Remember this?

Divide $x^3 - x^2 + x - 1$ by $x - 7$

We can do this

$$
\begin{array}{r}
x^2 + 6x + 43 \\
x - 7 \overline{)\, x^3 - x^2 + x - 1} \\
\underline{-(x^3 - 7x^2)} \\
6x^2 + x \\
\underline{-(6x^2 - 42x)} \\
43x - 1 \\
\underline{-(43x - 301)} \\
300
\end{array}
$$

Now there is a simpler way of doing this

So the remainder is 300

Divide $x^3 - x^2 + x - 1$ by $x - 7$

$x^2 + 6x + 43$ With a remainder of 300

Watch this. Can you figure out what's going on?

$$
\begin{array}{c|cccc}
7 & 1 & -1 & 1 & -1 \\
 & & 7 & 42 & 301 \\
\hline
 & 1 & 6 & 43 & 300
\end{array}
$$

And again the result is
300

Divide $1x^3 - 1x^2 + 1x - 1$ by $x - 7$

Let's try this again but slowly...

7	1	−1	1	−1
		7	42	301
	1	6	43	300

the coefficients of the polynomial

Start with the original coefficient

Multiply it by the number you're plugging in (7) and add the result to −1

This method is called <u>Synthetic Division</u>

It is also called <u>Synthetic Substitution</u> because...

$$f(x) = x^3 - x^2 + x - 1$$

$$f(7) = 300$$

So we can find function values using this method which is why it is also called <u>Synthetic Substitution</u>

How many roots (zeros) does

$$y = x^3 - x^2 + x - 1$$

have?

In other words, what value(s) for x will give us y = 0 when we plug them in?

Here is where Synthetic Substitution can help

$$0 = x^3 - x^2 + x - 1$$

After setting y = 0, we could use grouping to factor this and get

$$0 = (x-1)(x^2+1)$$

We could also use Synthetic Substitution since plugging in 1 would be so easy to do

We are going to choose 1 as our root since it is very easy to check if this is a good guess

Remember...

$$\begin{array}{c|cccc} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

Start with the original coefficient

Multiply by 1 and add to the next coefficient

Repeat the process and if your remainder is 0, then (x – 1) is a factor

$$y = x^3 - x^2 + x - 1$$

$$y = (x-1)\underbrace{(x^2+1)}$$

Notice something about this term

$$\begin{array}{r|rrrr} 1 & 1 & -1 & 1 & -1 \\ & & 1 & 0 & 1 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$y = (x-1)(x^2 + 0x + 1)$$

What remains is in descending order beginning with a degree of 2 (one smaller than what we started with)

Still not sure?

Let's do another one then...

$$y = 1x^3 + 1x^2 - 4x - 4$$

$$\begin{array}{r|rrrr} 1 & 1 & 1 & -4 & -4 \\ & & 1 & 2 & -2 \\ \hline & 1 & 2 & -2 & -6 \neq 0 \end{array}$$

Which means what?

That 1 is not a root so $(x - 1)$ is not a factor

It does mean that

$(1, -6)$ is a point on the graph of $y = x^3 + x^2 - 4x - 4$

It also means that

$$x^3 + x^2 - 4x - 4 = (x - 1)(x^2 + 2x - 2) - \frac{6}{x - 1}$$

Still not sure?

Let's go back and try –1

$$y = x^3 + x^2 - 4x - 4$$

$$\begin{array}{r|rrrr} -1 & 1 & 1 & -4 & -4 \\ & & -1 & 0 & 4 \\ \hline & 1 & 0 & -4 & 0 \end{array}$$

Which means what?

That –1 is a root so (x + 1) is a factor

and that

$$x^3 + x^2 - 4x - 4 = (x + 1)(x^2 - 4)$$

Since we now know that

$$x^3 + x^2 - 4x - 4 = (x+1)(x^2 - 4) = (x+1)(x-2)(x+2)$$

Let's go back and try 2 since we know it will work

$$y = x^3 + x^2 - 4x - 4$$

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -4 & -4 \\ & & 2 & 6 & 4 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

Which means what?

That 2 is a root so (x – 2) is a factor

and that

$$x^3 + x^2 - 4x - 4 = (x-2)(x^2 + 3x + 2) =$$

$$(x-2)(x+2)(x+1)$$

Show that -2 is a root of $y = x^4 - 16$

$$
\begin{array}{r|rrrrr}
-2 & 1 & 0 & 0 & 0 & -16 \\
 & & -2 & 4 & -8 & 16 \\
\hline
 & 1 & -2 & 4 & -8 & 0
\end{array}
$$

$$y = (x+2)\underbrace{(x^3 - 2x^2 + 4x - 8)}_{\text{Let's reduce this all the way}}$$

$$
\begin{array}{r|rrrr}
2 & 1 & -2 & 4 & -8 \\
 & & 2 & 0 & 8 \\
\hline
 & 1 & 0 & 4 & 0
\end{array}
$$

$$\boxed{y = (x+2)(x-2)(x^2+4)}$$