

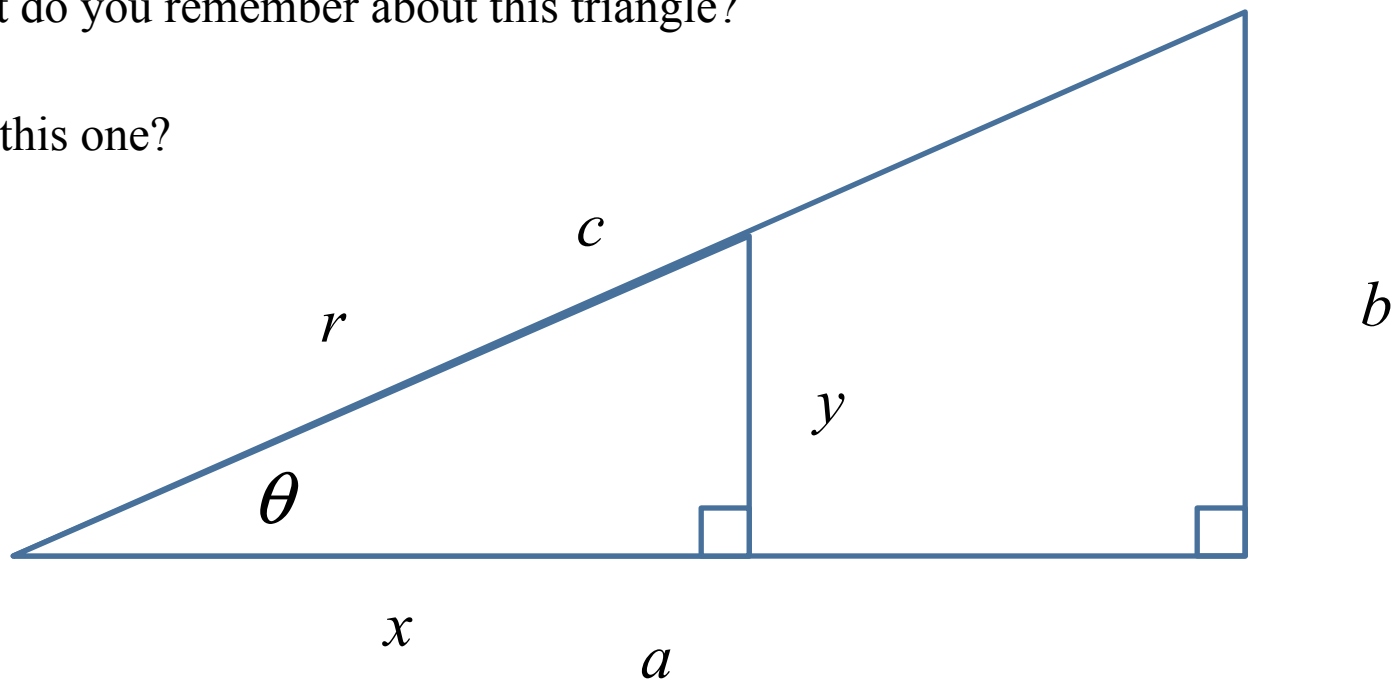
$$\frac{y}{r}$$

$$\frac{x}{r}$$

$$\frac{y}{x}$$

What do you remember about this triangle?

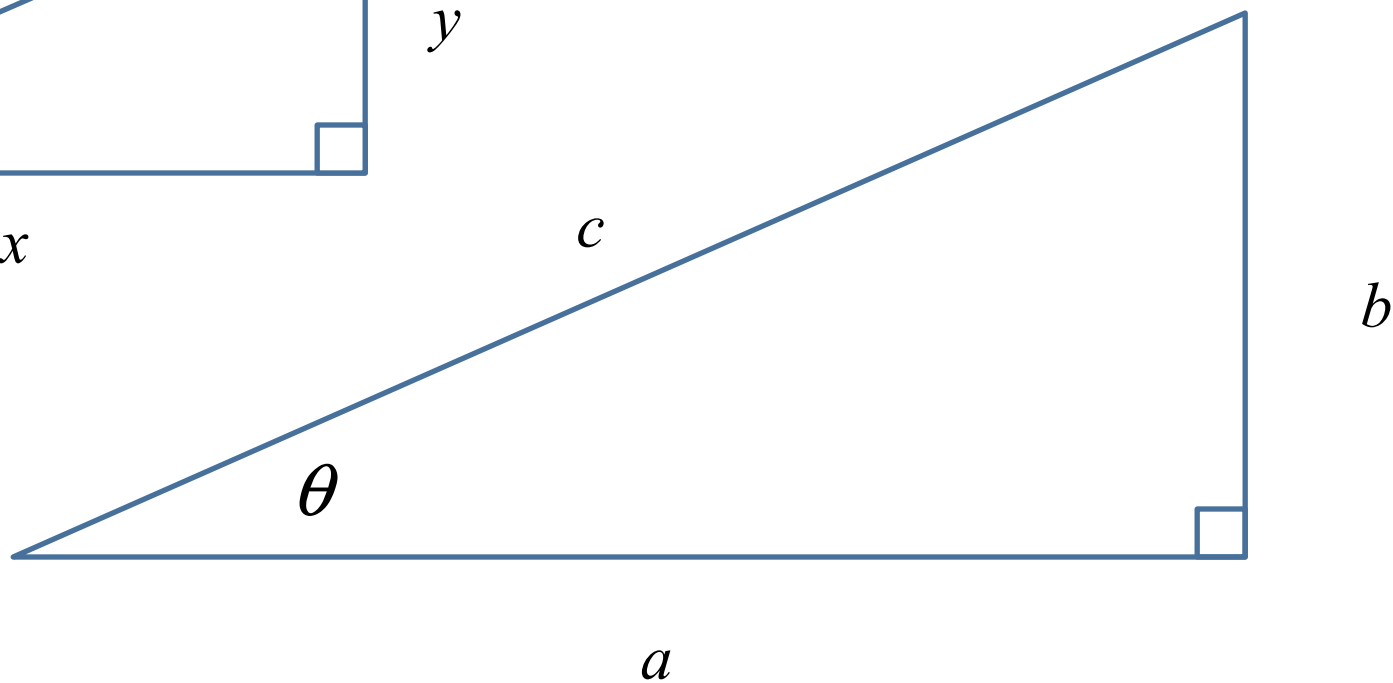
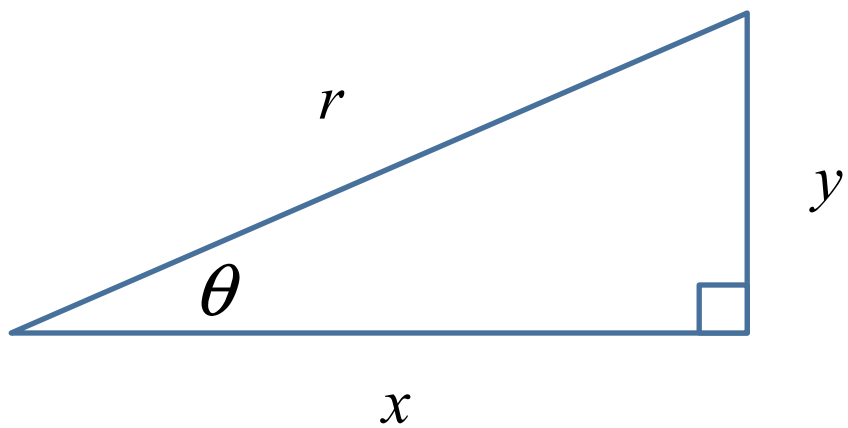
And this one?



$$\frac{y}{r} = \frac{b}{c}$$

$$\frac{x}{r} = \frac{a}{c}$$

$$\frac{y}{x} = \frac{b}{a}$$



$$\frac{y}{r}$$

$$\frac{b}{c}$$

$$\frac{x}{r}$$

$$\frac{a}{c}$$

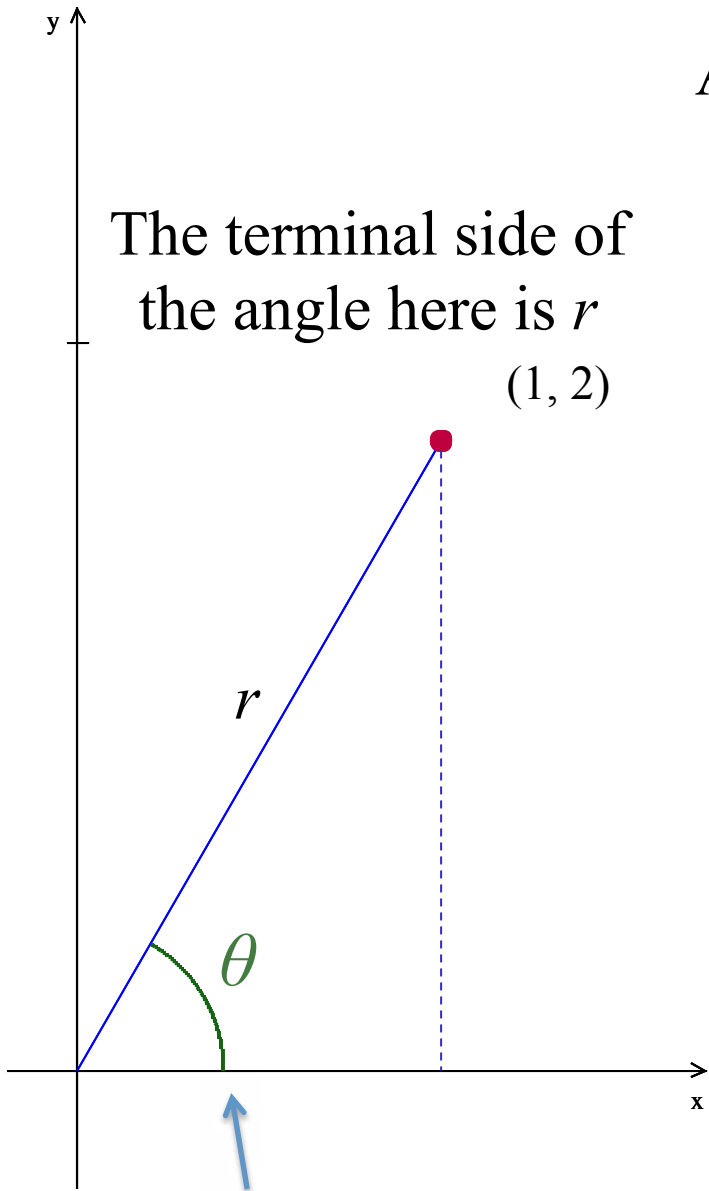
$$\frac{y}{x}$$

$$\frac{b}{a}$$

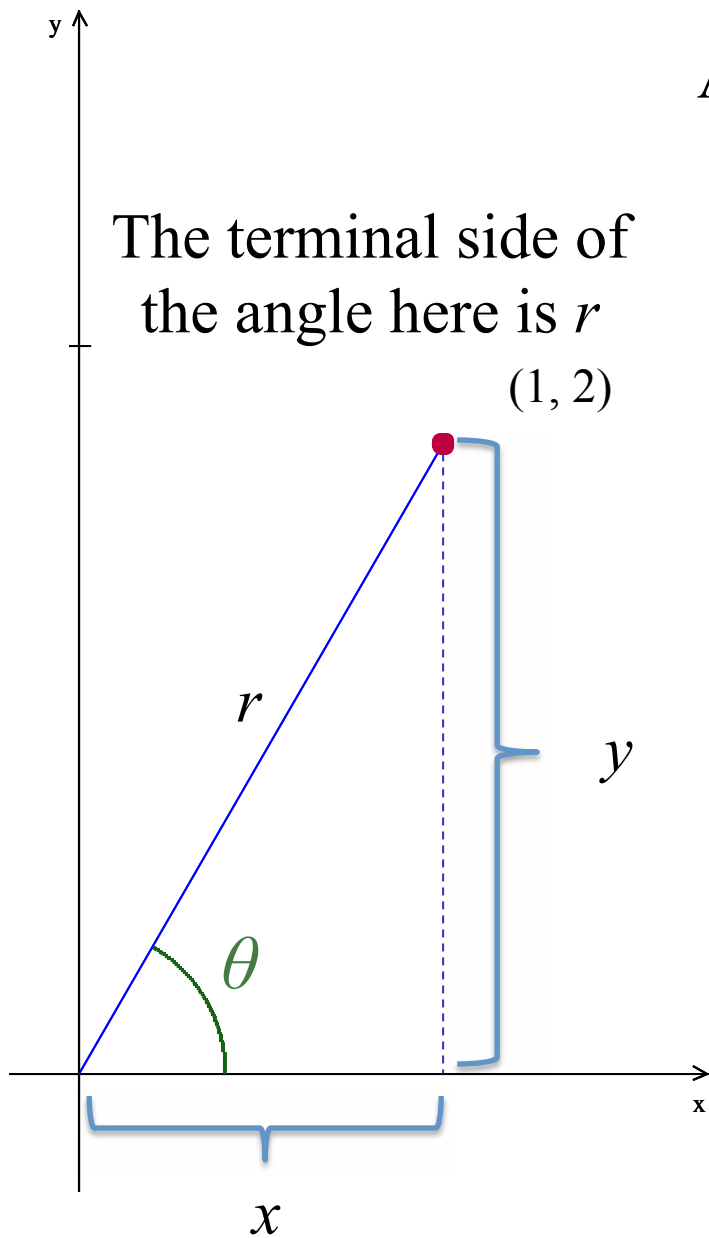
And r can be found
by...

The terminal side of
the angle here is r

(1, 2)



Starting θ from the positive x -axis
is called Standard Position



And r can be found by...

$$r^2 = x^2 + y^2$$

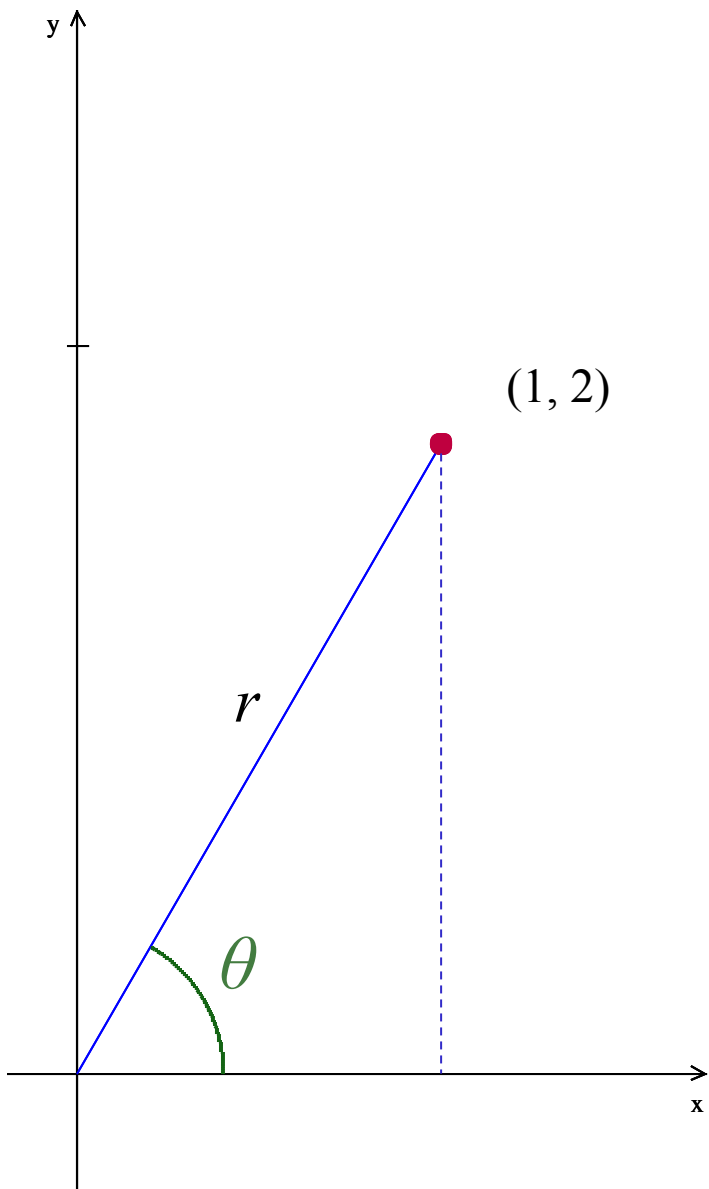
$$r^2 = 1^2 + 2^2$$

$$r = \sqrt{5}$$

Important Note: the value of r will always be positive

$$\sin \theta = ?$$

$$\sin \theta = \frac{y}{r}$$



$$r = \sqrt{5}$$

$$\sin \theta = \frac{2}{\sqrt{5}}$$

$$\csc \theta = \frac{\sqrt{5}}{2}$$

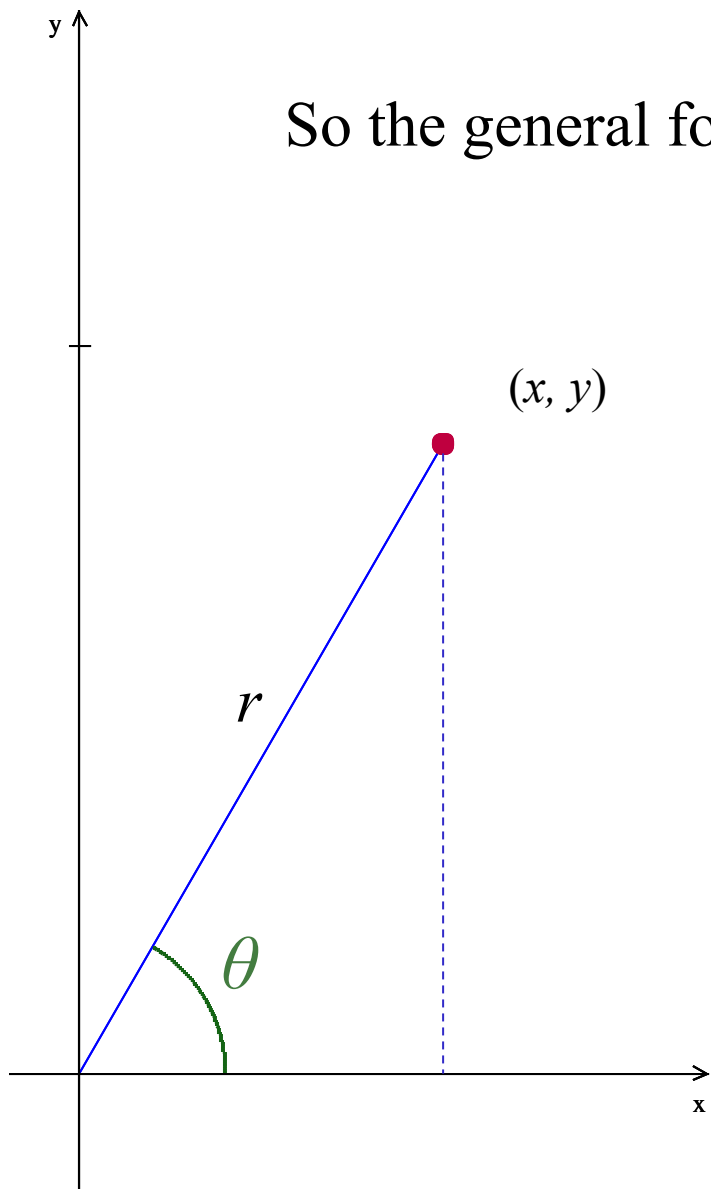
$$\cos \theta = \frac{1}{\sqrt{5}}$$

$$\sec \theta = \sqrt{5}$$

$$\tan \theta = 2$$

$$\cot \theta = \frac{1}{2}$$

So the general formulas look like this:



$$\sin \theta = \frac{y}{r}$$

$$\csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r}$$

$$\sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x}$$

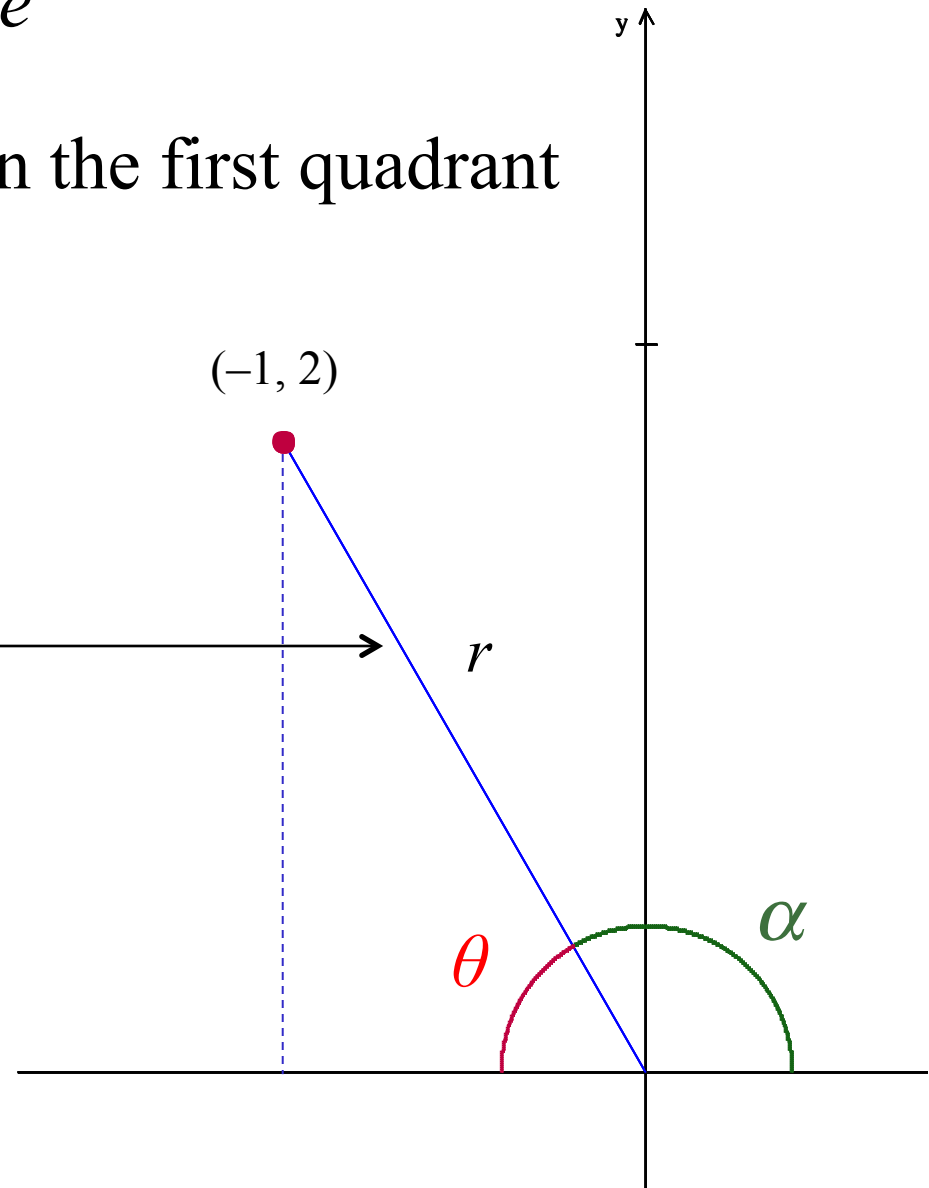
$$\cot \theta = \frac{x}{y}$$

Here θ is the *reference angle*

This is the same θ we saw in the first quadrant

reference angle

The acute angle
made between the
terminal side (r)
and the x -axis



Find all of the trig functions of α

$$x^2 + y^2 = r^2 \quad r^2 = 5$$

$$(-1)^2 + (2)^2 = r^2 \quad r = \sqrt{5}$$

$$\sin \alpha = \frac{2}{\sqrt{5}}$$

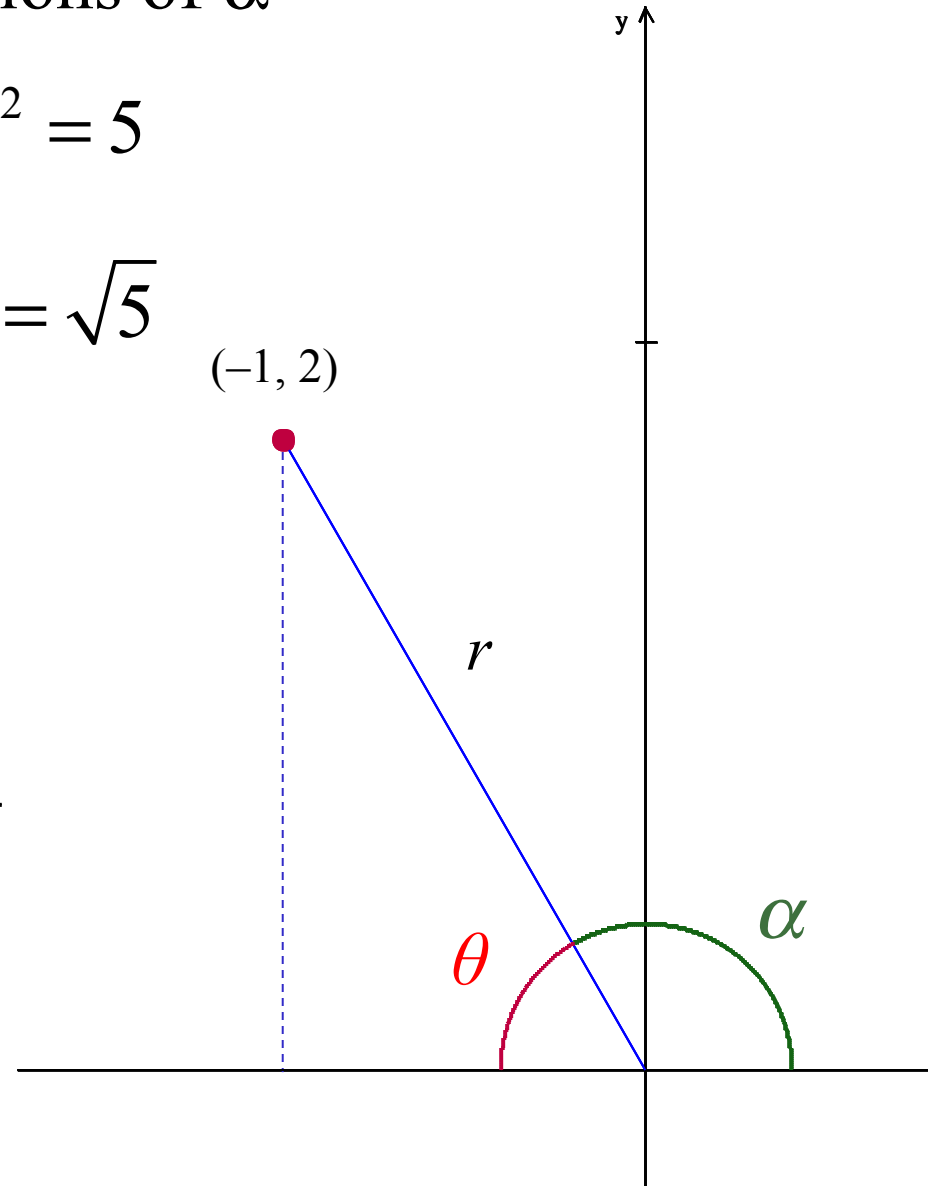
$$\csc \alpha = \frac{\sqrt{5}}{2}$$

$$\cos \alpha = -\frac{1}{\sqrt{5}}$$

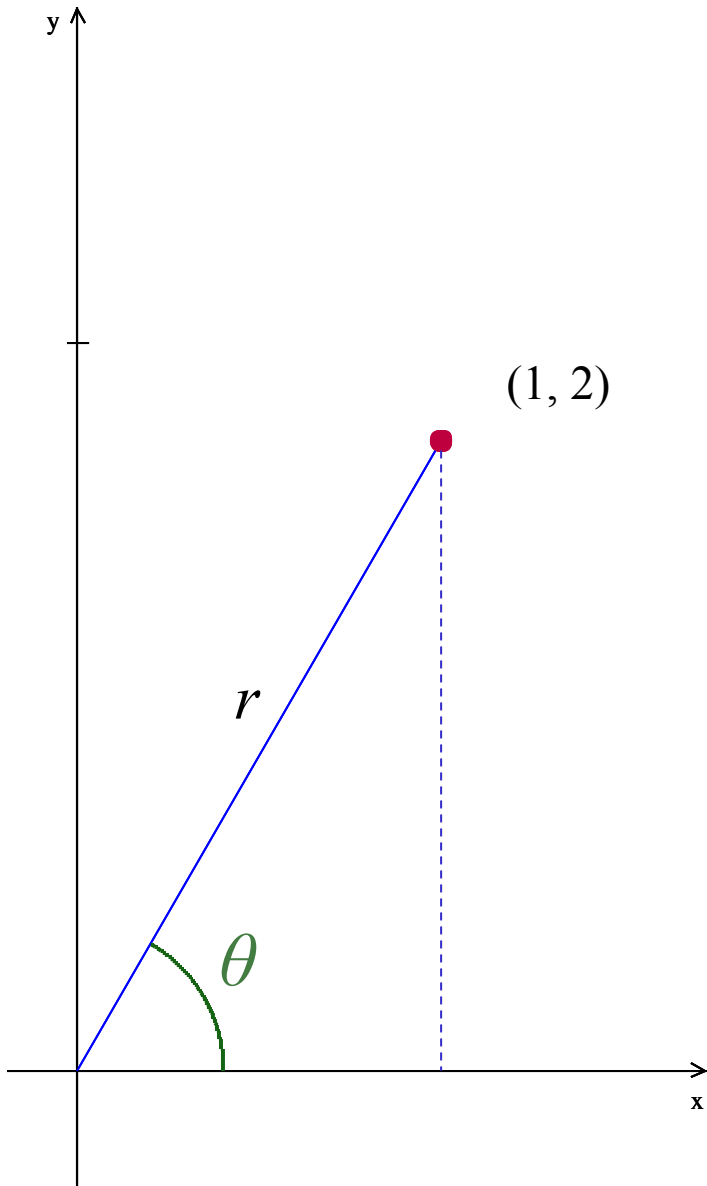
$$\sec \alpha = -\sqrt{5}$$

$$\tan \alpha = -2$$

$$\cot \alpha = -\frac{1}{2}$$



Compare these with our QI angle



$$r = \sqrt{5}$$

$$\sin \theta = \frac{2}{\sqrt{5}}$$

$$\csc \theta = \frac{\sqrt{5}}{2}$$

$$\cos \theta = \frac{1}{\sqrt{5}}$$

$$\sec \theta = \sqrt{5}$$

$$\tan \theta = 2$$

$$\cot \theta = \frac{1}{2}$$

Same trig values only the cosine and secant are negative in the QII case

Find all of the trig functions of α

$$x^2 + y^2 = r^2 \quad r^2 = 5$$

$$(-1)^2 + (2)^2 = r^2 \quad r = \sqrt{5}$$

$$\sin \alpha = \frac{2}{\sqrt{5}}$$

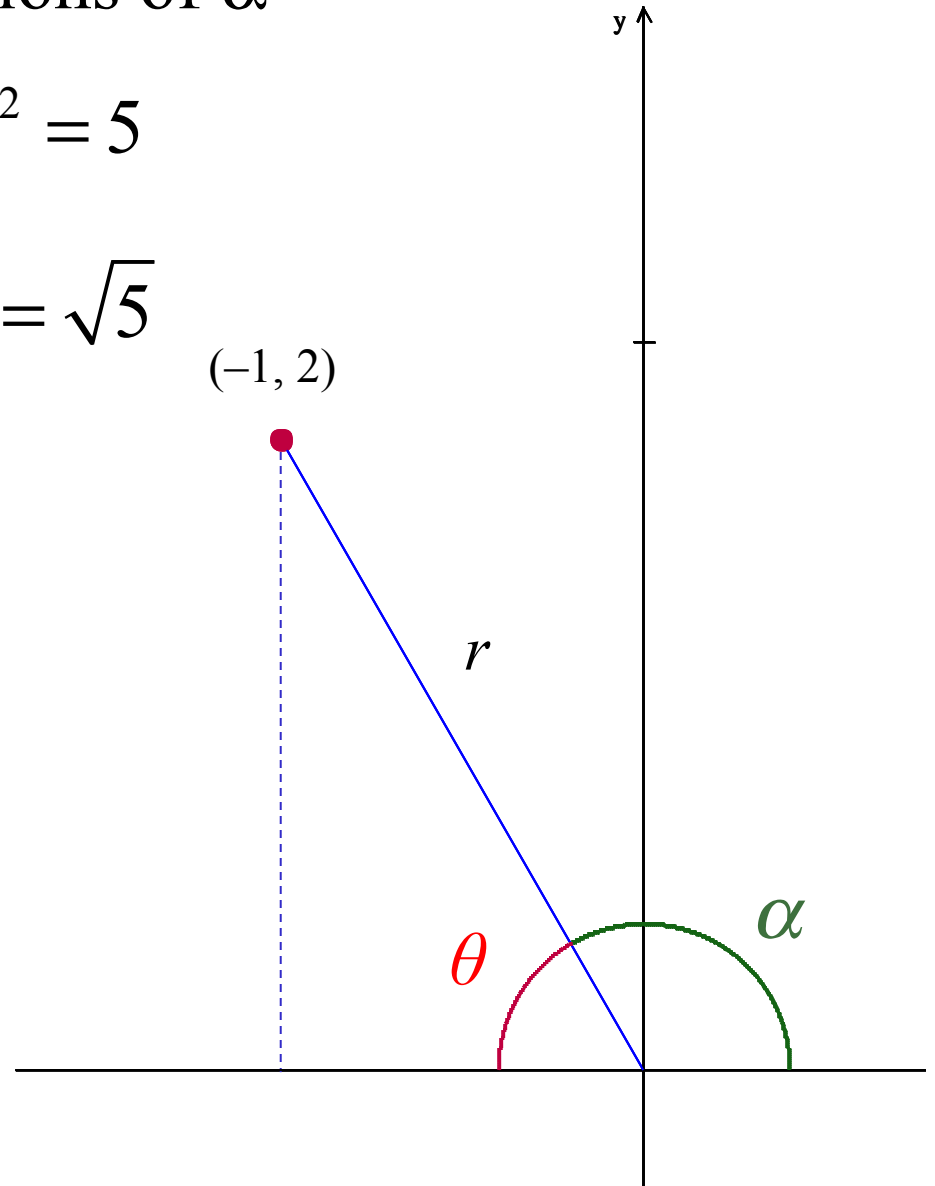
$$\csc \alpha = \frac{\sqrt{5}}{2}$$

$$\cos \alpha = -\frac{1}{\sqrt{5}}$$

$$\sec \alpha = -\sqrt{5}$$

$$\tan \alpha = -2$$

$$\cot \alpha = -\frac{1}{2}$$



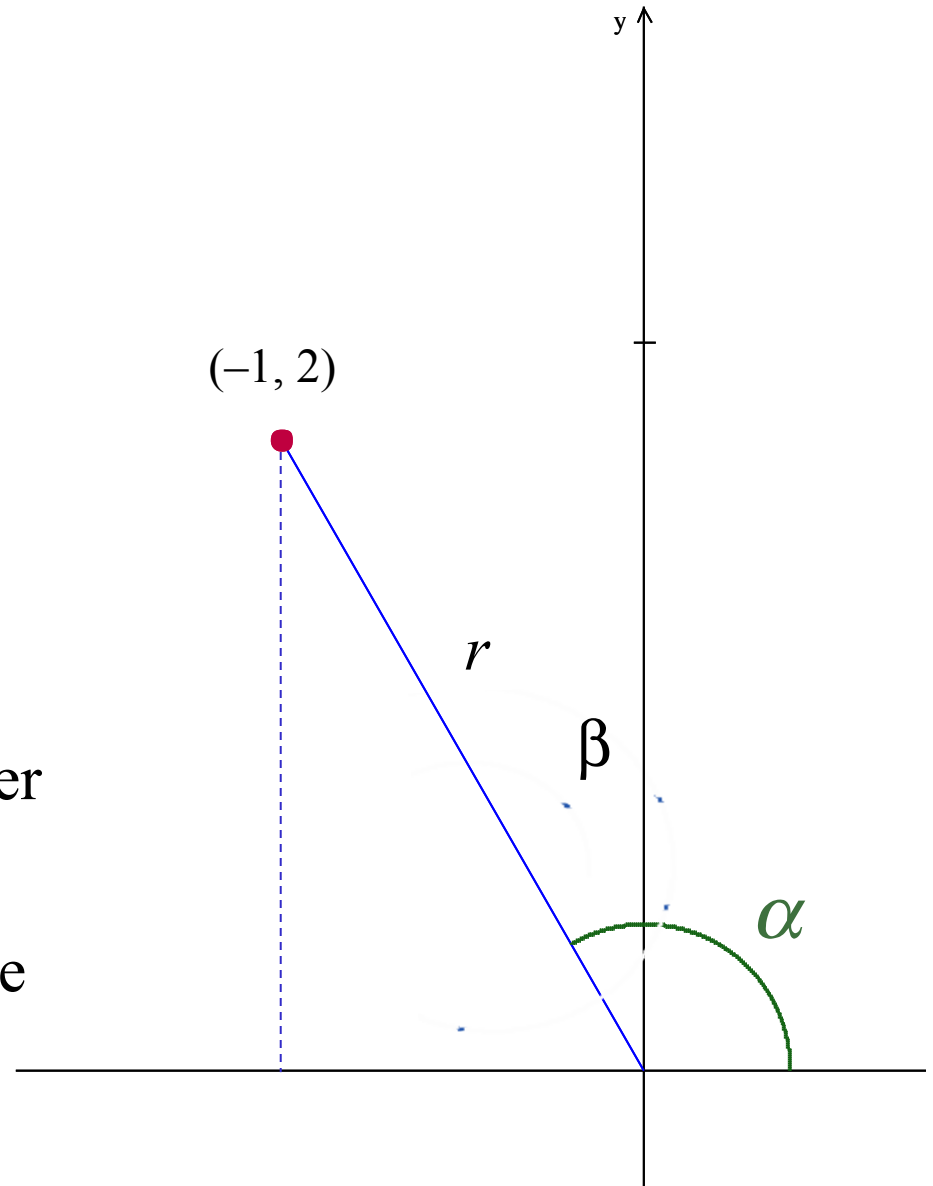
Co-terminal angles

Angles that have
the same initial and
terminal sides

α and β are
examples of co-
terminal angles.

Co-terminal angles always differ
by multiples of 360°

They also will have the same
sine, cosine, tangent, etc.



Find all of the trig functions of α given a terminal side through the point $(9, 40)$

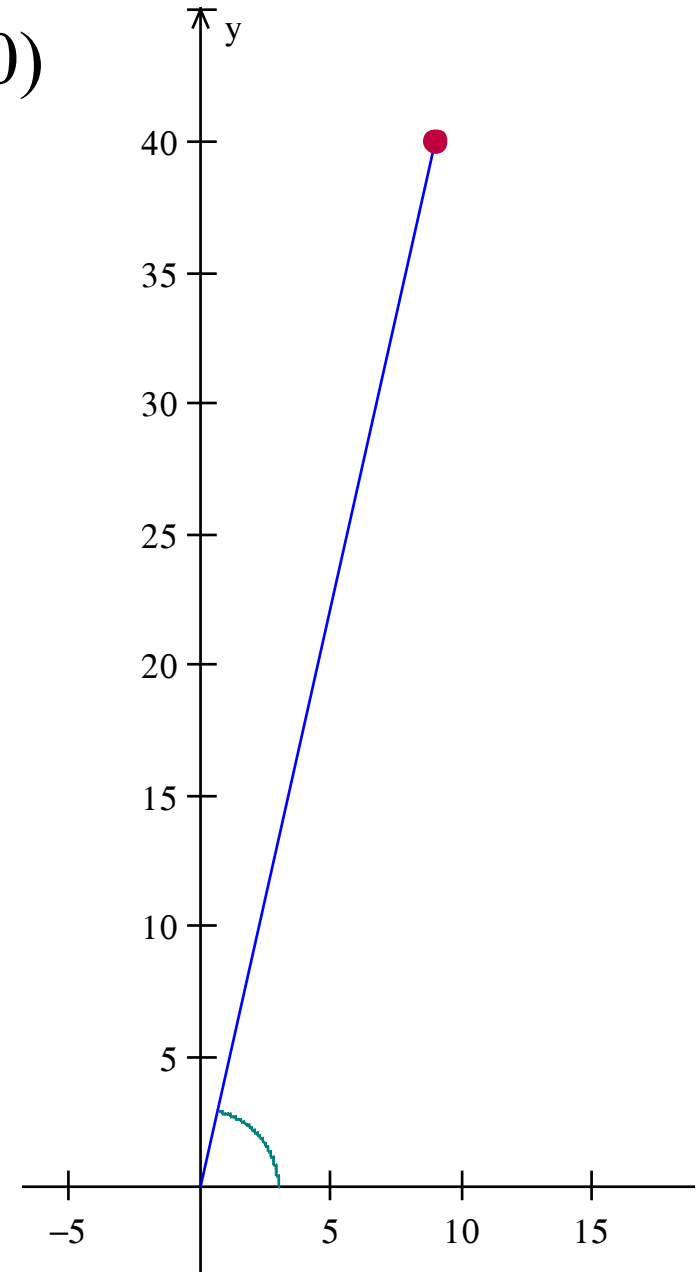
$$x^2 + y^2 = r^2 \quad r^2 = 1681$$

$$9^2 + 40^2 = r^2 \quad r = 41$$

$$\sin \alpha = \frac{40}{41} \quad \csc \alpha = \frac{41}{40}$$

$$\cos \alpha = \frac{9}{41} \quad \sec \alpha = \frac{41}{9}$$

$$\tan \alpha = \frac{40}{9} \quad \cot \alpha = \frac{9}{40}$$



Find all of the trig functions of α given a terminal side through the point $(-5, -12)$

$$x^2 + y^2 = r^2 \quad r^2 = 169$$

$$(-5)^2 + (-12)^2 = r^2 \quad r = 13$$

$$\sin \alpha = -\frac{12}{13}$$

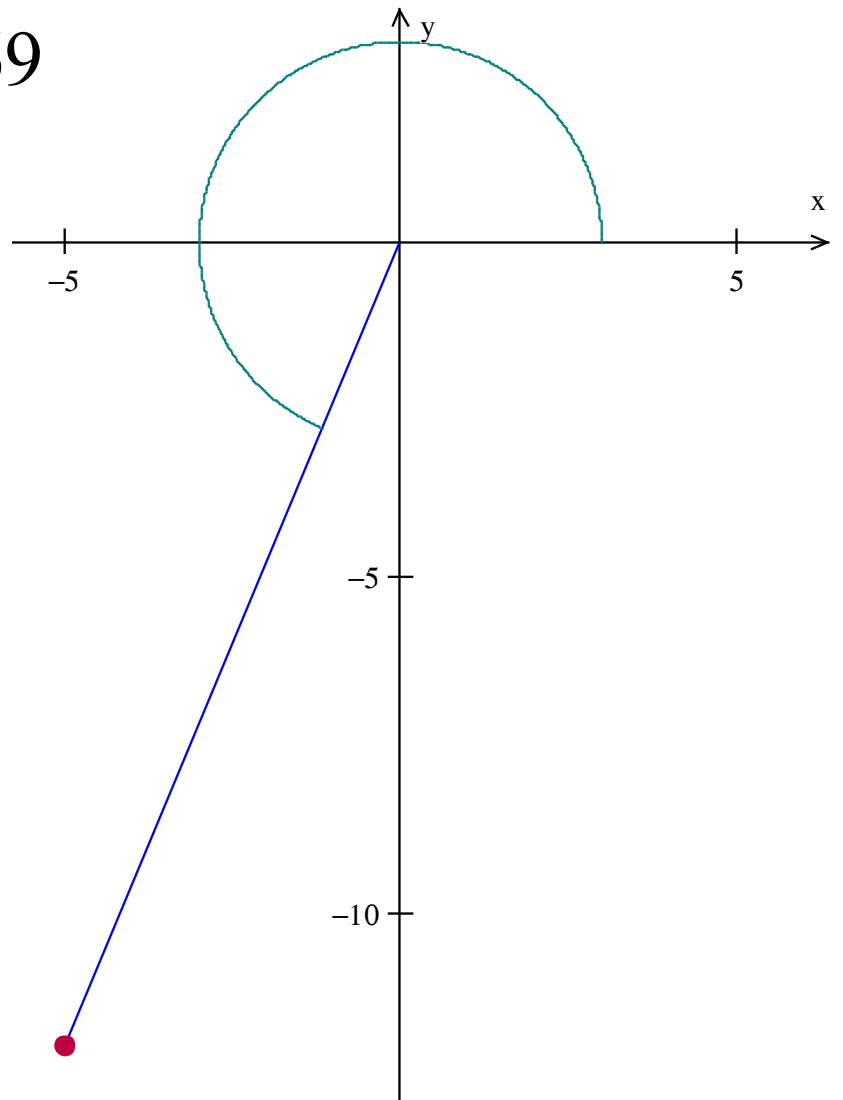
$$\csc \alpha = -\frac{13}{12}$$

$$\cos \alpha = -\frac{5}{13}$$

$$\sec \alpha = -\frac{13}{5}$$

$$\tan \alpha = \frac{12}{5}$$

$$\cot \alpha = \frac{5}{12}$$



Find all of the trig functions of θ given that $\sin \theta = -\frac{24}{25}$

...and θ is in Quadrant IV

$$x^2 + (-24)^2 = 25^2$$

$x = \pm 7$ But since we're in Quadrant IV

$$x = 7$$

$$\cos \theta = \frac{7}{25} \quad \sec \theta = \frac{25}{7}$$

$$\tan \theta = -\frac{24}{7} \quad \cot \theta = -\frac{7}{24}$$

$$\csc \theta = -\frac{25}{24}$$

