Rules of Logarithmic and Exponential Functions

$$5^3 = x \qquad x^3 = 125 \qquad 5^x = 125$$

Which of these would be hardest to solve algebraically?

$$5^3 = 125 \qquad \sqrt[3]{x^3} = \sqrt[3]{125} \qquad 5^x = 125$$

$$x = 5 \qquad ?$$

We do have a function that we can perform here that would actually help us solve for an <u>unknown exponent</u>.

$$5^x = 125 \longrightarrow \log_5 125 = x$$

Logarithmic Functions

$$\log_5 125 = x$$

Examples:

By the way, the log with base e ($\log_e$) has a special notation: $\ln$

Ex 1) $\log_3 81 = 4$

$$3^4 = 81$$

Ex 2) a) $9^{\frac{3}{2}} = 27$

$$\log_9 27 = \frac{3}{2}$$

b) $16^{-\frac{3}{4}} = \frac{1}{8}$

$$\log_{16} \frac{1}{8} = -\frac{3}{4}$$

Did You Know that this means this?

$$\log_{16}\frac{1}{8}=-\frac{3}{4}$$

$$\log_{16}8=\frac{3}{4}$$

We will find out why shortly

Laws of Logs

$$\boxed{\log_a M + \log_a N = \log_a MN}$$

Really?

$$\log_2 4 + \log_2 8 = \log_2 32$$

$$2 + 3 = 5 \quad \checkmark$$

$$\boxed{\log_a M - \log_a N = \log_a \frac{M}{N}}$$

$$\log_a M + \log_a M + \log_a M$$

$$\log_a M + \log_a M + \log_a M$$

and therefore...

$$\boxed{\log_a M^n = n \log_a M}$$

Did You Know This?

$$\log_{16}\frac{1}{8} = -\frac{3}{4}$$

$$\log_{16} 8 = \frac{3}{4}$$

$$\boxed{\log_a M^n = n\log_a M}$$

$$\log_{16} 8^{-1} = -\frac{3}{4} \longrightarrow (-1)\log_{16} 8 = -\frac{3}{4}$$

$$\log_{16} 8 = \frac{3}{4}$$

Ex 3 a) $\frac{1}{3}\log_4 64 - 4\log_4 2$

$$\log_4 64^{\frac{1}{3}} - \log_4 2^4$$

$$\log_4 4 - \log_4 16$$

$$\log_4 \frac{4}{16}$$

$$\log_4 \frac{1}{4} \quad = \log_4 4^{-1} \quad = (-1)\log_4 4$$

$$= -1$$

$$\log_{17} 1{,}419{,}857 = x$$

$$17^x = 1{,}419{,}857 \longrightarrow 17^5 = 1{,}419{,}857$$

Only the most recent versions of the TI-84 have a log function with a variable base.

But how would we do this on older calculators?

How about using a log function that is on there like base 10?

$$\log_{10} 17^x = \log_{10} 1{,}419{,}857$$

$$x \log_{10} 17 = \log_{10} 1{,}419{,}857$$

$$x = \frac{\log_{10} 1{,}419{,}857}{\log_{10} 17} = 5$$

Change of Base Formula

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$\log_{17} 1{,}419{,}857 = x$$

$$17^x = 1{,}419{,}857 \qquad 17^5 = 1{,}419{,}857$$

These two equations say the same thing which tells us something:

The log and exponential functions are inverses of each other

This also tells us that algebraically, each can be used to "undo" the other

What do I mean by this? Just watch…

$$17^5 = 1{,}419{,}857$$

We can also do this

$$\log_{17} 1{,}419{,}857 = x$$

$$\cancel{\log_{17} 17}^5 = \log_{17} 1{,}419{,}857$$

$$\cancel{17^{\log_{17}}}^{1{,}419{,}857} = 17^x$$

$$5 = \log_{17} 1{,}419{,}857$$

$$1{,}419{,}857 = 17^x$$

This gives us these two expressions to remember

$$\boxed{\log_a a^x = x} \quad \text{and} \quad \boxed{a^{\log_a x} = x}$$

So let's recap…

$$\log_5 125 = x$$

$$\log_a M + \log_a N = \log_a MN$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M^n = n \log_a M$$

$$\log_a a^x = x \qquad a^{\log_a x} = x$$

And for the calculator…

$$\log_a M = \frac{\log_b M}{\log_b a}$$

Which can be done using the two programmed bases

$$\log_{10} \;\; and \;\; \ln$$

More on this one soon…