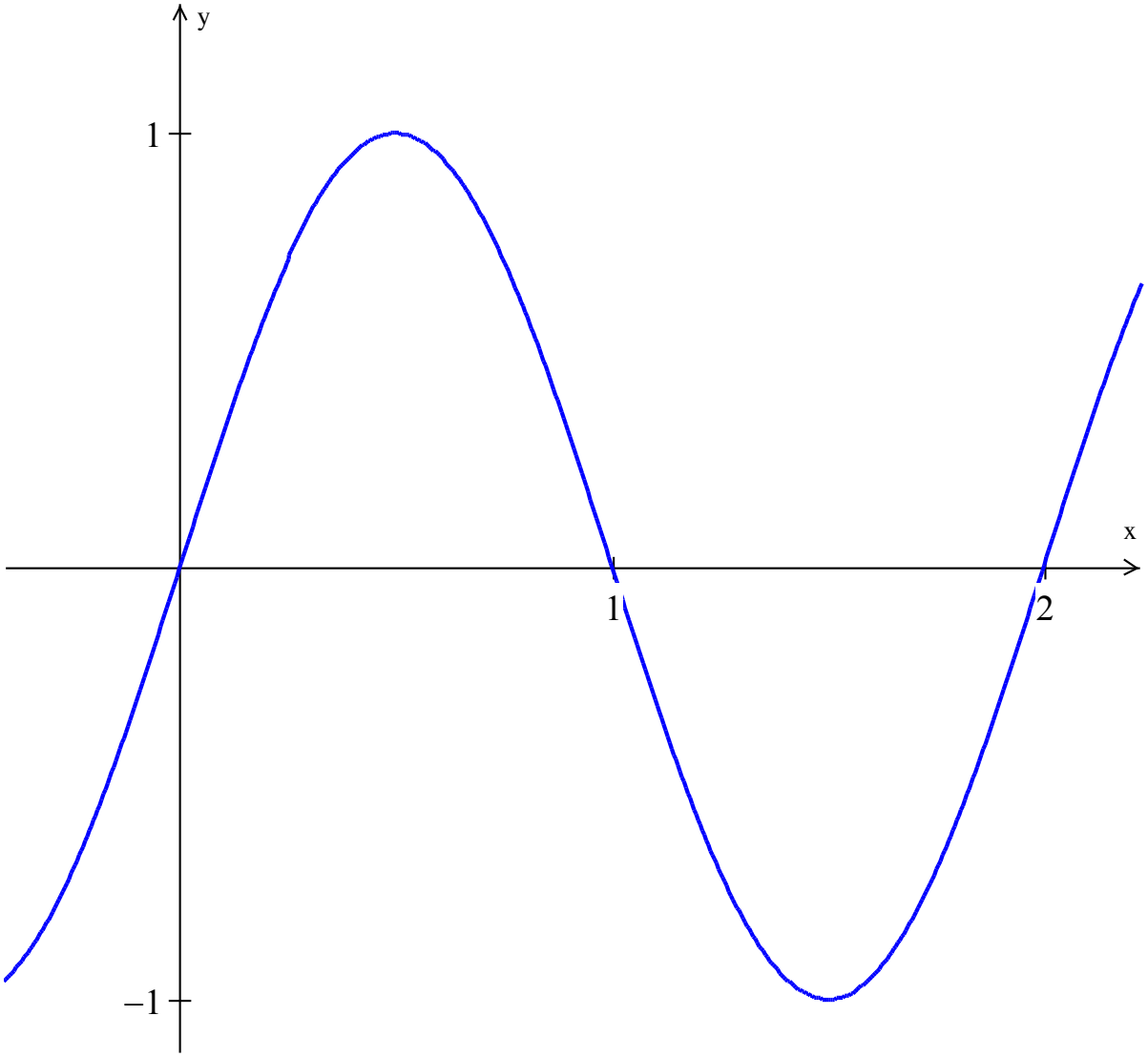
$$y = \sin 2x$$



$y = \sin 2x$

Notice that the length of the cycle is cut in half

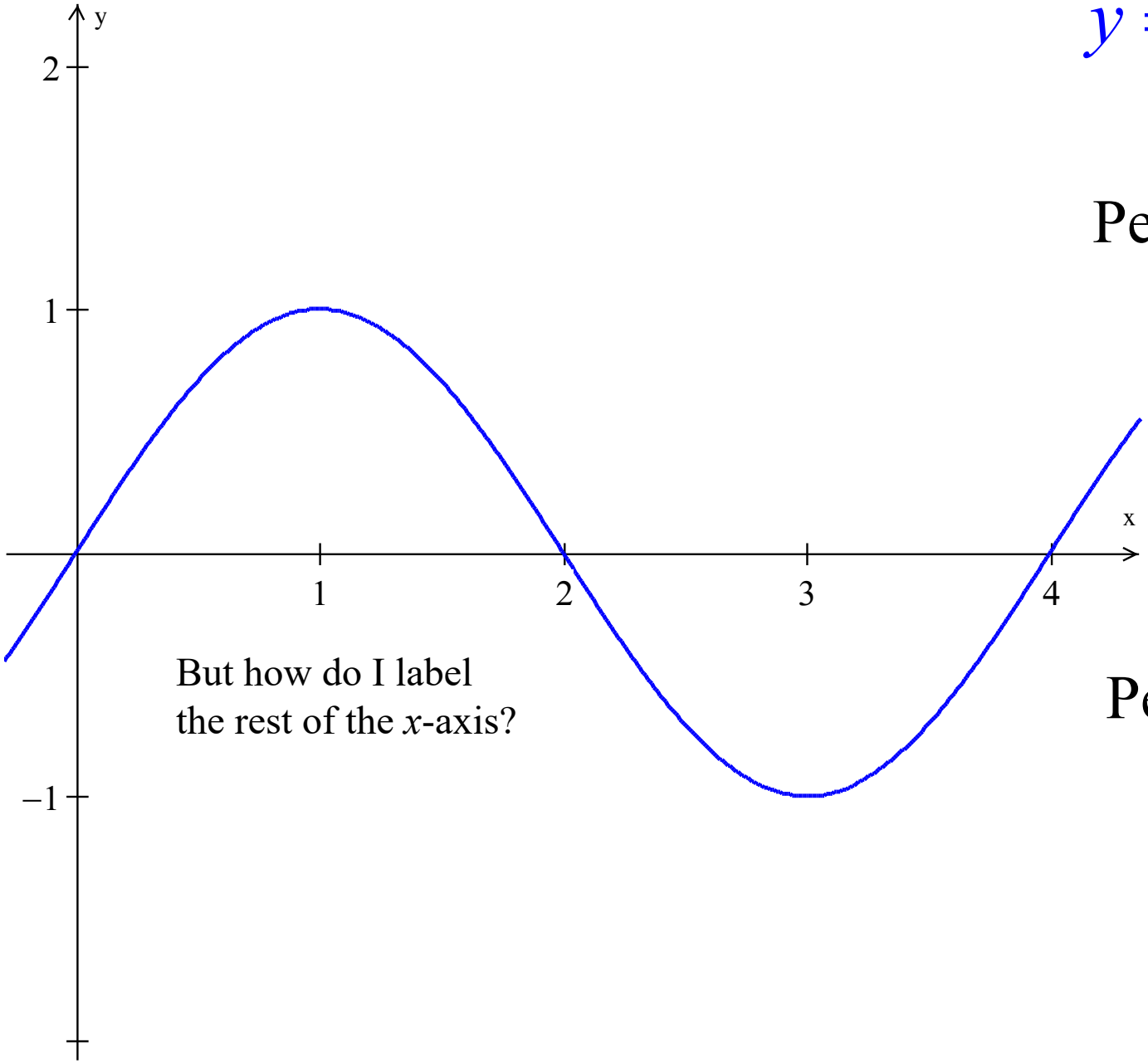
Try $y = \sin 3x$ on your calculator



$$y = \sin \pi x$$

$$\text{Period} = \frac{2\pi}{\pi}$$

$$= 2$$



$$y = \sin\frac{\pi}{2}x$$

$$\text{Period} = \frac{2\pi}{\frac{\pi}{2}}$$

$$= 2\pi\frac{2}{\pi}$$

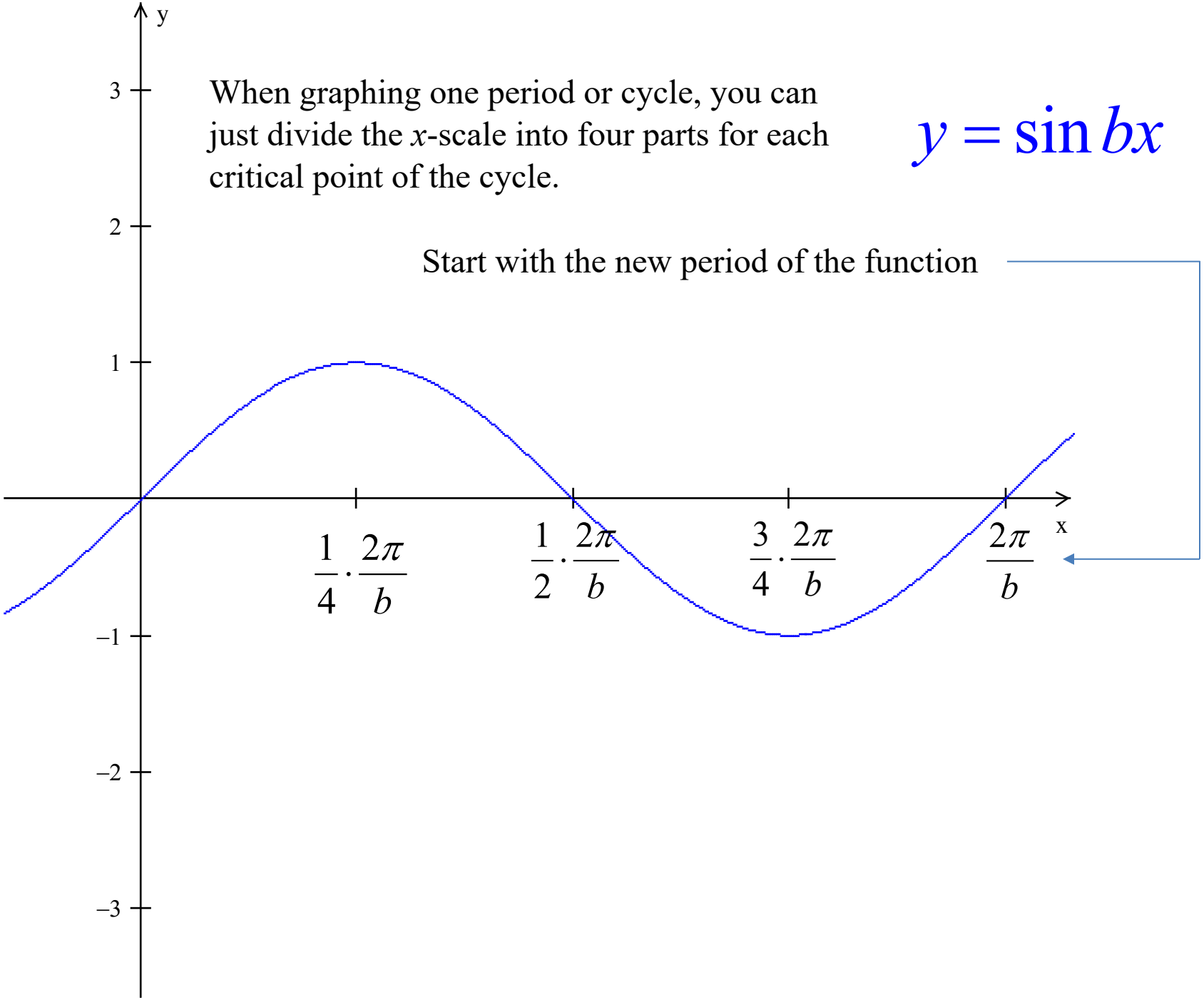
$$\text{Period} = 4$$

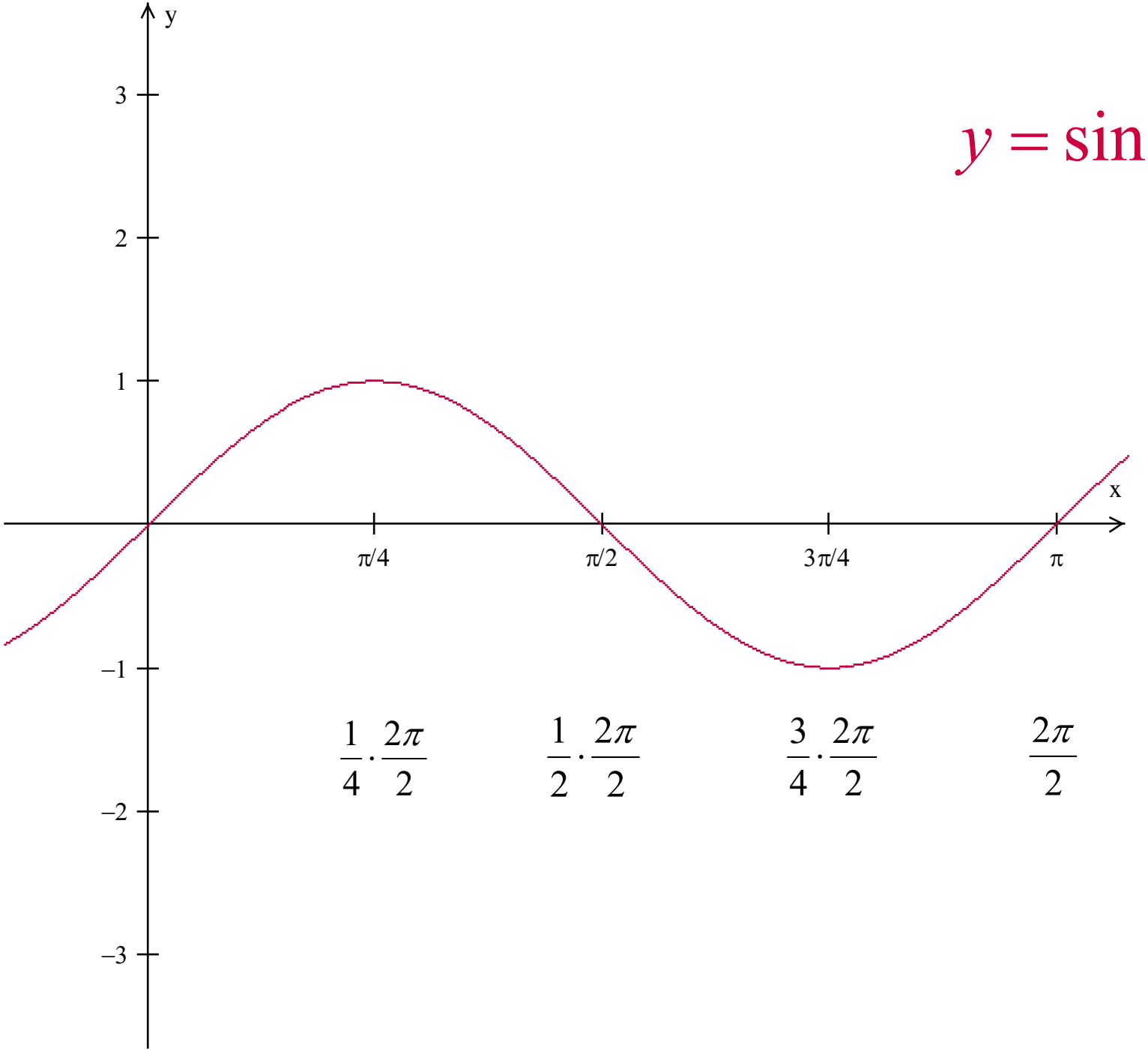
But how do I label the rest of the *x*-axis?

When graphing one period or cycle, you can just divide the *x*-scale into four parts for each critical point of the cycle.

$$y = \sin bx$$

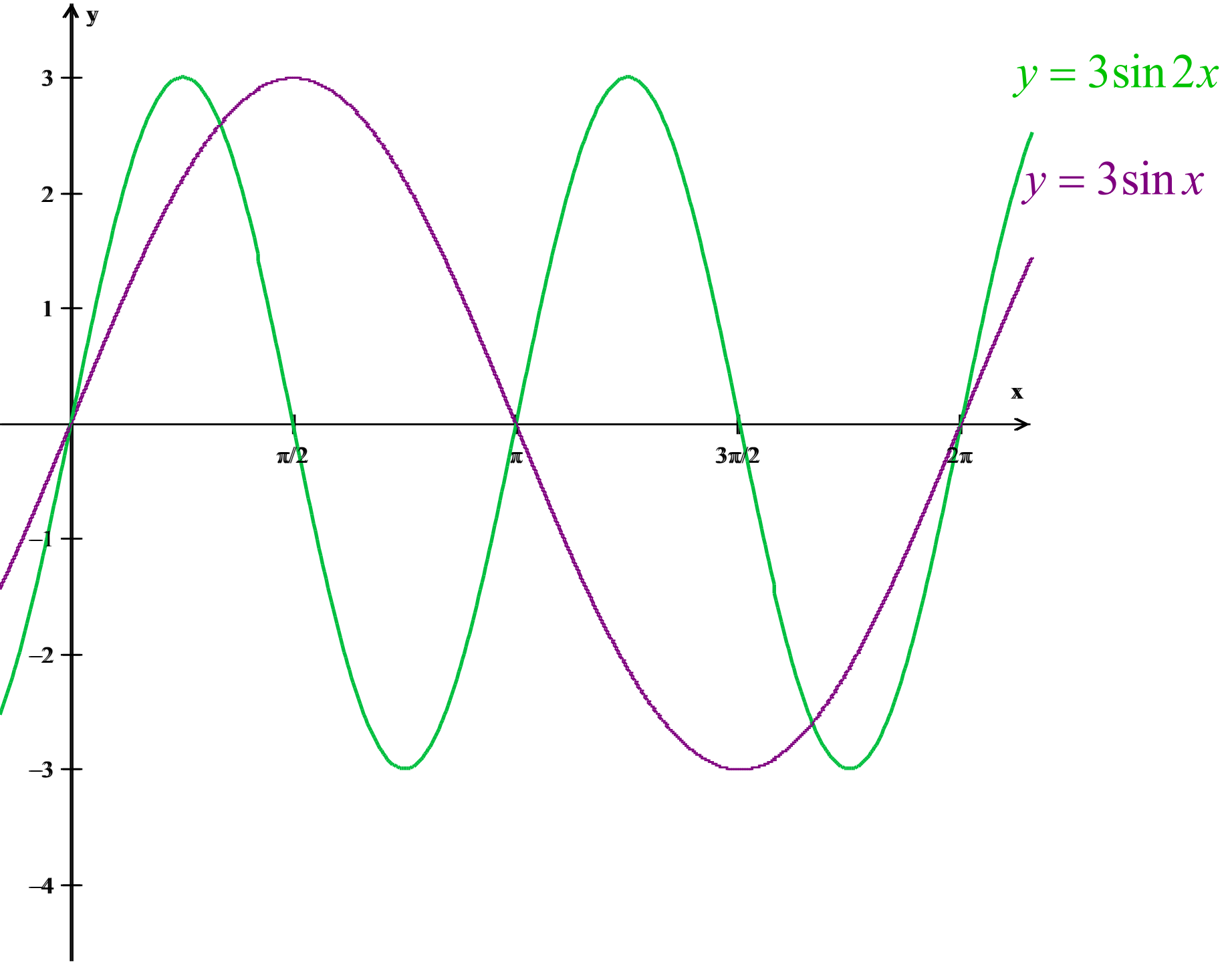
Start with the new period of the function

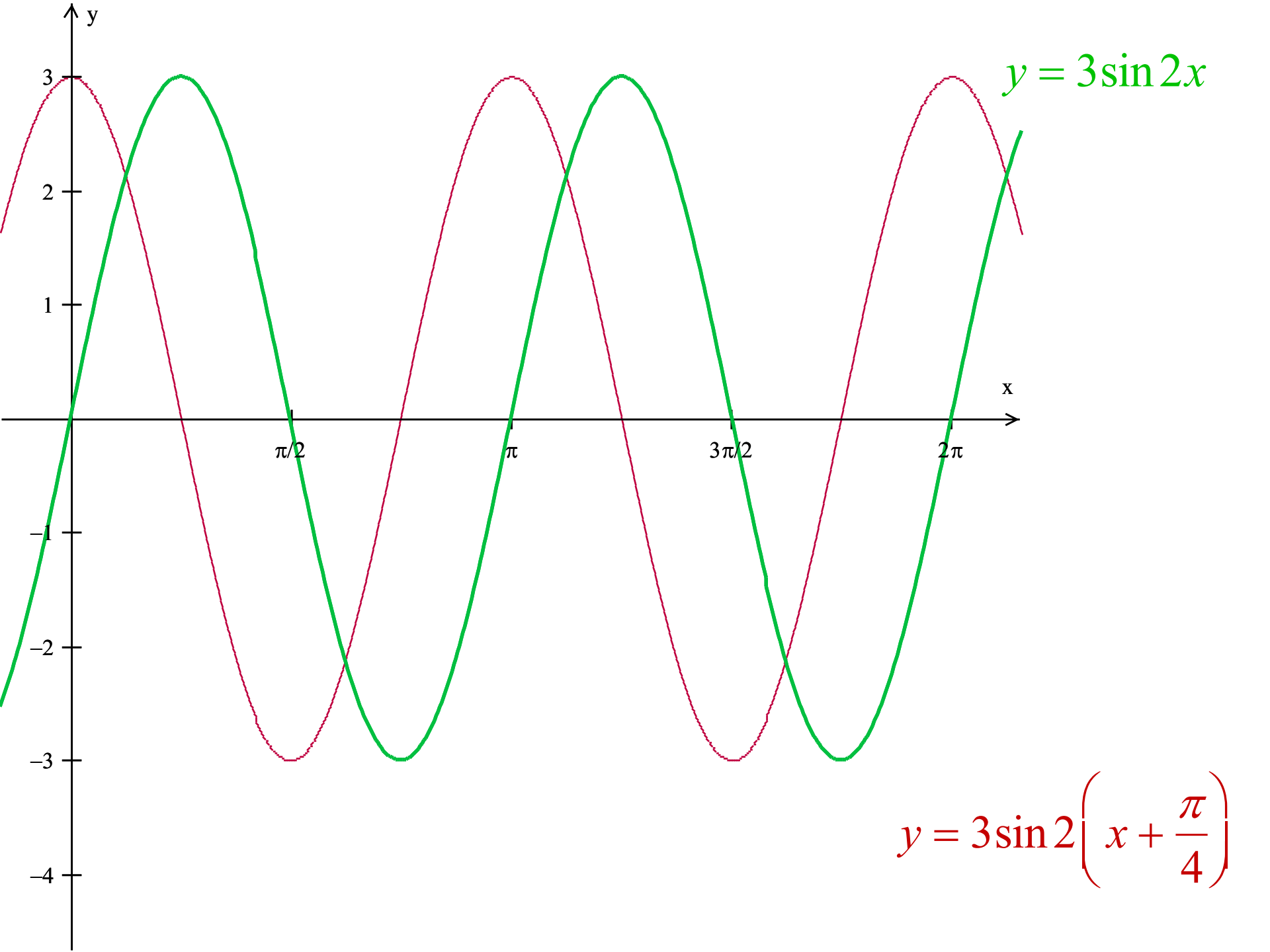


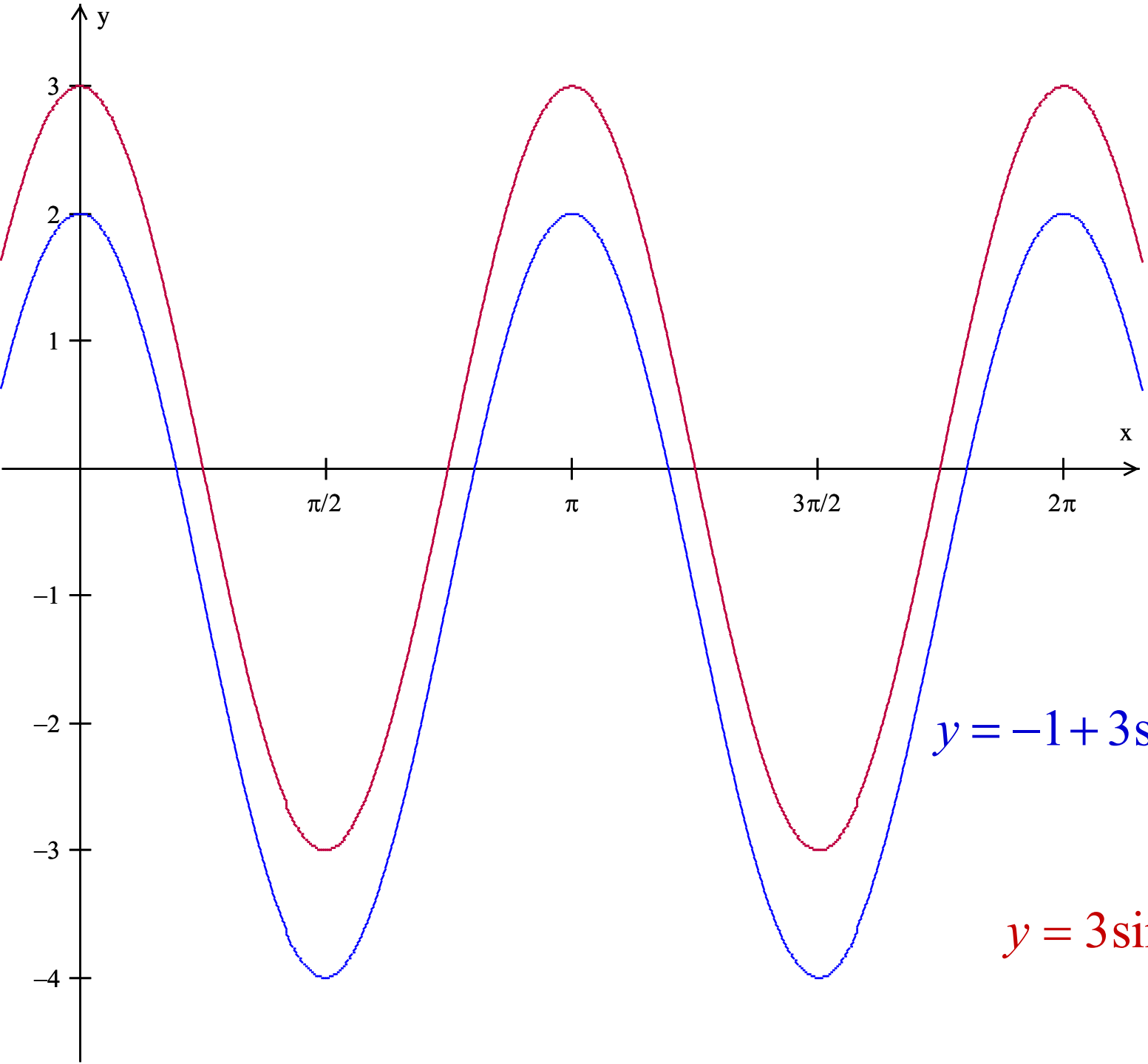


$$y = \sin 2x$$

$$\frac{1}{4} \cdot \frac{2\pi}{2} \qquad \frac{1}{2} \cdot \frac{2\pi}{2} \qquad \frac{3}{4} \cdot \frac{2\pi}{2} \qquad \frac{2\pi}{2}$$

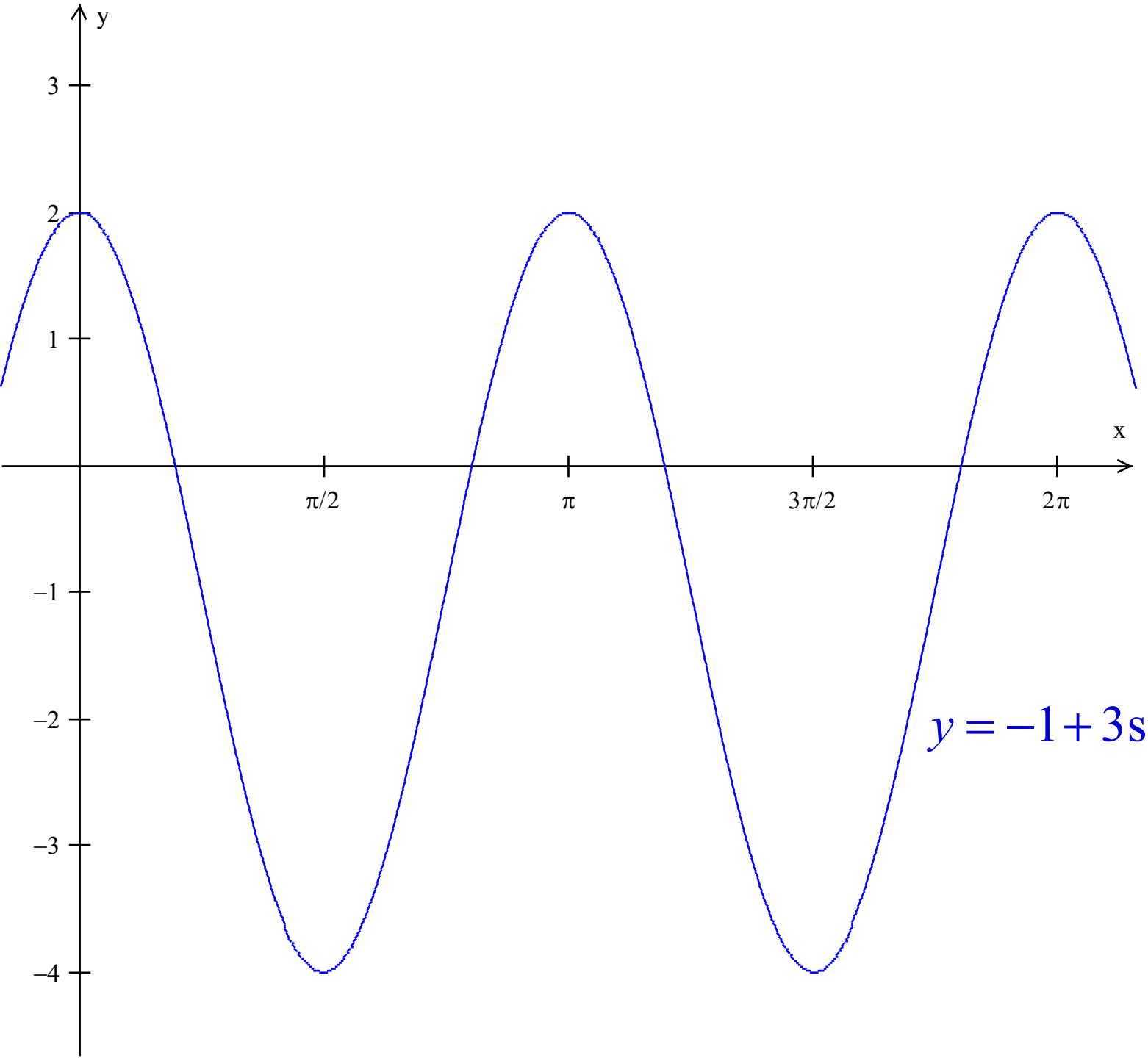

y
3
2
1
–1
–2
–3
–4
x
π/2
π
3π/2
2π
$y = 3\sin 2x$
$y = 3\sin x$


y
x
3
2
1
−1
−2
−3
−4
π/2
π
3π/2
2π
$y = 3\sin 2x$
$y = 3\sin 2\left(x + \frac{\pi}{4}\right)$



$$y = -1 + 3\sin 2\left(x + \frac{\pi}{4}\right)$$

$$y = 3\sin 2\left(x + \frac{\pi}{4}\right)$$



$$y = -1 + 3\sin 2\left(x + \frac{\pi}{4}\right)$$