

# Rules of Logarithmic and Exponential Functions

$$5^3 = x$$

$$x^3 = 125$$

$$5^x = 125$$

Which of these would be hardest to solve algebraically?

$$5^3 = 125$$

$$\sqrt[3]{x^3} = \sqrt[3]{125}$$

$$5^x = 125$$

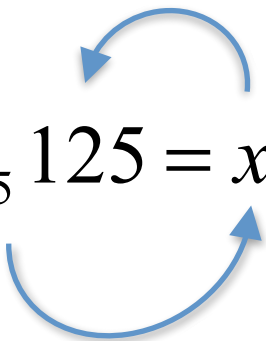
$$x = 5$$

?

We do have a function that we can perform here that would actually help us solve for an unknown exponent.

$$5^x = 125 \longrightarrow \log_5 125 = x$$

# Logarithmic Functions


$$\log_5 125 = x$$

Examples:

Ex 1)  $\log_3 81 = 4$

$$3^4 = 81$$

Ex 2) a)  $9^{\frac{3}{2}} = 27$

$$\log_9 27 = \frac{3}{2}$$

b)  $16^{-\frac{3}{4}} = \frac{1}{8}$

$$\log_{16} \frac{1}{8} = -\frac{3}{4}$$

By the way, the log with base  $e$  ( $\log_e$ ) has a special notation: **ln**

Did You Know that this means this?

$$\log_{16} \frac{1}{8} = -\frac{3}{4} \qquad \log_{16} 8 = \frac{3}{4}$$

We will find out why shortly

## Laws of Logs

$$\log_a M + \log_a N = \log_a MN$$

Really?

$$\log_2 4 + \log_2 8 = \log_2 32$$

$$2 + 3 = 5 \quad \checkmark$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M + \log_a M + \log_a M$$

$$\log_a M + \log_a M + \log_a M$$

and therefore...

$$\log_a M^n = n \log_a M$$

Did You Know This?

$$\log_{16} \frac{1}{8} = -\frac{3}{4} \qquad \log_{16} 8 = \frac{3}{4}$$

$$\log_a M^n = n \log_a M$$

$$\log_{16} 8^{-1} = -\frac{3}{4} \longrightarrow (-1) \log_{16} 8 = -\frac{3}{4}$$

$$\log_{16} 8 = \frac{3}{4}$$

$$\text{Ex 3 a) } \frac{1}{3} \log_4 64 - 4 \log_4 2$$

$$\log_4 64^{\frac{1}{3}} - \log_4 2^4$$

$$\log_4 4 - \log_4 16$$

$$\log_4 \frac{4}{16}$$

$$\begin{aligned} \log_4 \frac{1}{4} &= \log_4 4^{-1} = (-1) \log_4 4 \\ &= -1 \end{aligned}$$

$$\log_{17} 1,419,857 = x$$

$$17^x = 1,419,857 \longrightarrow 17^5 = 1,419,857$$

Only the most recent versions of the TI-84 have a log function with a variable base.

But how would we do this on older calculators?

How about using a log function that is on there like base 10?

$$\log_{10} 17^x = \log_{10} 1,419,857$$


$$x \log_{10} 17 = \log_{10} 1,419,857$$

$$x = \frac{\log_{10} 1,419,857}{\log_{10} 17} = 5$$

**Change of Base Formula**

$$\log_a M = \frac{\log_b M}{\log_b a}$$



$$\log_{17} 1,419,857 = x$$

$$17^x = 1,419,857$$

$$17^5 = 1,419,857$$

These two equations say the same thing which tells us something:

The log and exponential functions are inverses of each other

This also tells us that algebraically, each can be used to “undo” the other

What do I mean by this? Just watch...

$$17^5 = 1,419,857 \quad \text{We can also do this} \quad \log_{17} 1,419,857 = x$$

$$\cancel{\log_{17}} 17^5 = \log_{17} 1,419,857$$

$$\cancel{17^{\log_{17}}} 1,419,857 = 17^x$$

$$5 = \log_{17} 1,419,857$$

$$1,419,857 = 17^x$$

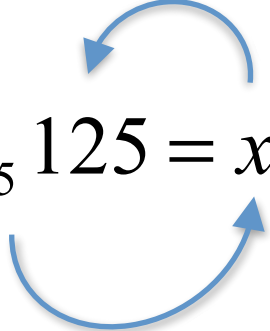
This gives us these two expressions to remember

$$\log_a a^x = x$$

and

$$a^{\log_a x} = x$$

# So let's recap...

$$\log_5 125 = x$$


$$\log_a M + \log_a N = \log_a MN$$

$$\log_a M - \log_a N = \log_a \frac{M}{N}$$

$$\log_a M^n = n \log_a M$$

$$\log_a a^x = x$$

$$a^{\log_a x} = x$$

And for the calculator...

$$\log_a M = \frac{\log_b M}{\log_b a}$$

Which can be done using the two programmed bases

$\log_{10}$  *and*  $\ln$



More on this one soon...

