The Slope of a Curve
or
How Secant Lines become Tangent Lines

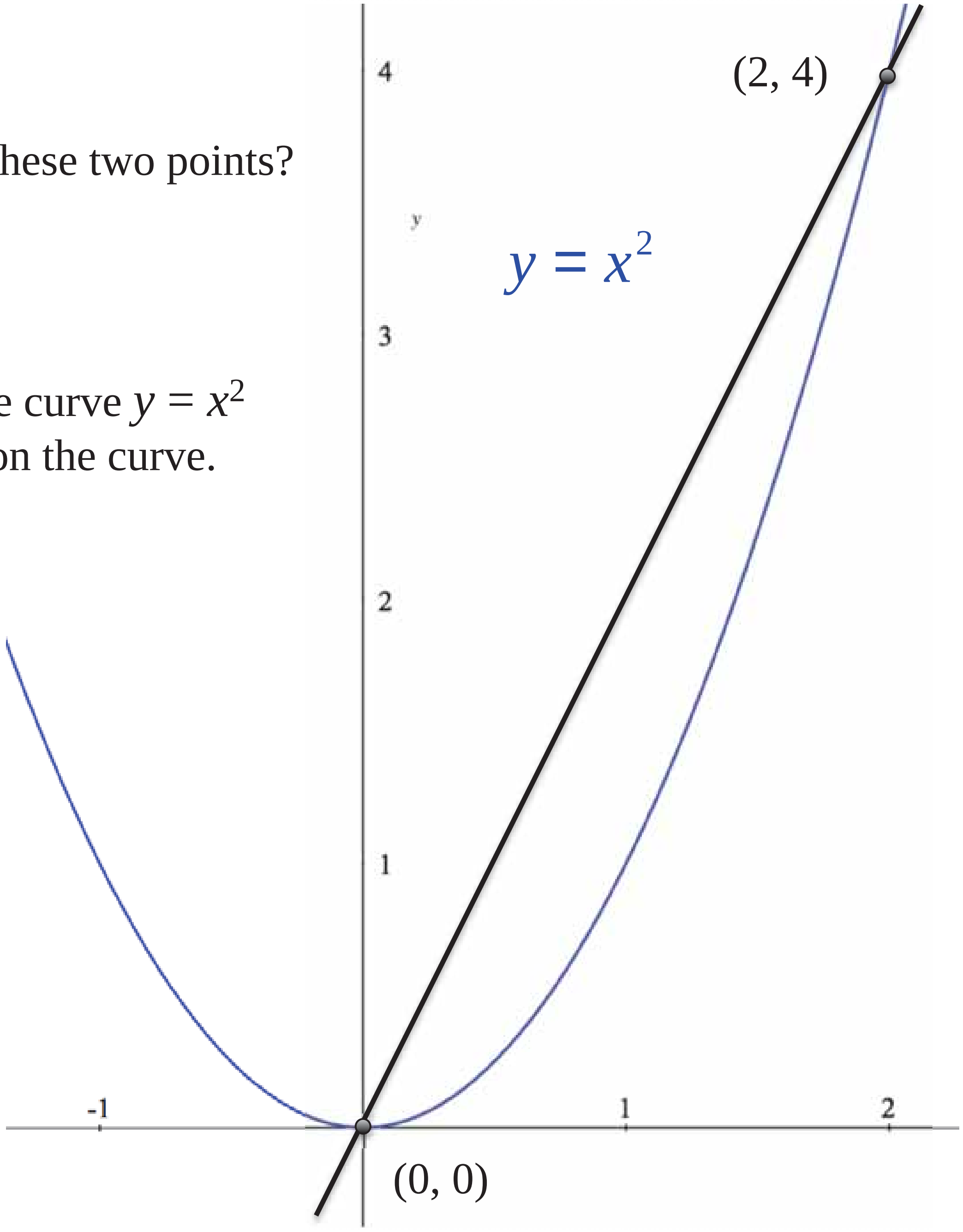
What is the slope of the line through these two points?

$$m = \frac{4-0}{2-0} = 2$$

This is called a *secant line* through the curve $y = x^2$ because it passes through two points on the curve.

Finding the slope of a line is easy

What about finding the slope of a *curve*?

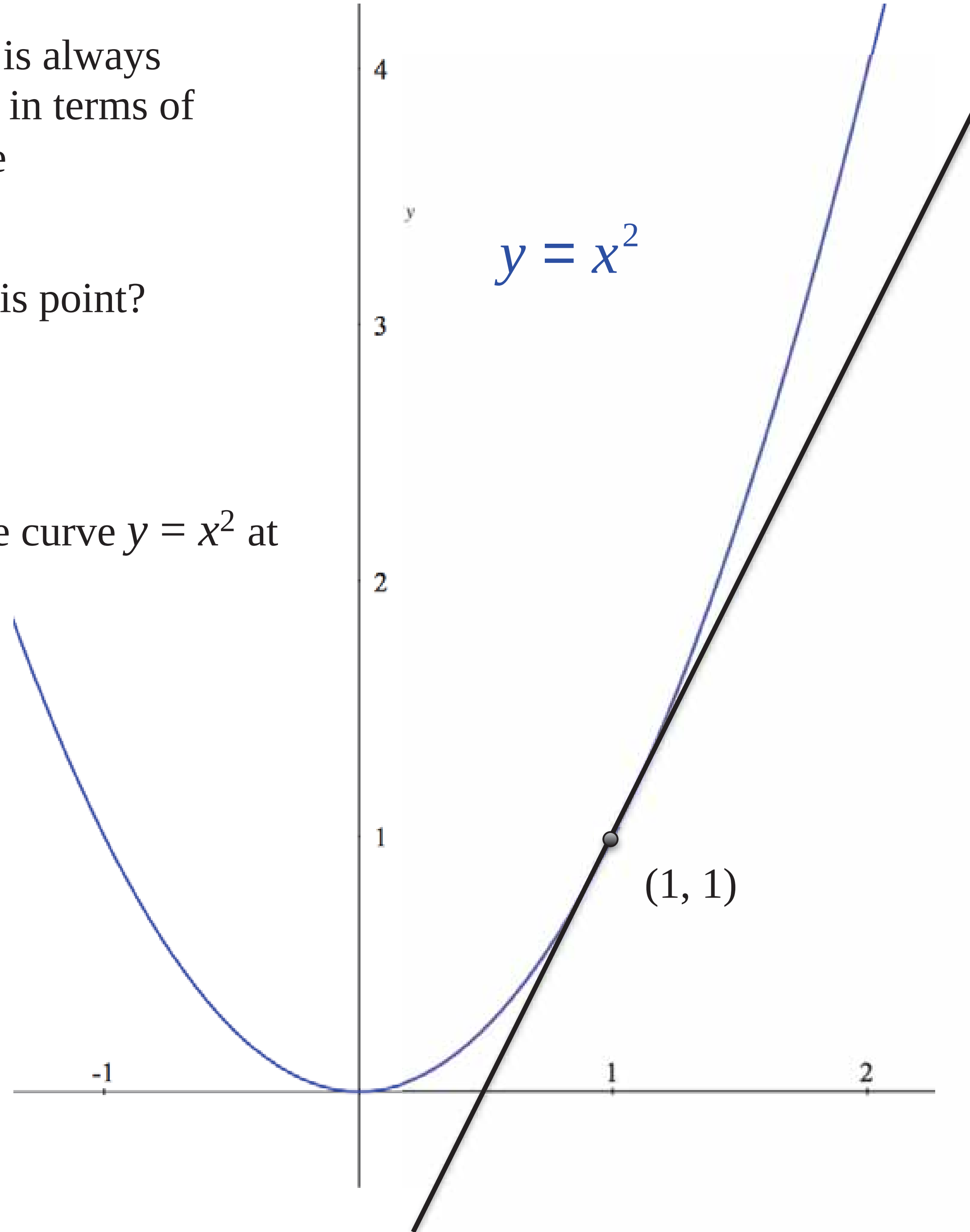


Since the slope of a curve like this one is always changing, we can only talk about slope in terms of specific points or intervals on the curve

What is the slope of the line through this point?

$$m = ?$$

This is called a *tangent line* through the curve $y = x^2$ at the point (1, 1).



Since the slope of a curve like this one is always changing, we can only talk about slope in terms of specific points or intervals on the curve

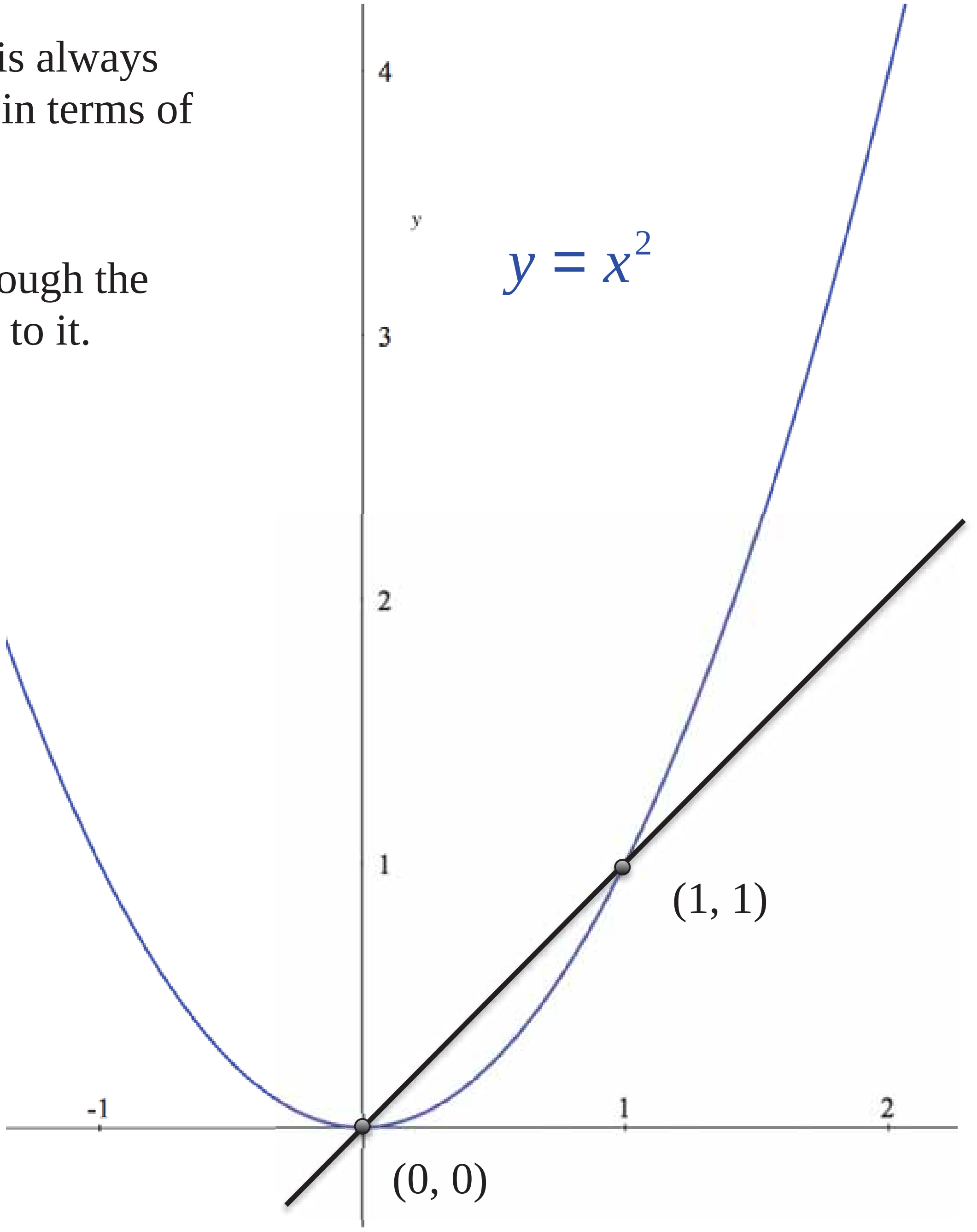
Let's start by drawing a *secant line* through the point (1, 1) and some other point close to it.

$$m = \frac{1-0}{1-0} = 1$$

This is clearly not the slope at (1, 1)

So now what do we do?

Try a point even closer…



Since the slope of a curve like this one is always changing, we can only talk about slope in terms of specific points or intervals on the curve

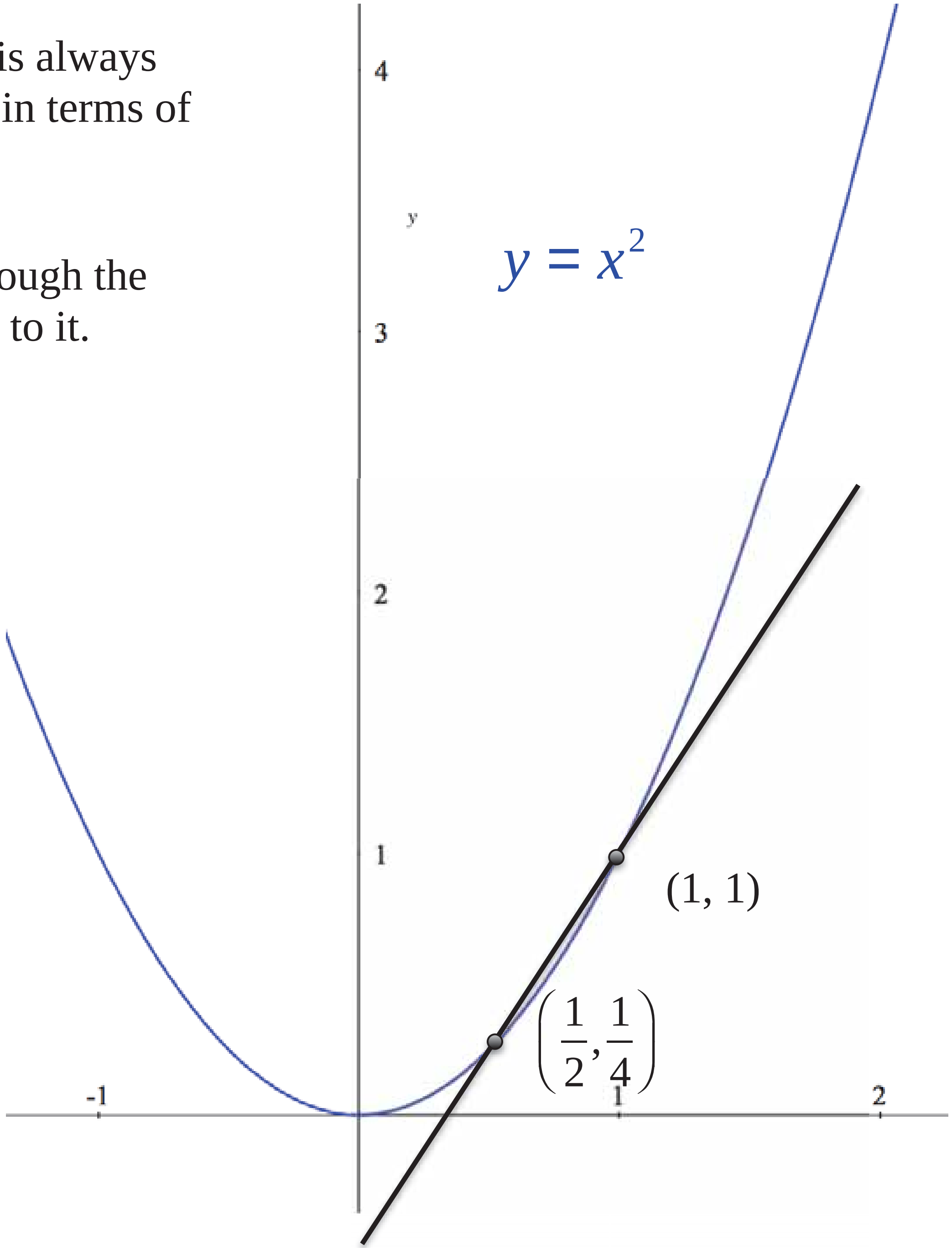
Let's start by drawing a *secant line* through the point (1, 1) and some other point close to it.

$$m = \frac{1 - \frac{1}{4}}{1 - \frac{1}{2}} = \frac{3}{2}$$

This is a lot closer to the slope at (1, 1)

How much closer can we get?

If we use limits, we can get as close to the point as we want.



Since the slope of a curve like this one is always changing, we can only talk about slope in terms of specific points or intervals on the curve

$$m = \frac{y-1}{x-1}$$

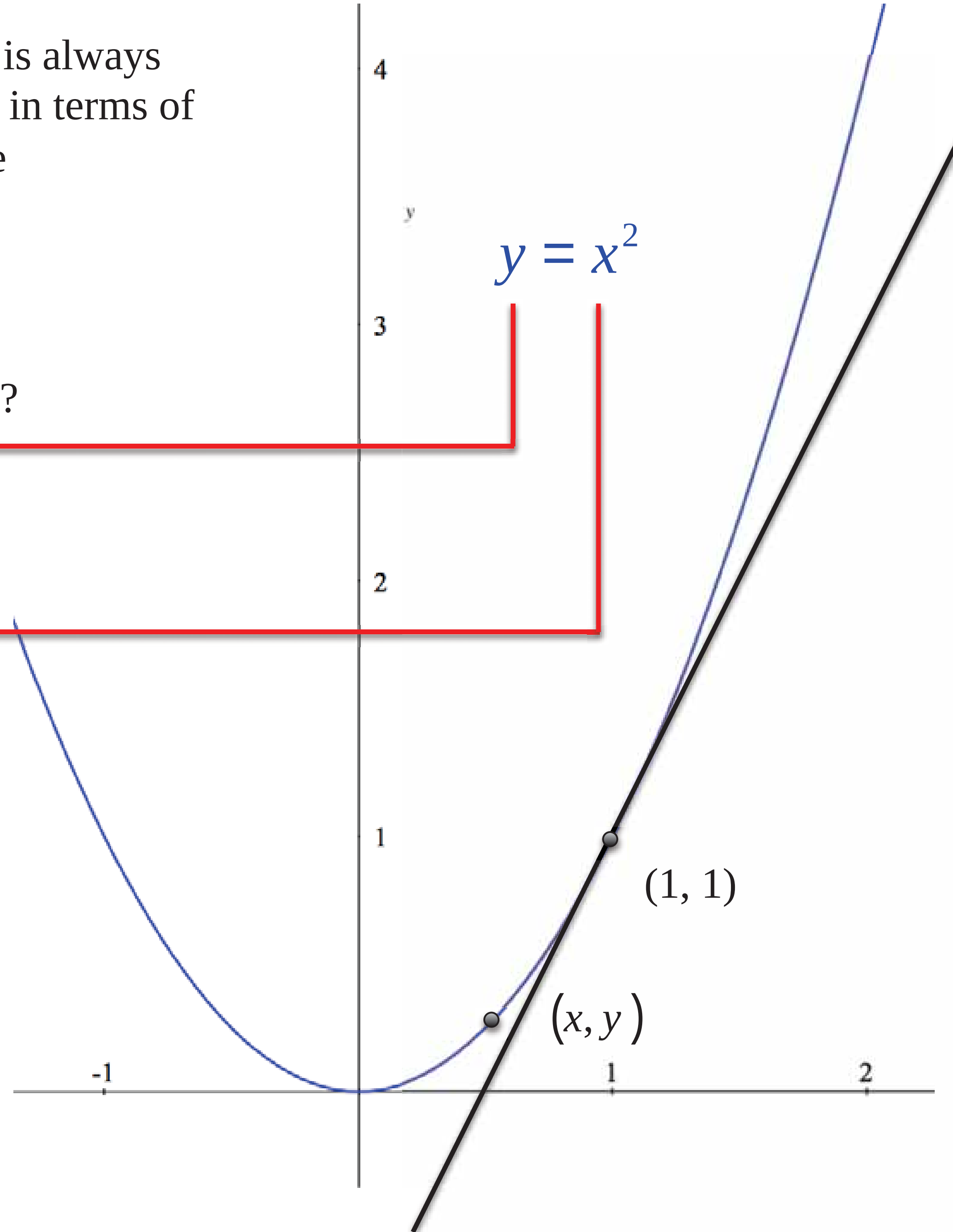
What is the slope of the line at (1, 1)?

$$m_{\tan} = \lim_{x\to 1} \frac{y-1}{x-1}$$

$$m_{\tan} = \lim_{x\to 1} \frac{x^2-1}{x-1}$$

And the first thing we do when taking a limit is…

Plug it in $\frac{0}{0}$ Now what?



Since the slope of a curve like this one is always changing, we can only talk about slope in terms of specific points or intervals on the curve

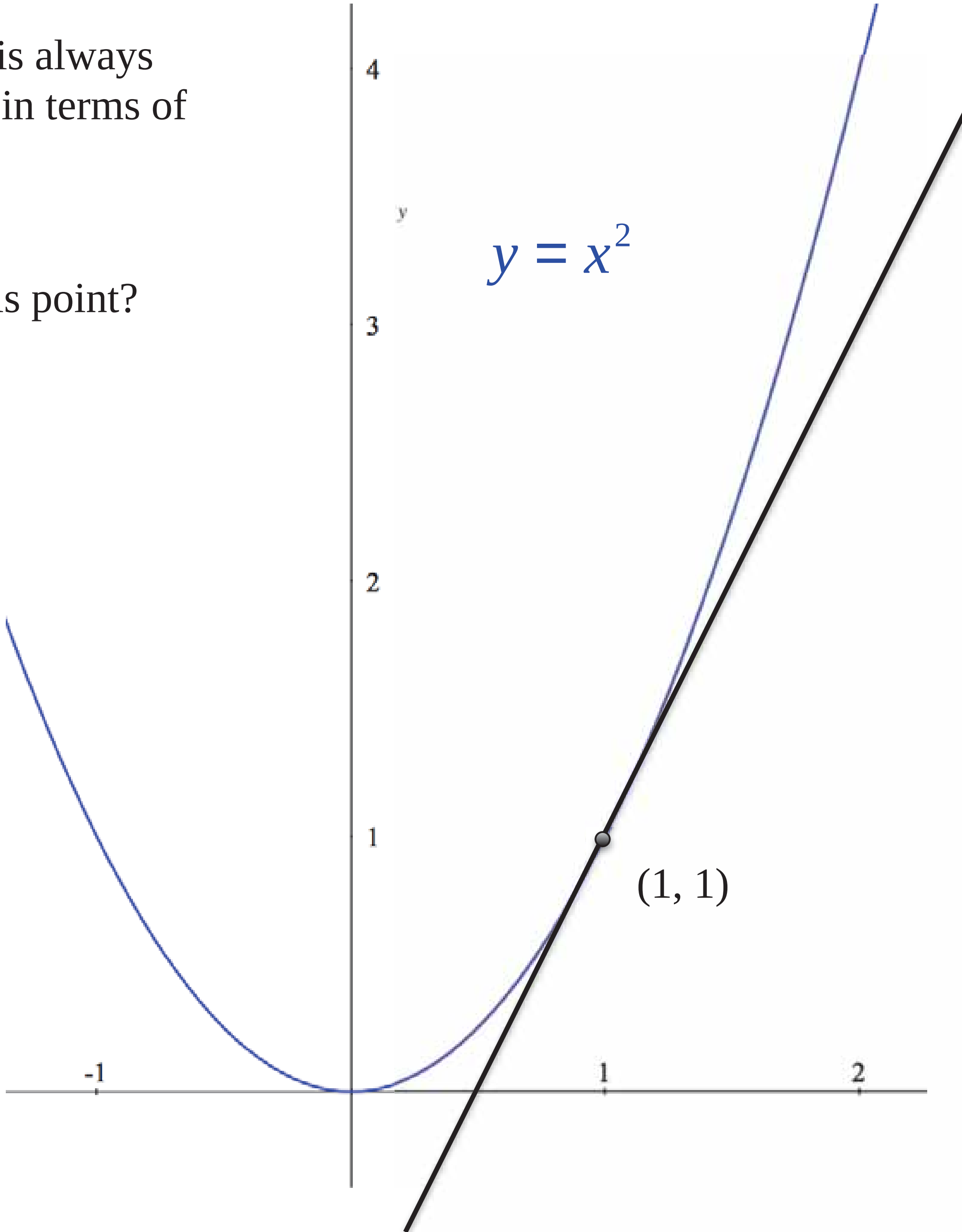
What is the slope of the line through this point?

$$m_{\tan} = \lim_{x\to 1} \frac{x^2 - 1}{x - 1}$$

$$m_{\tan} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}$$

$$m_{\tan} = \lim_{x\to 1} x + 1$$

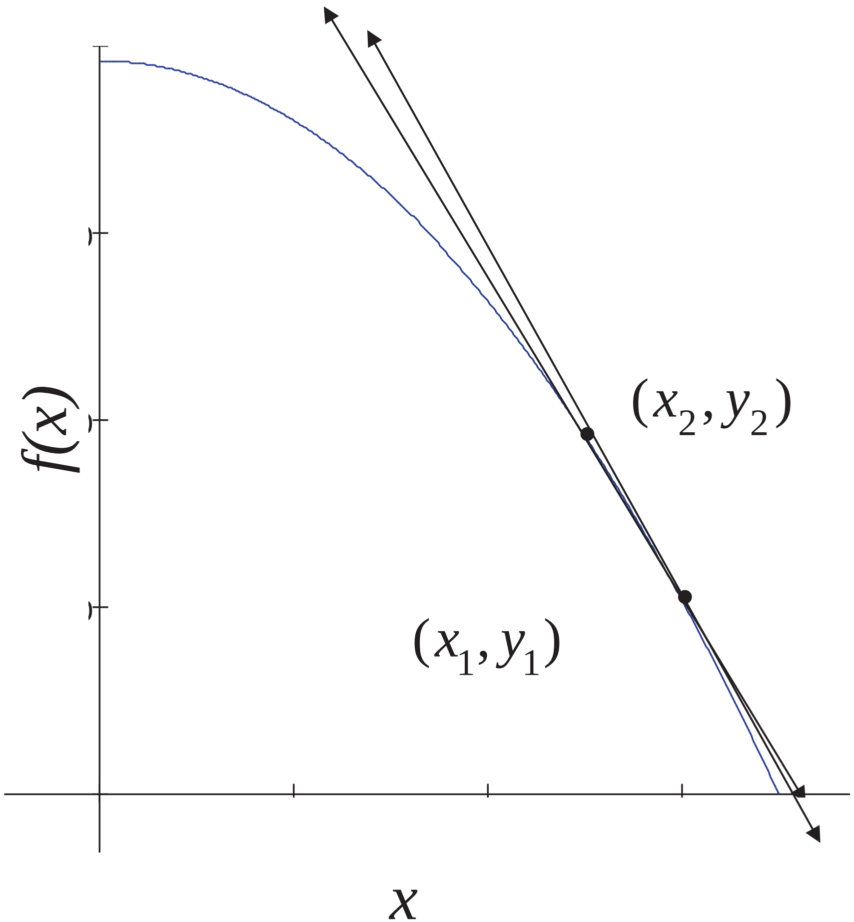
$$m_{\tan} = 2$$



So the slope of the *<u>tangent line</u>* through the curve $y = x^2$ at the point (1, 1) is 2

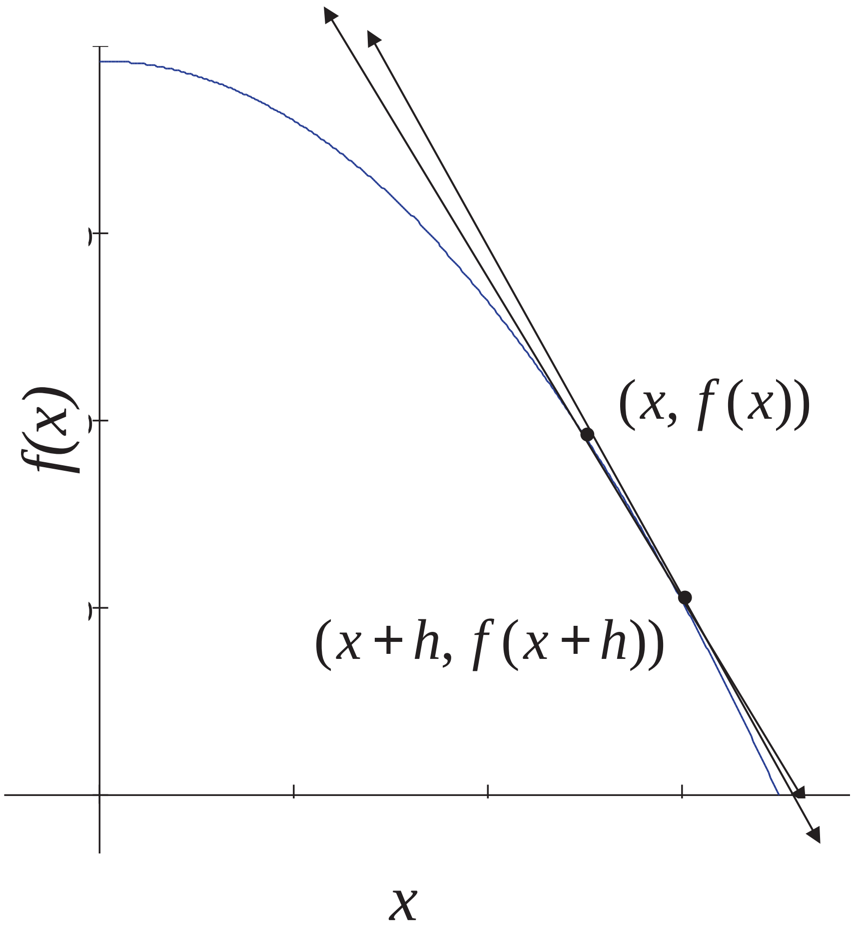
$$m_{sec} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{\tan} = \lim_{x_2 \to x_1} \frac{y_2 - y_1}{x_2 - x_1}$$



But we need to be able to write and read this formula in text book terms (which means replacing y with $f(x)$)

$$m_{sec} = \frac{f\left(x+h\right) - f(x)}{h}$$



$$m_{tan} = \lim_{h\to 0} \frac{f\left(x+h\right) - f(x)}{h}$$

$$\boxed{f'(x) = \lim_{h\to 0} \frac{f\left(x+h\right) - f(x)}{h}}$$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$ is called the derivative of f at a

We write: $$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

"The derivative of f with respect to x is …"

There are two formulas for the derivative of $y=f(x)$

→

The Definition of the Derivative:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$y_2 \quad - \quad y_1$

$x_2 \quad - \quad x_1$

The Derivative at a Point:

(Also called the Numerical Derivative in your text)

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$x_2 \quad - \quad x_1$

There are many ways to write the derivative of $y = f(x)$

$f'(x)$ "f prime x" or "the derivative of f with respect to x"

y' "y prime" or

$$\frac{dy}{dx}$$

$$D_x$$

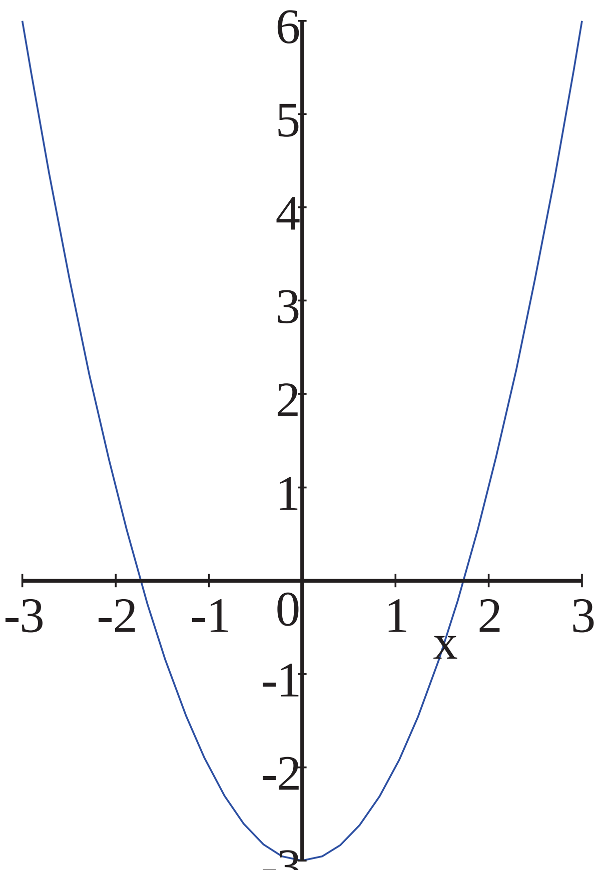
$$\frac{df}{dx}$$

$$\frac{d}{dx} f(x)$$

Here’s how to find the general derivative of a polynomial function.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 - 3$$



$$f'(x) = \lim_{h \to 0} \frac{\overbrace{(x+h)^2 - 3}^{f(x+h)} - \overbrace{(x^2 - 3)}^{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}(2x + h)}{\cancel{h}}$$

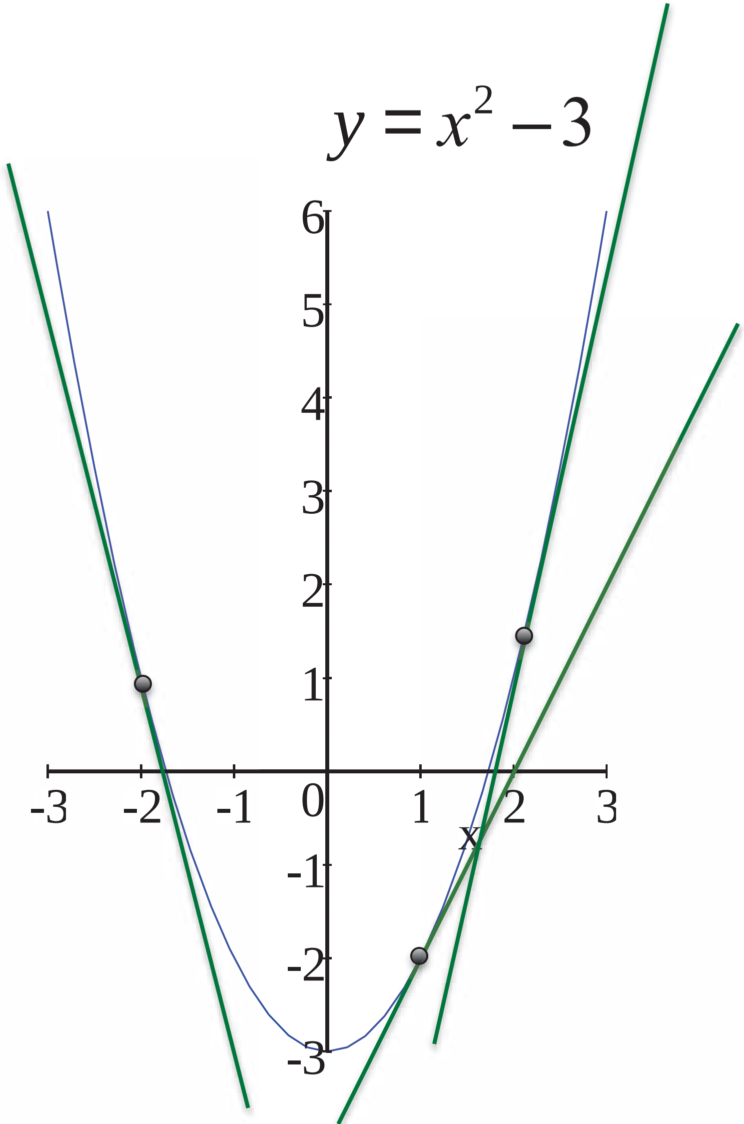
$$f'(x) = \lim_{h \to 0} 2x + \cancelto{0}{h} \qquad f'(x) = 2x$$

Here’s how to find the general derivative of a polynomial function.

$$f(x) = x^2 - 3$$

So in this case the derivative of $f(x)$ is

$$f'(x) = 2x$$



And what this means is

The slope of $y = x^2 - 3$ at

$x = 1$ is $f'(1) = 2(1) = 2$

$x = 2$ is $f'(2) = 2(2) = 4$

$x = -2$ is $f'(-2) = 2(-2) = -4$