Rational Function Traits

Standard 7g: Find all the traits and sketch a fairly accurate rational curve algebraically.

Rational Traits

1. Domain (or when the denominator = 0)
2. y-intercept
3. Zero(s)
4. Point(s) of Exclusion
5. Vertical Asymptote(s)
6. End Behavior
7. Extreme Point(s)
8. Range

Find the extreme points of

$$y = \frac{x^2 + 6x}{x - 2}$$

VA's, zeros, POE's first…

$$y = \frac{x(x+6)}{x-2} \quad = 0 \text{ or undefined}$$

Numerator = 0 or Denominator = 0

Numerator = 0 <u>and</u> Denominator = 0

Never

Because the denominator is 0 at $x = 2$ the **<u>domain</u>** of this function is

$$x \in (-\infty, 2) \cup (2, \infty) \quad \text{or} \quad x \neq 2$$

Zeros: <u>Numerator = 0</u>

$$x(x+6) = 0$$

$$x = -6, 0$$

VA's: <u>Denominator = 0</u>

$$x - 2 = 0$$

$$x = 2$$

Zero: at $x = -6$, 0

Vertical Asymptote: at $x = 2$

Point of Exclusion: None

Because the function is not defined at $x = 2$ the domain of this function is

$$x \in (-\infty, 2) \cup (2, \infty) \quad \text{or} \quad x \neq 2$$

An even simpler way to look at domain in a rational function:

The domain is every value of x <u>except</u> where there are POE's and VA's

Zero: at $x = -6, 0$

Vertical Asymptote: at $x = 2$

Point of Exclusion: None

Find the extreme points of

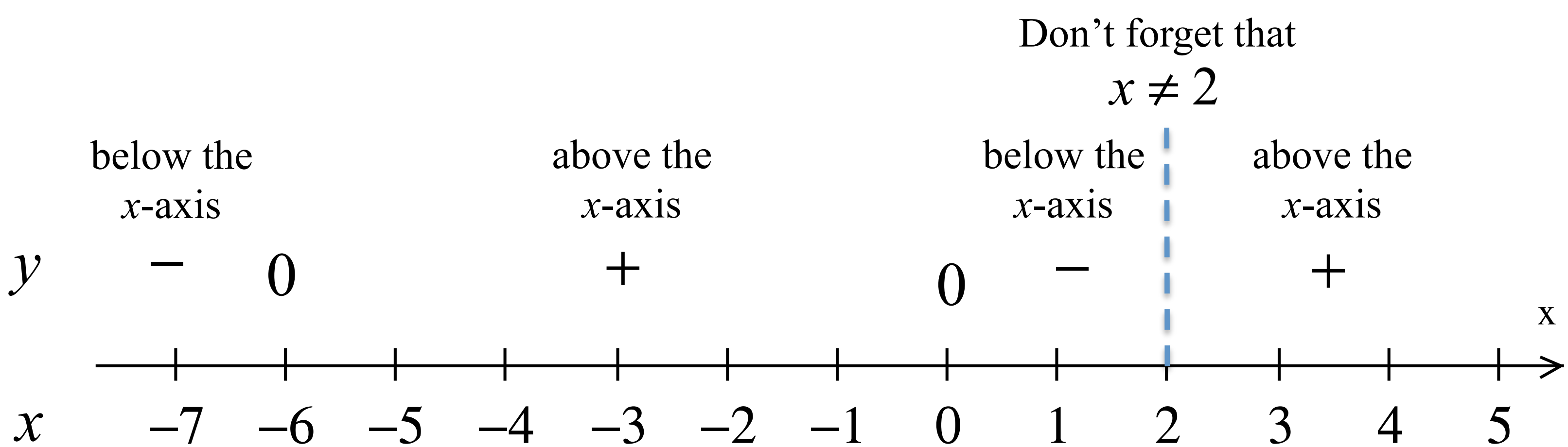
$$y = \frac{x^2 + 6x}{x - 2}$$

Let’s also find the values of x for which

y is positive (above the x-axis) or negative (below)

$$y = \frac{x(x+6)}{x-2} \begin{cases} > 0 \text{ (above the } x\text{-axis)} \\ < 0 \text{ (below the } x\text{-axis)} \end{cases}$$

For this what should we do? Let’s make a sign pattern for y



Find the extreme points of

$$y=\frac{x^2+6x}{x-2}$$

$$f=x^2+6x \qquad g=x-2$$

$$f'=2x+6 \qquad g'=1$$

Now let's take the derivative

$$y'=\frac{(x-2)(2x+6)-(x^2+6x)(1)}{(x-2)^2}$$

$$y'=\frac{2x^2-4x+6x-12-x^2-6x}{(x-2)^2}$$

$$y'=\frac{x^2-4x-12}{(x-2)^2} = 0 \text{ or undefined}$$

Numerator = 0 or Denominator = 0

<u>Numerator = 0</u>

$$x^2-4x-12=0$$

$$(x-6)(x+2)=0$$

$$x=-2,\ 6$$

<u>Denominator = 0</u>

$$(x-2)^2=0$$

$$x=2$$

Find the extreme points of

$$y = \frac{x^2 + 6x}{x - 2}$$

$$y' = \frac{x^2 - 4x - 12}{(x-2)^2}$$

<u>Numerator = 0</u>

$$x^2 - 4x - 12 = 0$$

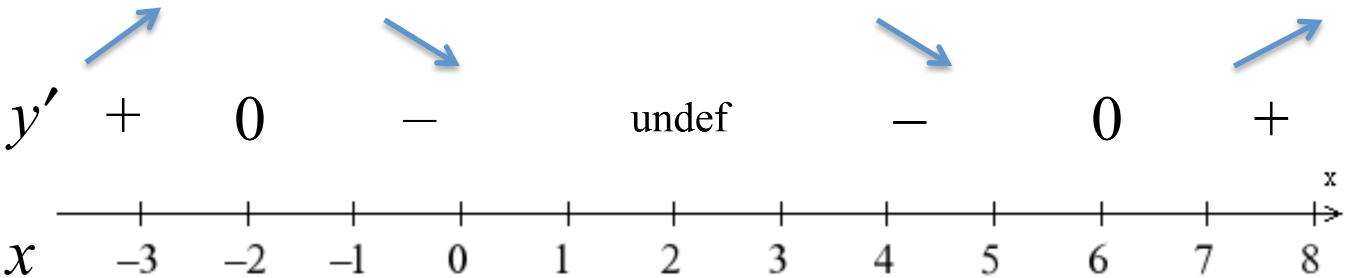
$$(x-6)(x+2) = 0$$

$$x = -2, 6$$

<u>Denominator = 0</u>

$$(x-2)^2 = 0$$

$$x = 2$$



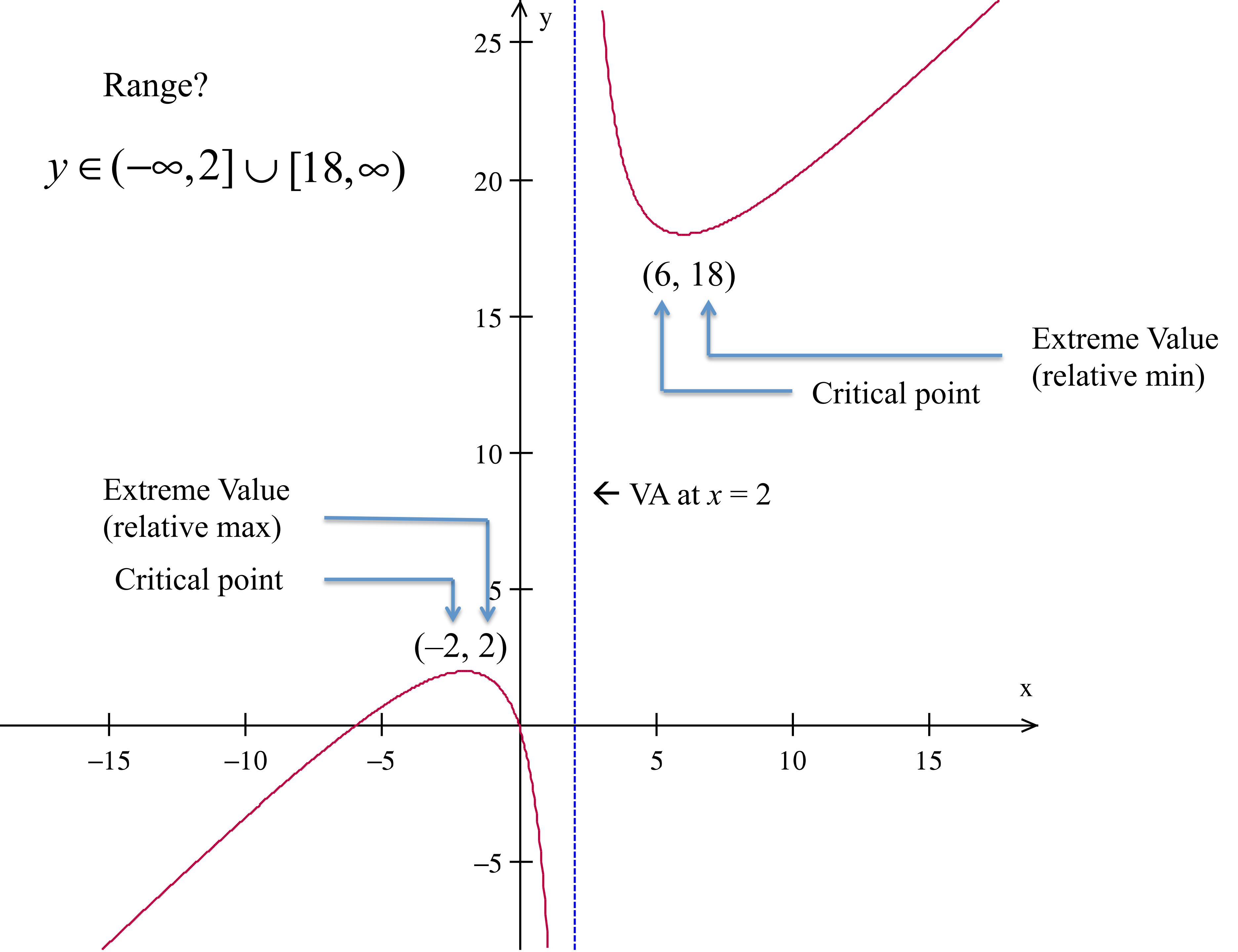
To find the extreme points we plug our critical points back into the original function:

When $x = -2$ $\quad y = \frac{(-2)^2 + 6(-2)}{-2-2} = \frac{-8}{-4} = 2$

When $x = 6$ $\quad y = \frac{(6)^2 + 6(6)}{6-2} = \frac{72}{4} = 18$

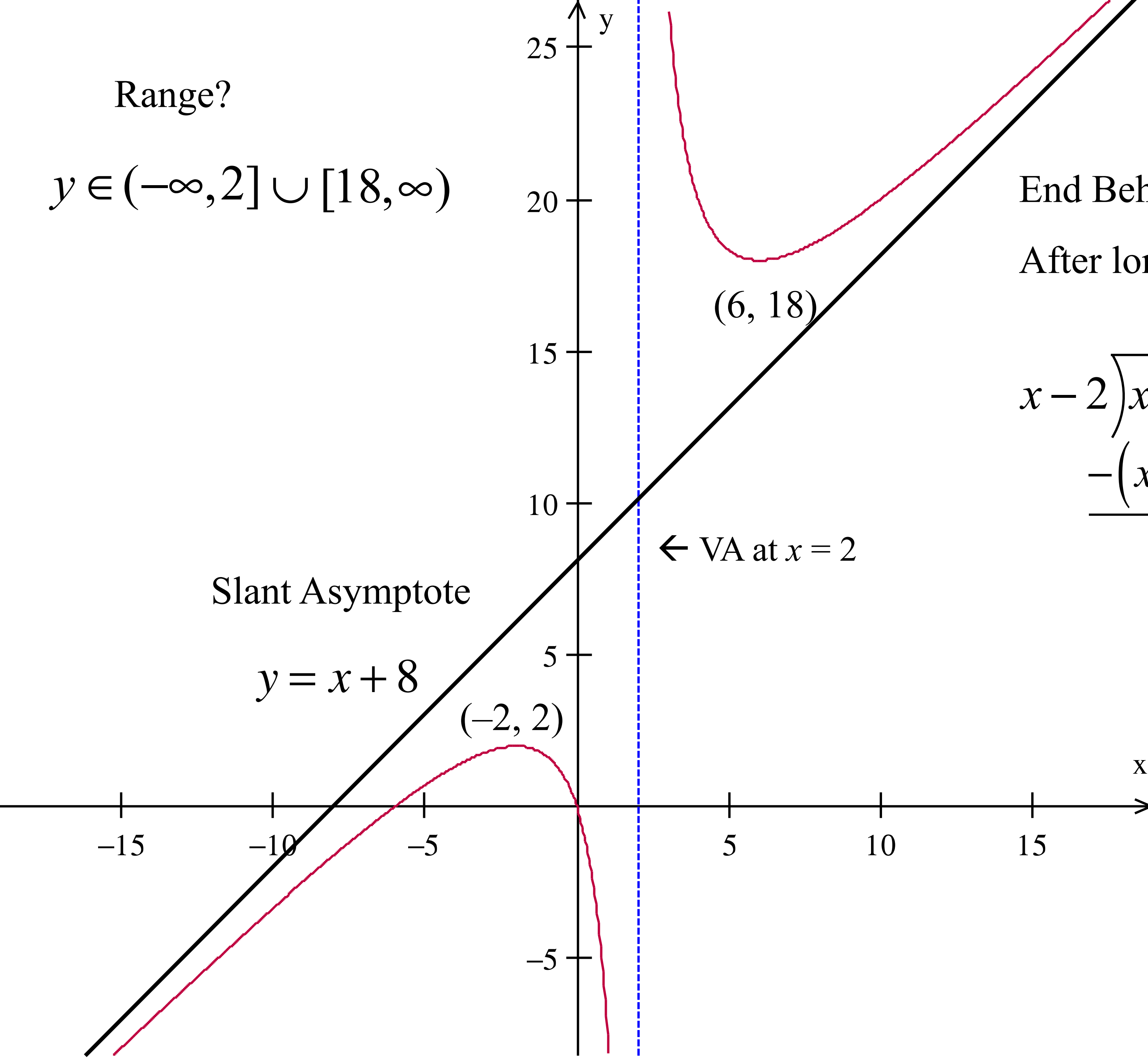
Range?

$$y \in (-\infty, 2] \cup [18, \infty)$$



Range?

$$y \in (-\infty, 2] \cup [18, \infty)$$



End Behavior?

After long division

$$\begin{array}{r} x+8 \\ x-2\,\overline{)\,x^2+6x} \\ \underline{-(x^2-2x)} \\ 8x \\ \underline{-(8x-16)} \end{array}$$