Rational Function Extreme Points

Standard 4e: Find the extreme points of a rational function

(remember that extreme points are *y* values)

We’ve found a number of traits already in Standards 4a, 4b, 4d, and 4e

We did not yet discuss the specifics of the domain of a rational function

We also talked about critical values but not extreme points.

- **Extreme Points**

- Use the Quotient Rule to find the derivative
- Find the critical points (where the derivative is 0 or undefined)
- Make your sign pattern using this information
- Determine which critical points are maximum, minimum, or neither
- Find the extreme points by plugging the critical points back into the original function
- Check your work on the graphing calculator

Find the extreme points of

$$y = \frac{x^2 + 6x}{x - 2}$$

VA's, zeros, POE's first…

$$y = \frac{x(x+6)}{x-2}$$ = 0 or undefined

Numerator = 0 or Denominator = 0

Numerator = 0 and Denominator = 0

Never

Because the function is not defined at $x = 2$ the **domain** of this function is

$$x \in (-\infty, 2) \cup (2, \infty) \quad \text{or} \quad x \neq 2$$

Zeros: Numerator = 0

$$x(x+6) = 0$$

$$x = -6, 0$$

VA's: Denominator = 0

$$x - 2 = 0$$

$$x = 2$$

Zero: at $x = -6, 0$

Vertical Asymptote: at $x = 2$

Point of Exclusion: None

Because the function is not defined at $x = 2$ the domain of this function is

$x \in (-\infty, 2) \cup (2, \infty)$ or $x \neq 2$

An even simpler way to look at domain in a rational function:

The domain is every value of x except where there are POE's and VA's

Zero: at $x = -6, 0$

Vertical Asymptote: at $x = 2$

Point of Exclusion: None

Find the extreme points of

$$y = \frac{x^2 + 6x}{x - 2}$$

$$f = x^2 + 6x \qquad g = x - 2$$

$$f' = 2x + 6 \qquad g' = 1$$

$$y' = \frac{(x-2)(2x+6) - (x^2+6x)(1)}{(x-2)^2}$$

$$y' = \frac{2x^2 - 4x + 6x - 12 - x^2 - 6x}{(x-2)^2}$$

$$y' = \frac{x^2 - 4x - 12}{(x-2)^2} = 0 \text{ or undefined}$$

Numerator = 0 or Denominator = 0

<u>Numerator = 0</u>

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = -2, 6$$

<u>Denominator = 0</u>

$$(x-2)^2 = 0$$

$$x = 2$$

Find the extreme points of

$$y = \frac{x^2 + 6x}{x - 2}$$

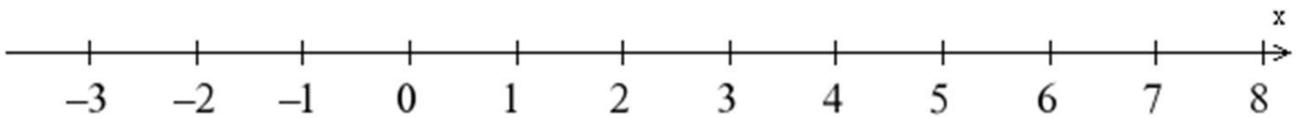
Numerator = 0

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = -2,\ 6$$

$+$ $\quad 0 \quad$ $-$ $\quad$ undef $\quad$ $-$ $\quad 0 \quad$ $+$



Denominator = 0

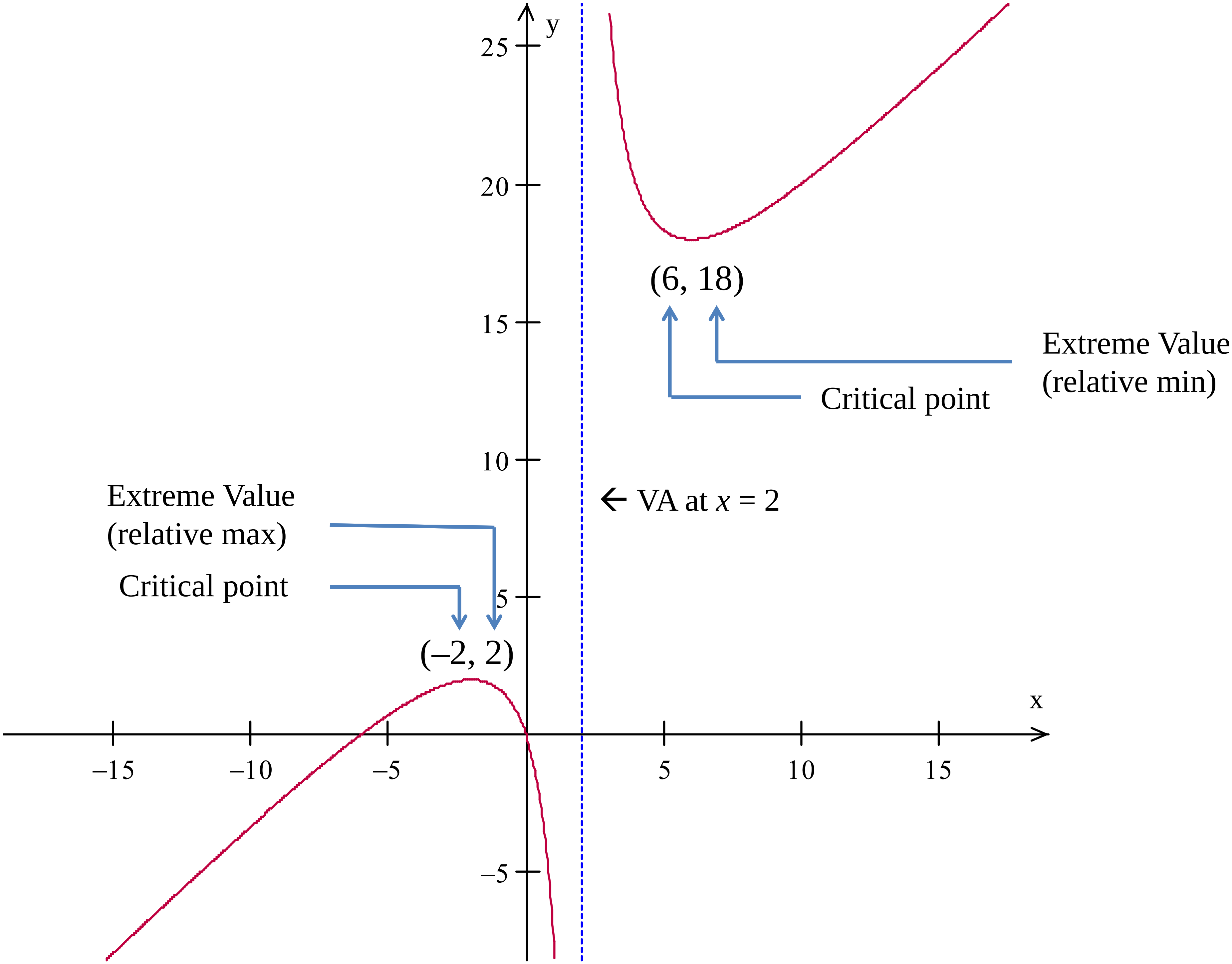
$$(x-2)^2 = 0$$

$$x = 2$$

To find the extreme points we plug our critical points back into the original function:

When $\quad x = -2 \quad$ $$y = \frac{(-2)^2 + 6(-2)}{-2-2} = \frac{-8}{-4} = 2$$

When $\quad x = 6 \quad$ $$y = \frac{(6)^2 + 6(6)}{6-2} = \frac{72}{4} = 18$$



y
25
20
15
10
5
–5
x
–15
–10
–5
5
10
15
(6, 18)
Extreme Value
(relative min)
Critical point
VA at x = 2
Extreme Value
(relative max)
Critical point
(–2, 2)

[redacted]