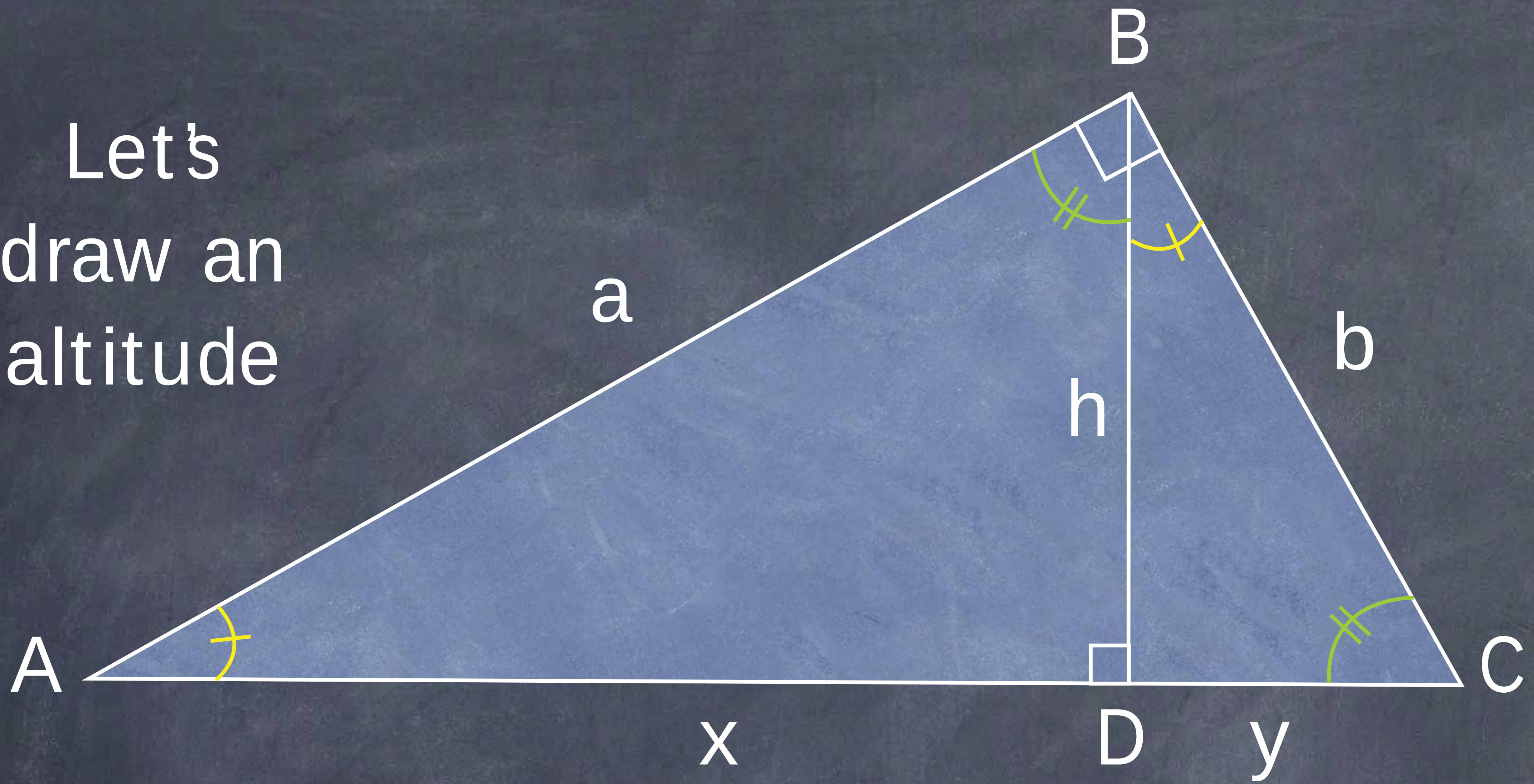
Similar Right Triangles & Geometric Mean

Let's draw an altitude



How many similar triangles are there here?

Three

Given both right angles

$m\angle A + m\angle C = 90°$ so

$\angle A$ is common to $\triangle ABC$ and $\triangle ADB$

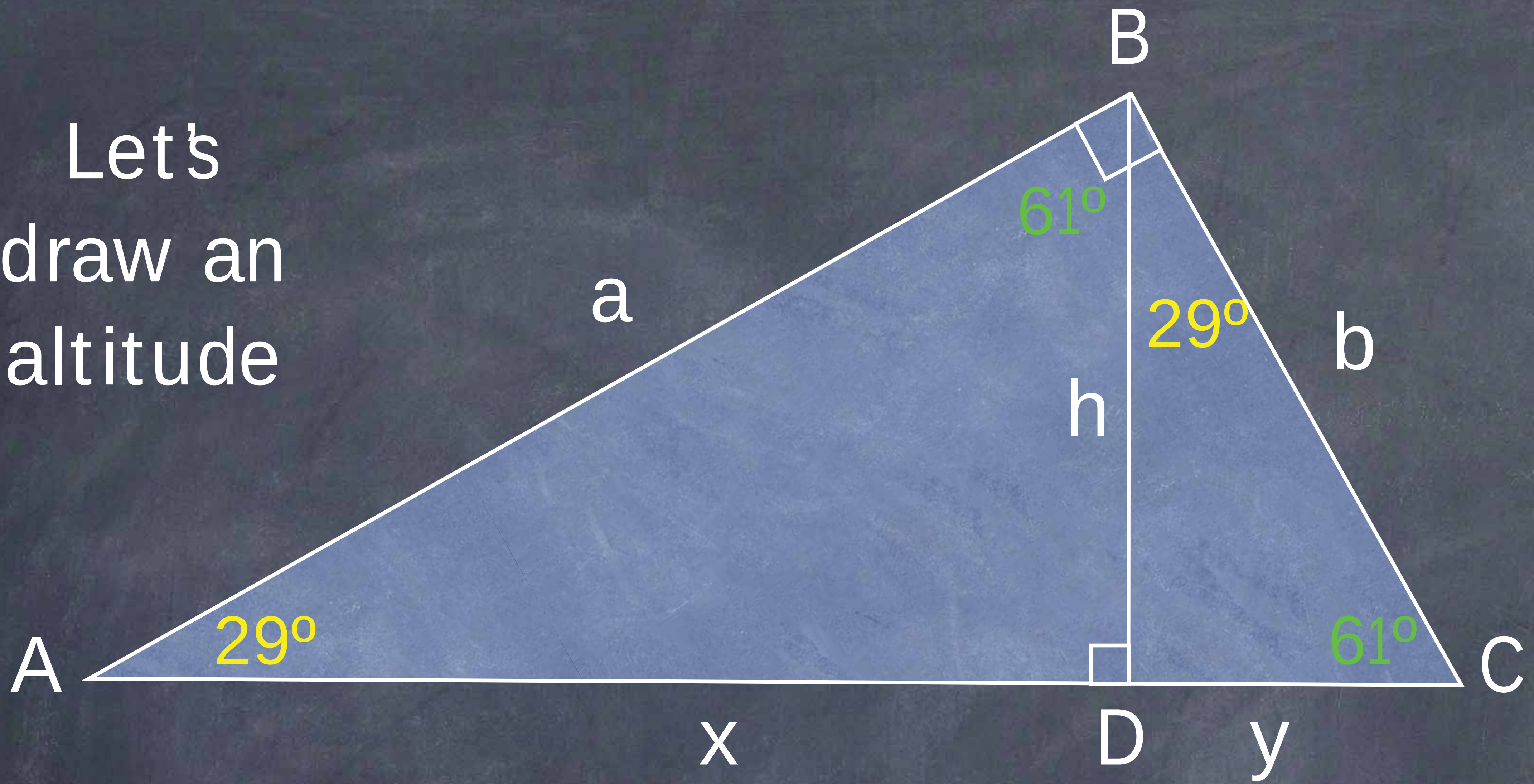
$\angle A \cong \angle DBC$

$\angle C$ is common to $\triangle ABC$ and $\triangle BDC$

$\angle C \cong \angle ABD$

Still not convinced?

Let's draw an altitude



How many similar triangles are there here?

Three

Given both right angles

$\angle A$ is common to $\triangle ABC$ and $\triangle ADB$

$\angle C$ is common to $\triangle ABC$ and $\triangle BDC$

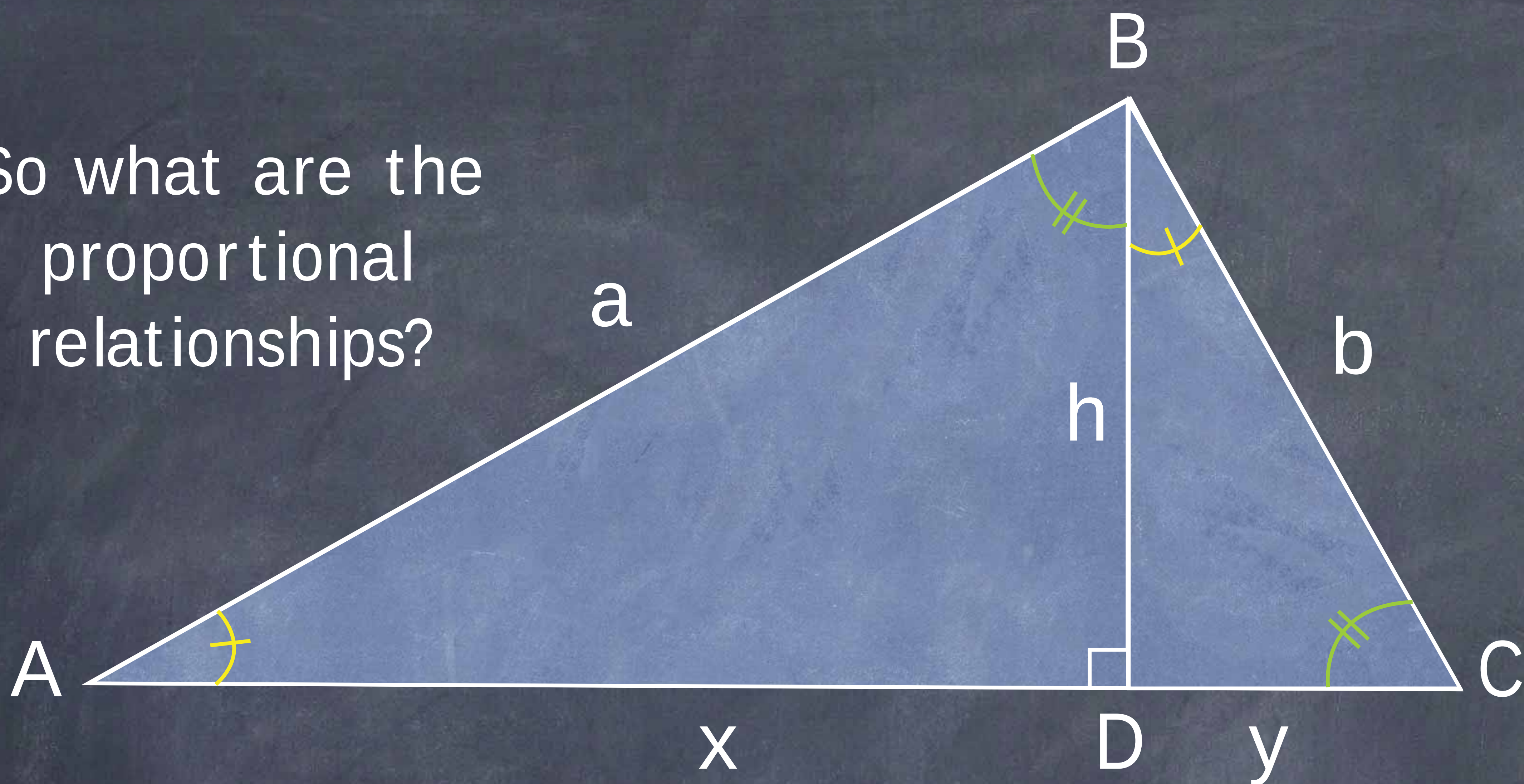
Let's assign some angle measurements

$m\angle A = 29°$ $m\angle C = 61°$

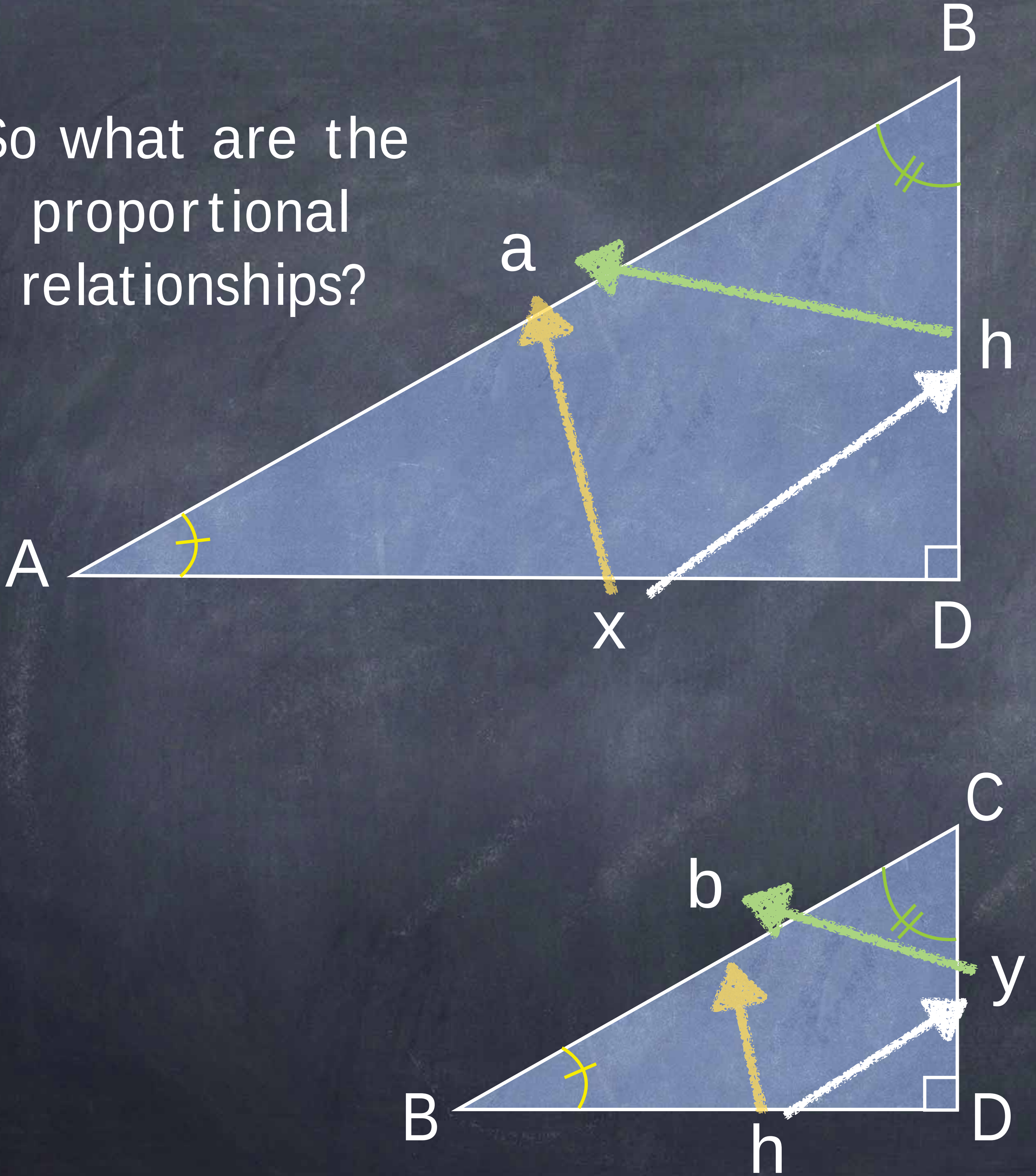
$m\angle DBC = 29°$ $m\angle ABD = 61°$

so

So what are the proportional relationships?



So what are the proportional relationships?

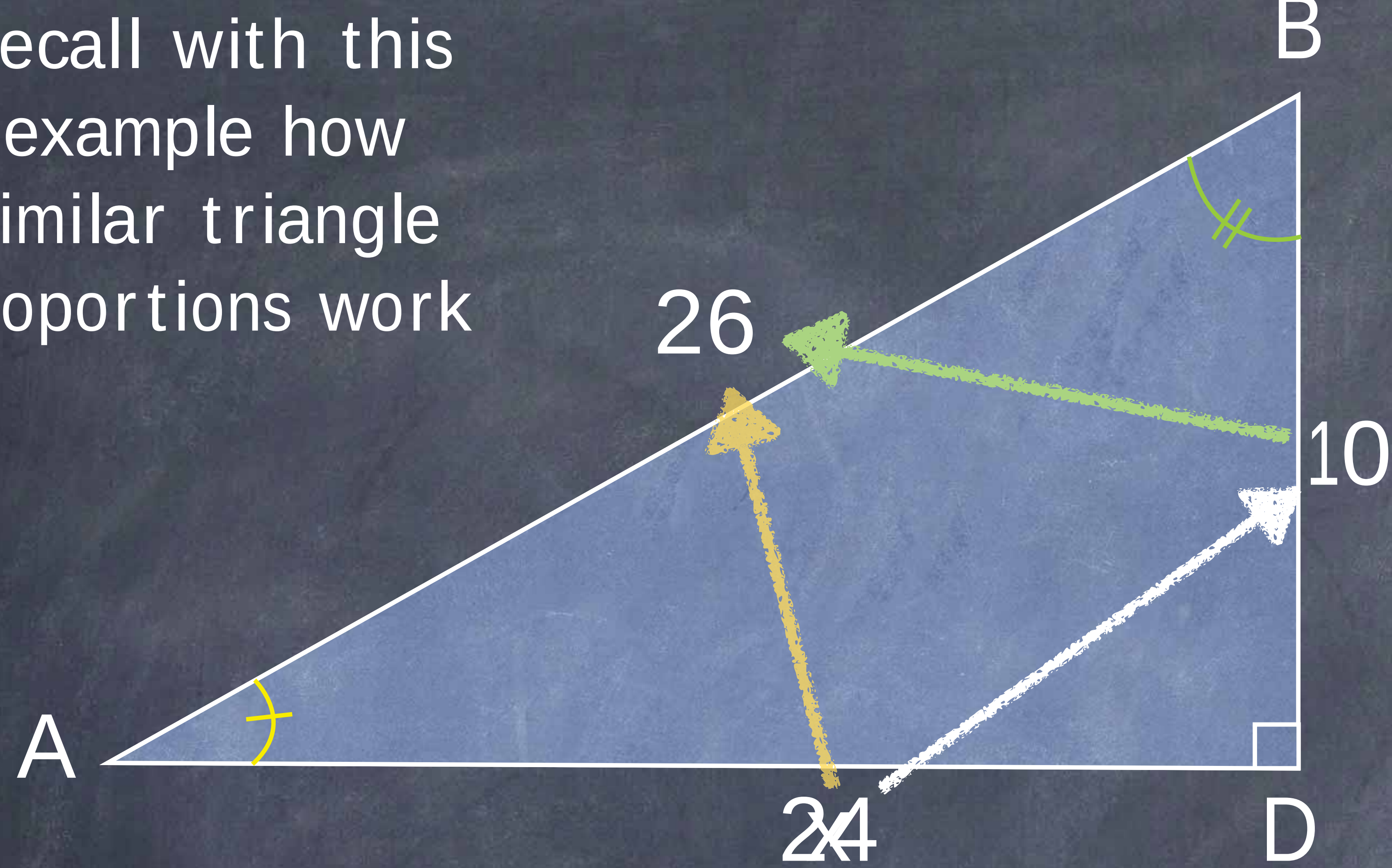


$$\frac{x}{h} = \frac{h}{y}$$

$$\frac{x}{a} = \frac{h}{b}$$

$$\frac{h}{a} = \frac{y}{b}$$

Recall with this example how similar triangle proportions work



$$\frac{x}{10} = \frac{h}{5}$$

Not helpful

$$\frac{x}{26} = \frac{h}{b}$$

Not helpful

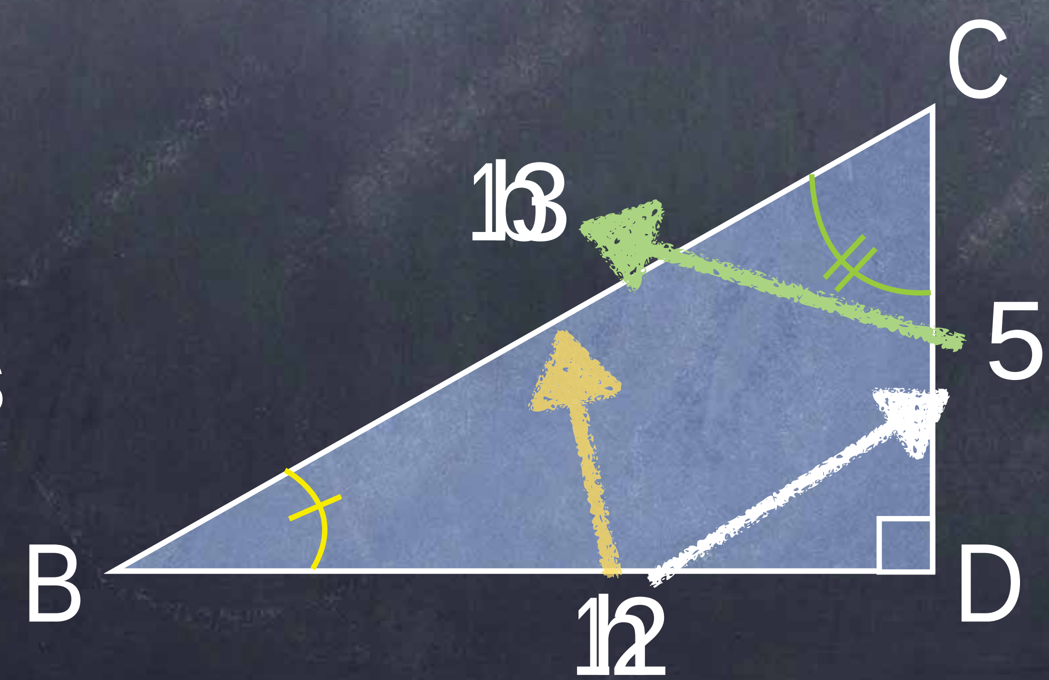
$$\frac{10}{26} = \frac{5}{b}$$

Now we can solve for b

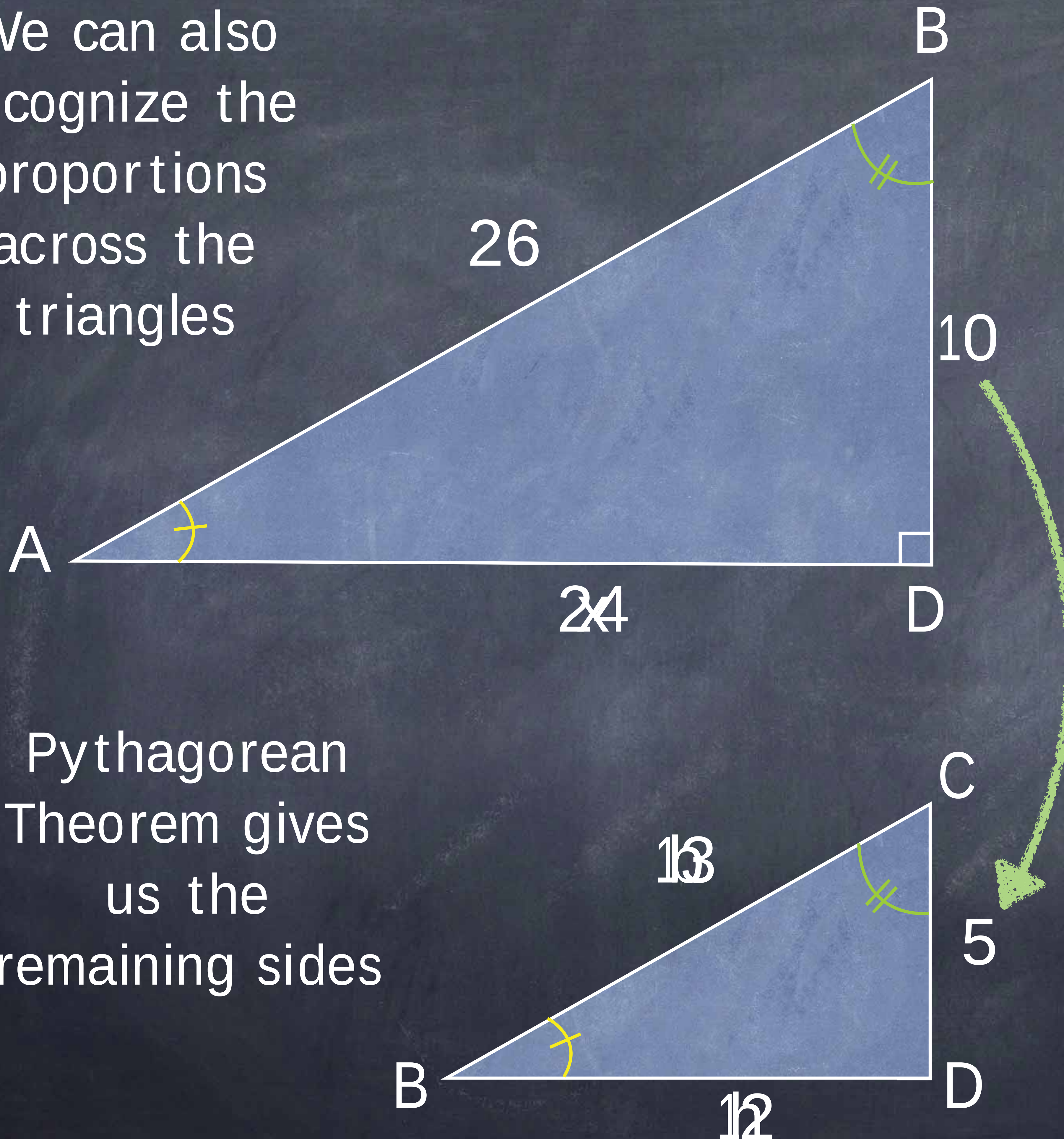
$$\frac{\cancel{10}\ 5}{\cancel{26}\ 13} = \frac{5}{b}$$

$$b = 13$$

Pythagorean Theorem gives us the remaining sides



We can also recognize the proportions across the triangles



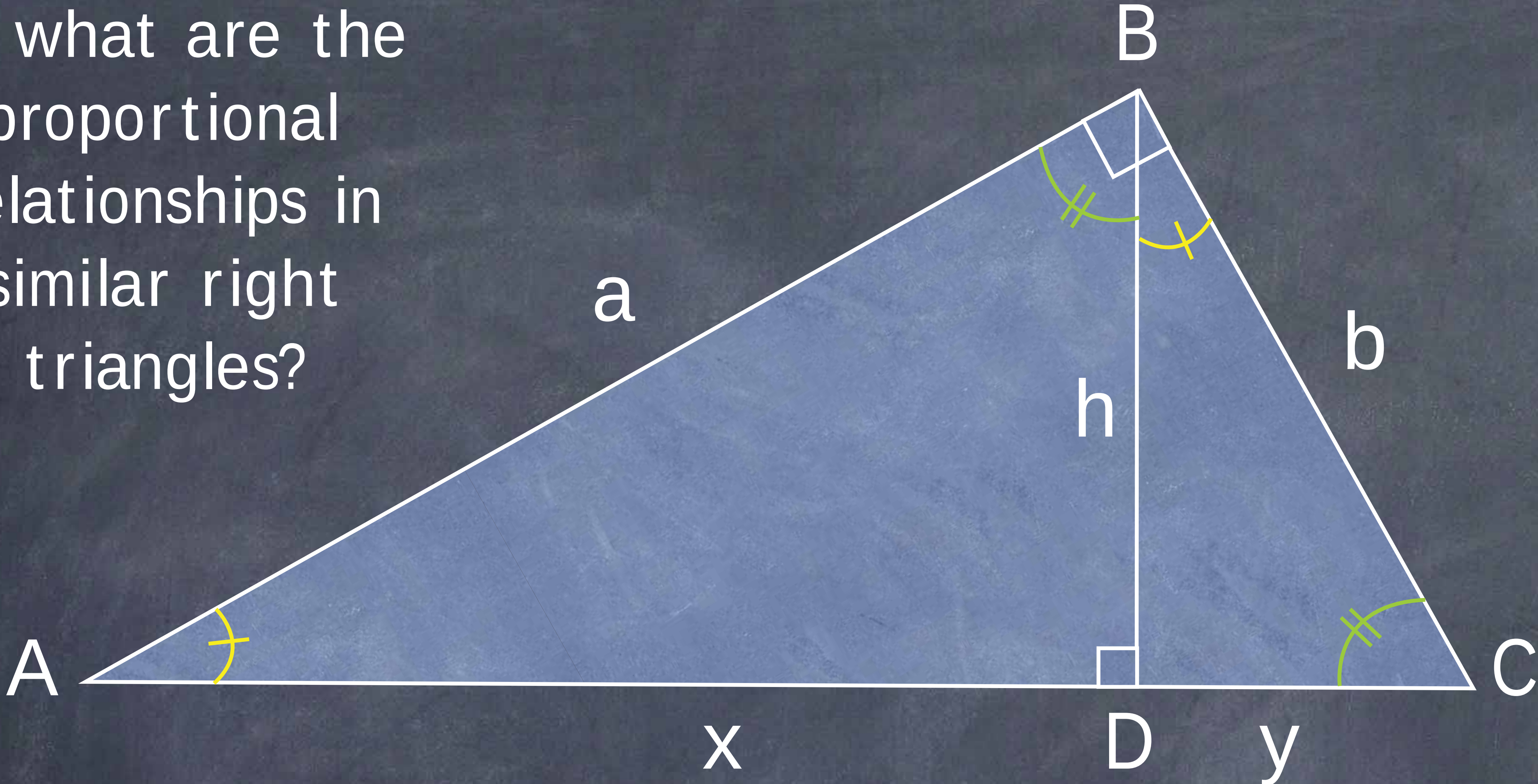
A ratio of 2 to 1

We know that b will be half of 26

Pythagorean Theorem gives us the remaining sides

This also shows that we can choose whatever proportions work as long as they are corresponding sides

So what are the proportional relationships in similar right triangles?



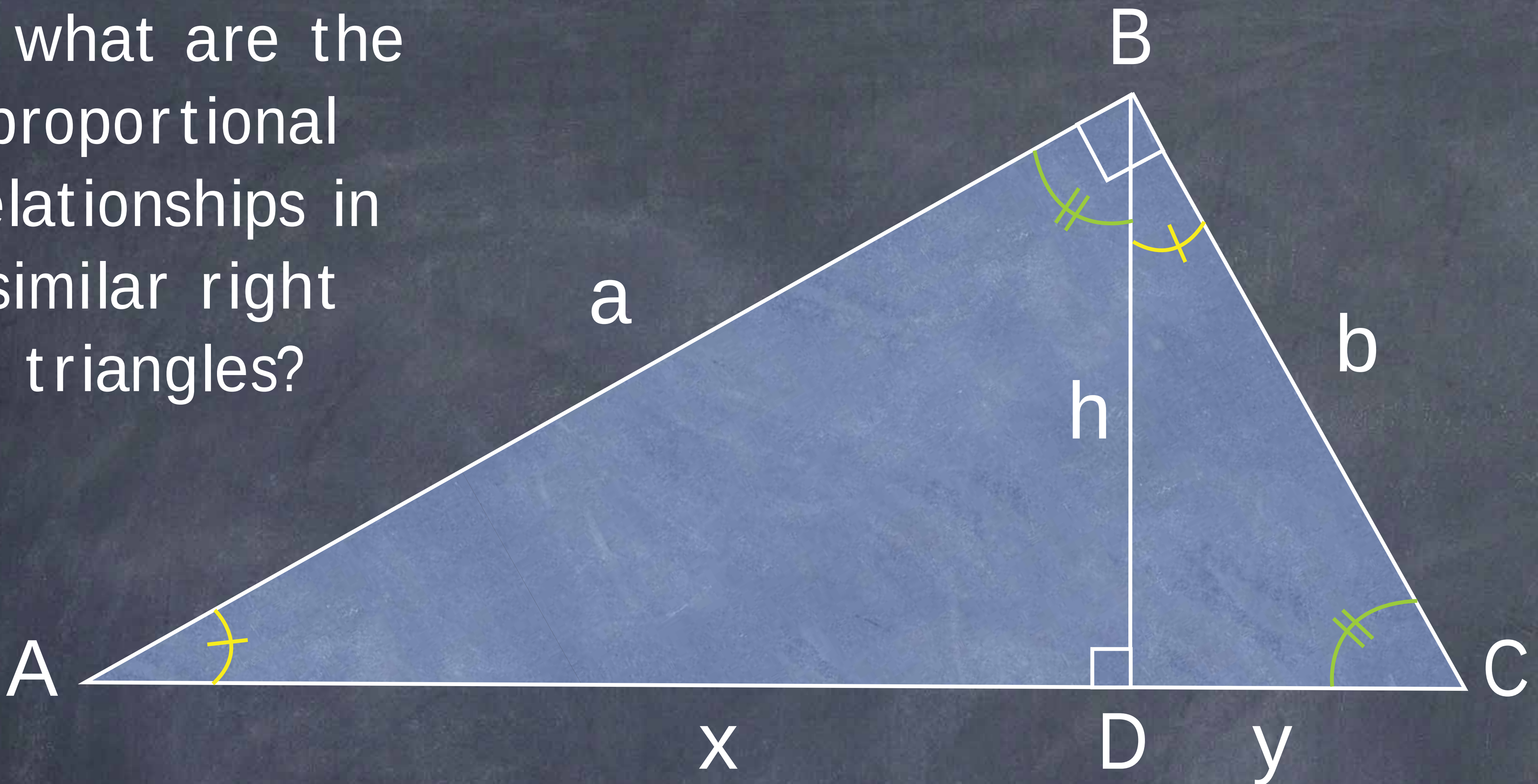
$$\boxed{\frac{x}{h} = \frac{h}{y}} = \frac{a}{b} \qquad \boxed{\frac{a}{x+y} = \frac{x}{a}} = \frac{h}{b} \qquad \boxed{\frac{b}{x+y} = \frac{y}{b}} = \frac{h}{a}$$

Let's just focus on these highlighted proportions

Cross multiplying these proportions gives us

$$h^2 = xy \qquad a^2 = x(x+y) \qquad b^2 = y(x+y)$$

So what are the proportional relationships in similar right triangles?

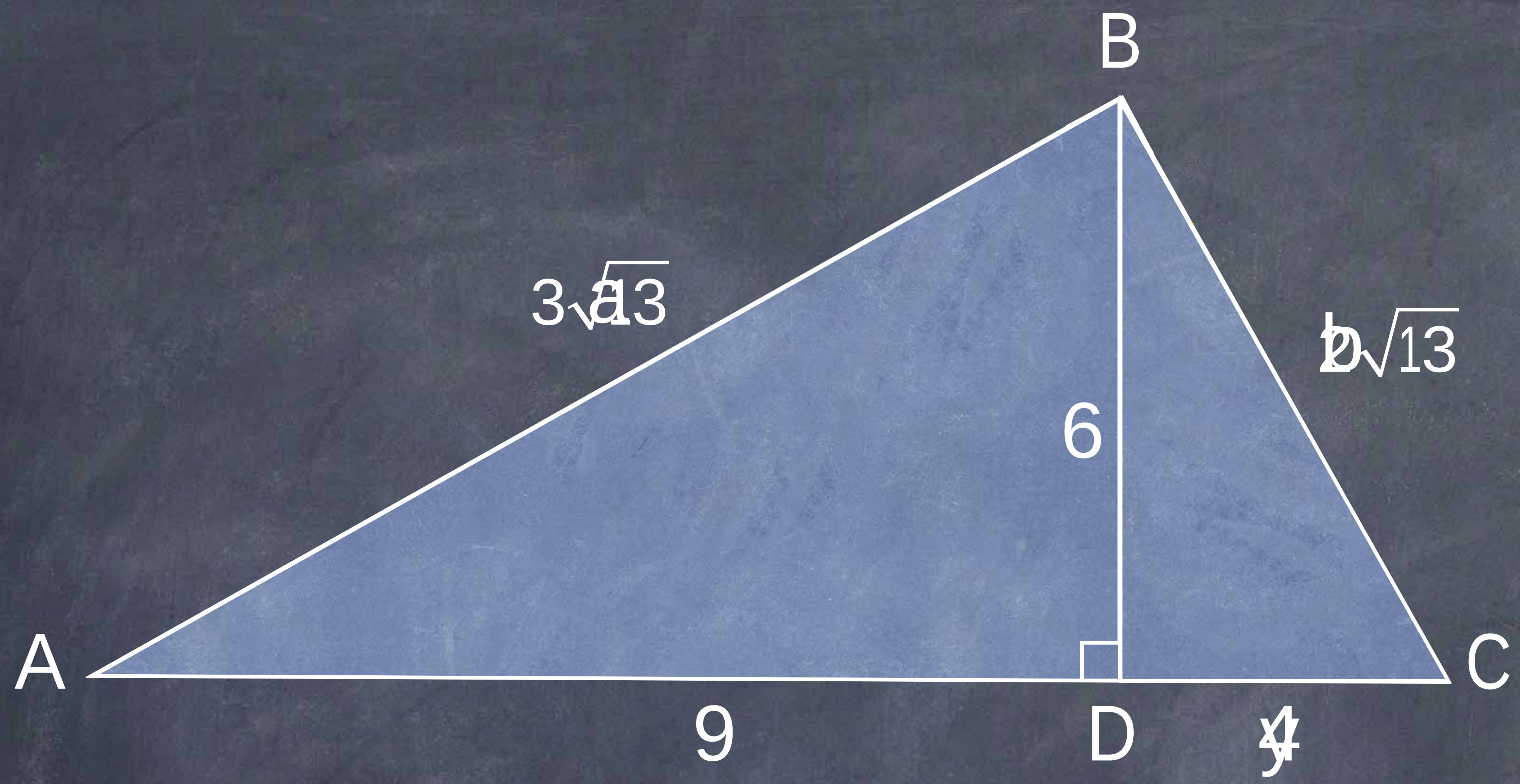


This is the one most useful in solving for missing lengths

$h^2 = xy$ In this case, h is considered the Geometric Mean of x and y

$a^2 = x(x + y)$ a is considered the Geometric Mean of x and x+y

$b^2 = y(x + y)$ and b is considered the Geometric Mean of y and x+y



$$9^2 + 6^2 = a^2$$
$$81 + 36 = a^2$$
$$117 = a^2$$
$$a = \sqrt{117}$$

$$a = \sqrt{9 \cdot 13}$$
$$a = 3\sqrt{13}$$

$$h^2 = xy$$
$$6^2 = 9y$$
$$36 = 9y$$
$$y = 4$$

$$4^2 + 6^2 = b^2$$
$$16 + 36 = b^2$$
$$52 = b^2$$
$$b = \sqrt{52}$$

$$b = \sqrt{4 \cdot 13}$$
$$b = 2\sqrt{13}$$