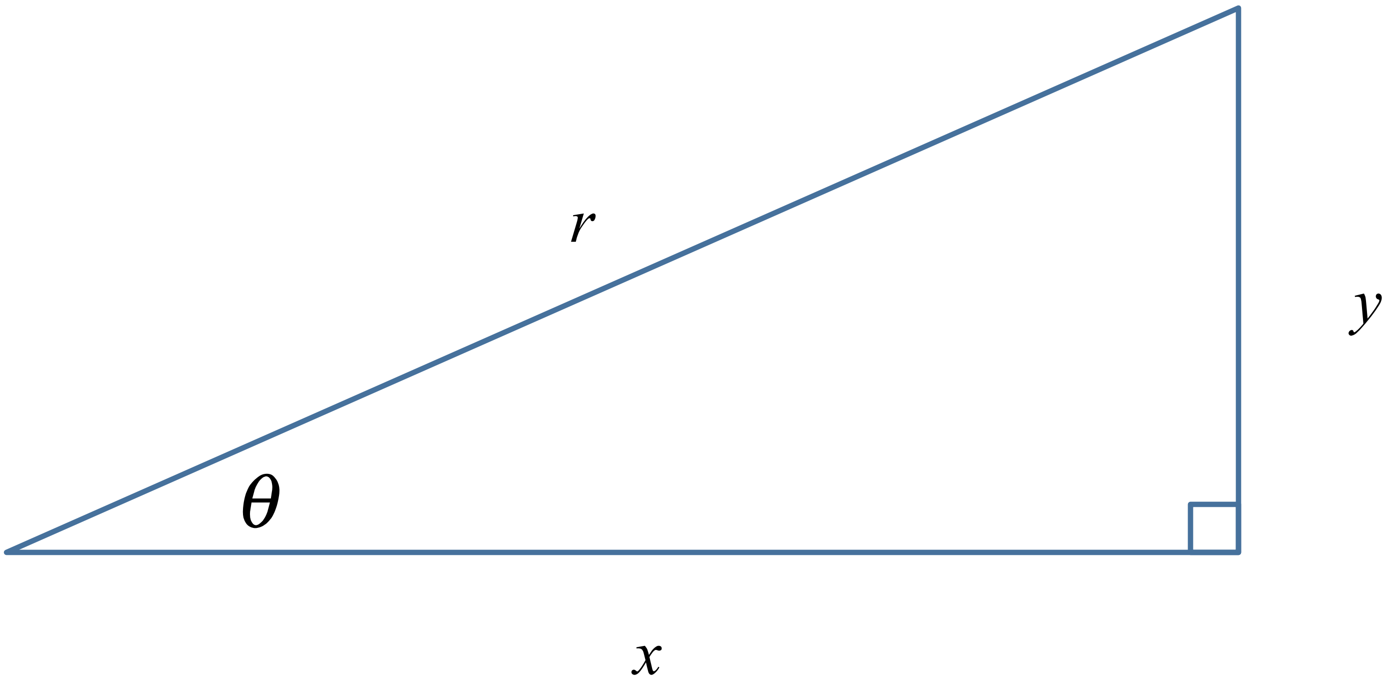
Standard A5: Prove Trigonometric Identities and use them to simplify Trigonometric equations



$$\frac{y}{r} \qquad \frac{x}{r} \qquad \frac{y}{x}$$

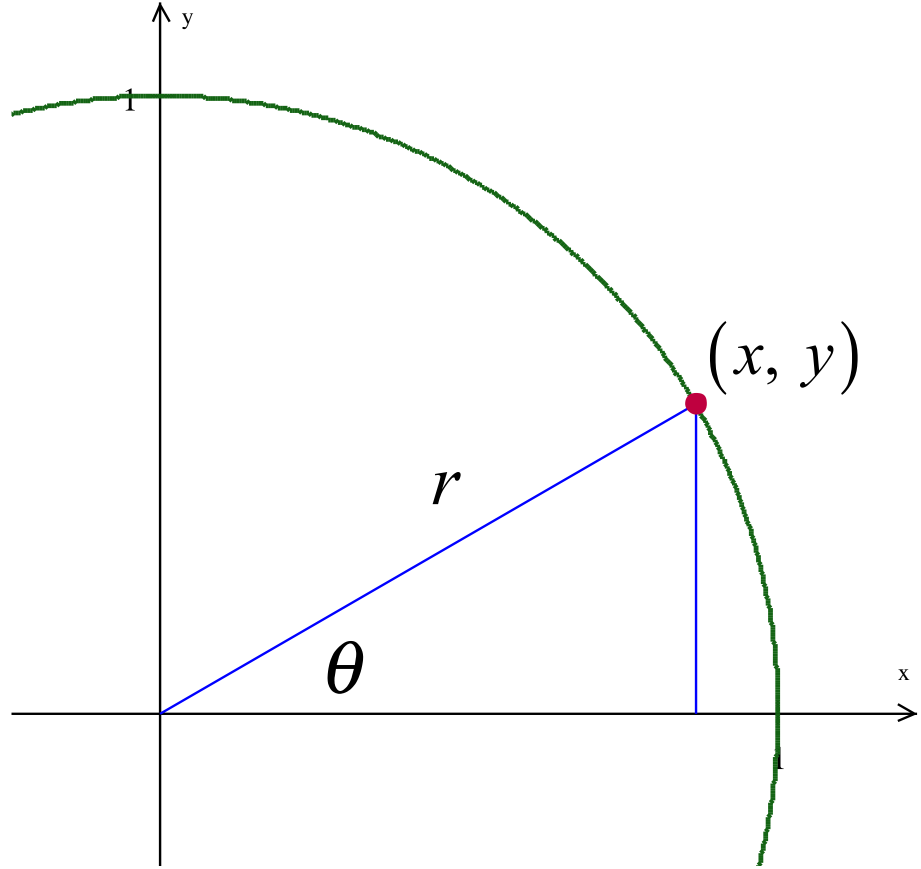
$$x^2 + y^2 = r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2}$$

$$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 = 1$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

This is the first of three
Pythagorean Identities



Reciprocal Trig Functions

$$\csc\theta = \frac{1}{\sin\theta}$$

How did we get this, you wonder?

$$\sin\theta = \frac{y}{r} \qquad \frac{1}{\sin\theta} = \frac{1}{\left(\dfrac{y}{r}\right)} \qquad \frac{1}{\sin\theta} = \frac{r}{y}$$

Which according to page 21 is $\csc\theta$

$$\csc\theta = \frac{1}{\sin\theta}$$

Reciprocal Trig Functions

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

And don't forget…

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

These are the three
Pythagorean Identities

These and the other identities on Pg 21 and 22…

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

…will have to be memorized

The Pythagorean identities have alternative versions as well:

$$\begin{array}{lll} \sin^2\theta+\cos^2\theta=1 & \tan^2\theta+1=\sec^2\theta & 1+\cot^2\theta=\csc^2\theta \\ 1-\cos^2\theta=\sin^2\theta & \sec^2\theta-1=\tan^2\theta & \csc^2\theta-1=\cot^2\theta \\ 1-\sin^2\theta=\cos^2\theta & \sec^2\theta-\tan^2\theta=1 & \csc^2\theta-\cot^2\theta=1 \end{array}$$

Pg 22 is important as well

These alternative forms are very useful because they are "difference of squares" binomials that can be factored. For example,

$$1-\sin^2\theta=(1-\sin\theta)(1+\sin\theta)$$

The Pythagorean identities have alternative versions as well:

$$\begin{array}{lll} \sin^2\theta+\cos^2\theta=1 & \tan^2\theta+1=\sec^2\theta & 1+\cot^2\theta=\csc^2\theta \\ 1-\cos^2\theta=\sin^2\theta & \sec^2\theta-1=\tan^2\theta & \csc^2\theta-1=\cot^2\theta \\ 1-\sin^2\theta=\cos^2\theta & \sec^2\theta-\tan^2\theta=1 & \csc^2\theta-\cot^2\theta=1 \end{array}$$

LEARNING OUTCOME

Prove basic trigonometric identities.

The proofs of the identities are algebraic in nature, meaning that the use of multiplication, addition, and common denominators will cause one side of the equation to simplify to the other. Or, similar to proofs in Geometry, that both sides simplify to the same thing.

EX 1 Prove $\csc x\tan x\cos x=1$

$$\begin{gathered} \csc x\tan x\cos x= \\ \frac{1}{\sin x}\cdot\frac{\sin x}{\cos x}\cdot\cos x= \\ 1 \end{gathered}$$

Notice that the answer is the process, not the final line; the final line was given.

Show that

$$\sin\theta\cot\theta = \cos\theta$$

Rewrite in terms of sine and cosine

$$\cancel{\sin\theta\frac{\cos\theta}{\sin\theta}} = \cos\theta$$

$$\cos\theta = \cos\theta$$

These are proofs but not as rigorous. Here are some tips on how you can approach them.

- ***Write everything in terms of sine and cosine***

  This often works though not always. Still, it can be a good way to start as you saw in the first example.

- ***Look for squares*** - Check for Pythagorean Identity substitutions (squared trig functions). If a direct substitution is there, use it.

$$\cos^2 x(1+\tan^2 x)=$$

- ***Parentheses*** - Distribute if parentheses get in the way. Factor if parentheses can be helpful

$$\cos x(\sec x+\tan x)=$$

- ***Common Denominators*** - If you have fractions that need to be added or subtracted, look for common denominators

$$\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}=$$

Show that

$$\frac{\csc\theta}{\sec\theta} = \cot\theta$$

$$\frac{\dfrac{1}{\sin\theta}}{\dfrac{1}{\cos\theta}} = \cot\theta$$

$$\frac{\cos\theta}{\sin\theta} = \cot\theta$$

Show that

$$\sin\theta(1+\cot^2\theta)=\csc\theta$$

Notice the identity first

$$\sin\theta(\csc^2\theta)=\csc\theta$$

$$\cancel{\sin\theta}\frac{1}{\sin^{\cancel{2}}\theta}=\csc\theta$$

$$\frac{1}{\sin\theta}=\csc\theta$$