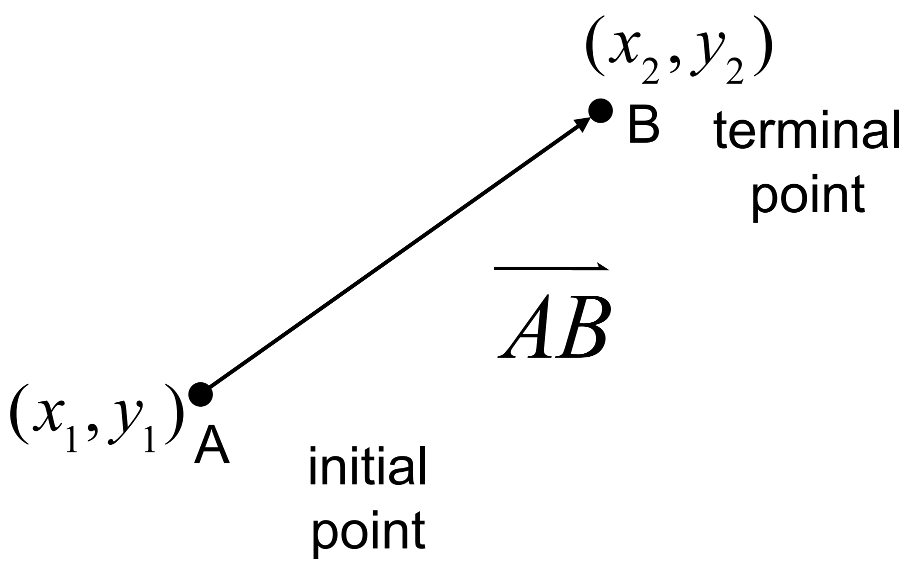
Vectors in a Plane

In the past we've only worked with lines that have slopes but not necessarily direction.

Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by <u>directed line segments</u>.

There's a difference between going 50 mph north and 50 mph south



The length is written as

$$\left|\overrightarrow{AB}\right|$$

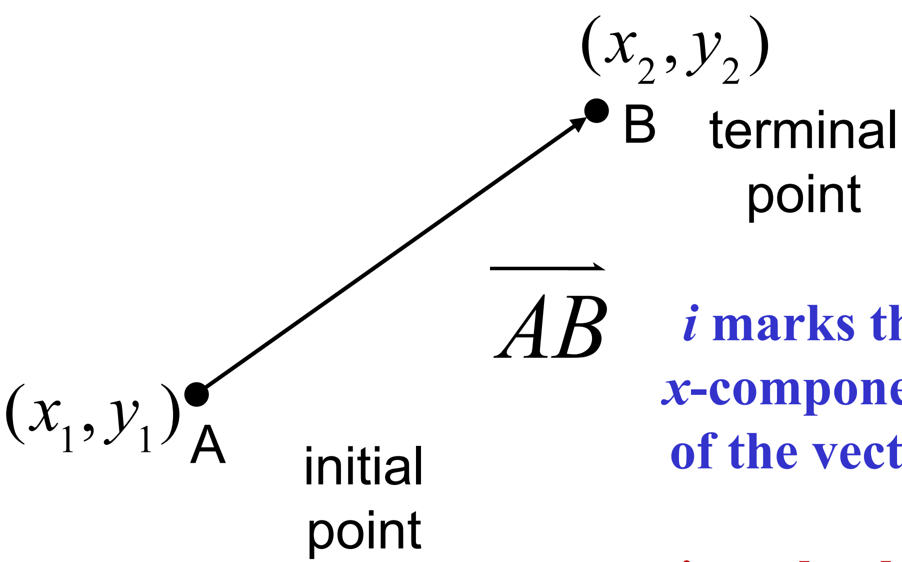
$$\left|\overrightarrow{AB}\right| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Look, it's just the distance formula

In the past we've only worked with lines that have slopes but not necessarily direction.

Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by directed line segments.

There's a difference between going 50 mph north and 50 mph south



$$\overrightarrow{AB}$$

The component form of this vector is:

$$\overrightarrow{AB} = (x_2 - x_1)i + (y_2 - y_1)j$$

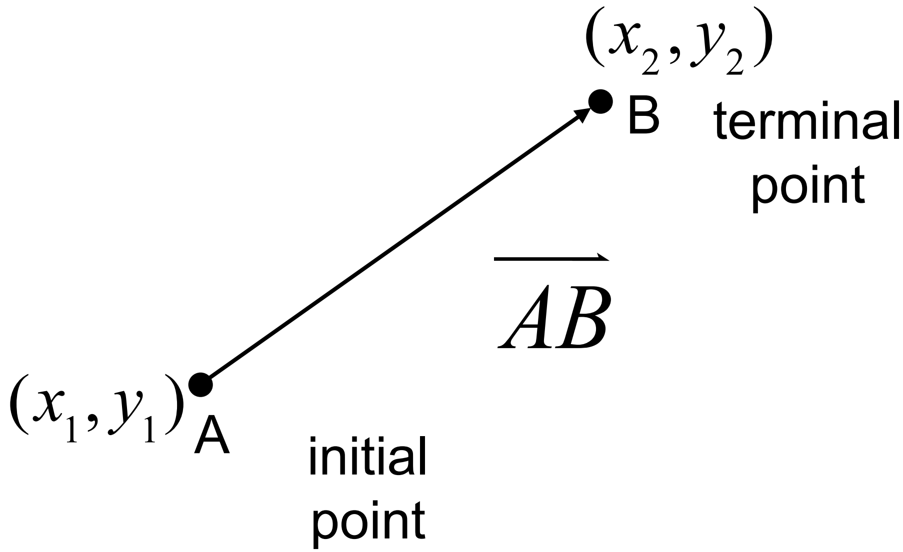
***i* marks the *x*-component of the vector**

***j* marks the *y*-component of the vector**

In the past we've only worked with lines that have slopes but not necessarily direction.

Quantities such as force, displacement or velocity that have direction as well as magnitude are represented by directed line segments.

There's a difference between going 50 mph north and 50 mph south



The component form of this vector is:

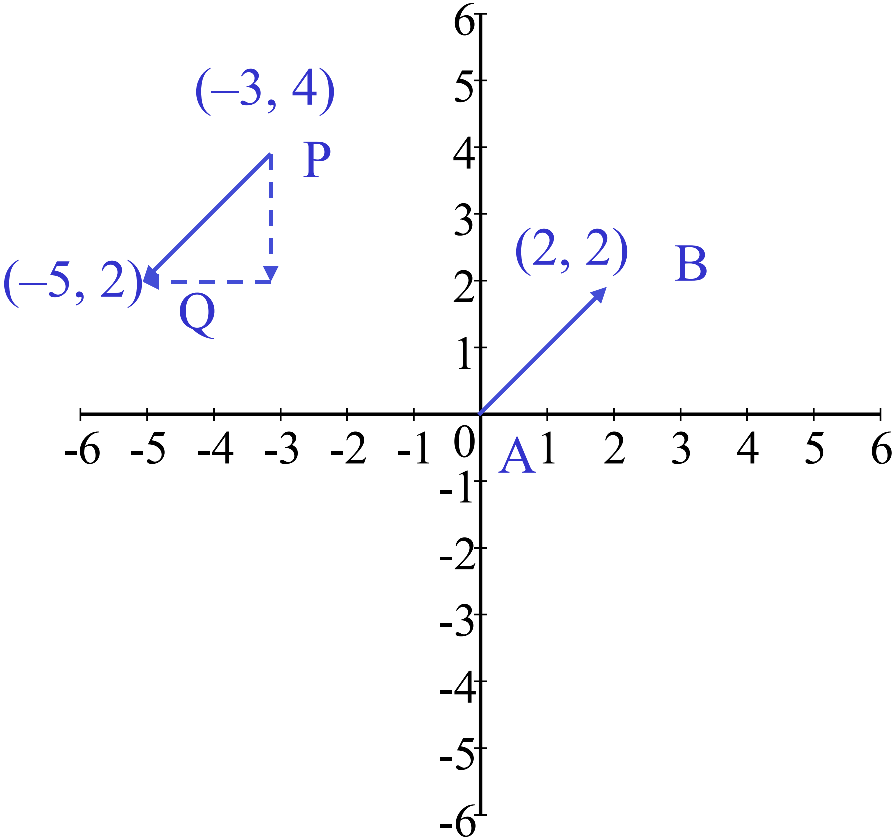
$$\overrightarrow{AB} = (x_2 - x_1)i + (y_2 - y_1)j$$

or

$$\overrightarrow{AB} = \left\langle x_2 - x_1, y_2 - y_1 \right\rangle$$

Notice here that we always subtract the initial point from the terminal point because we need to establish direction

Find the component form of each vector



$$\overrightarrow{AB} = (2-0)i + (2-0)j$$

$$\overrightarrow{AB} = 2i + 2j \;\; or \; \langle 2,2 \rangle$$

$$\overrightarrow{PQ} = (-5+3)i + (2-4)j$$

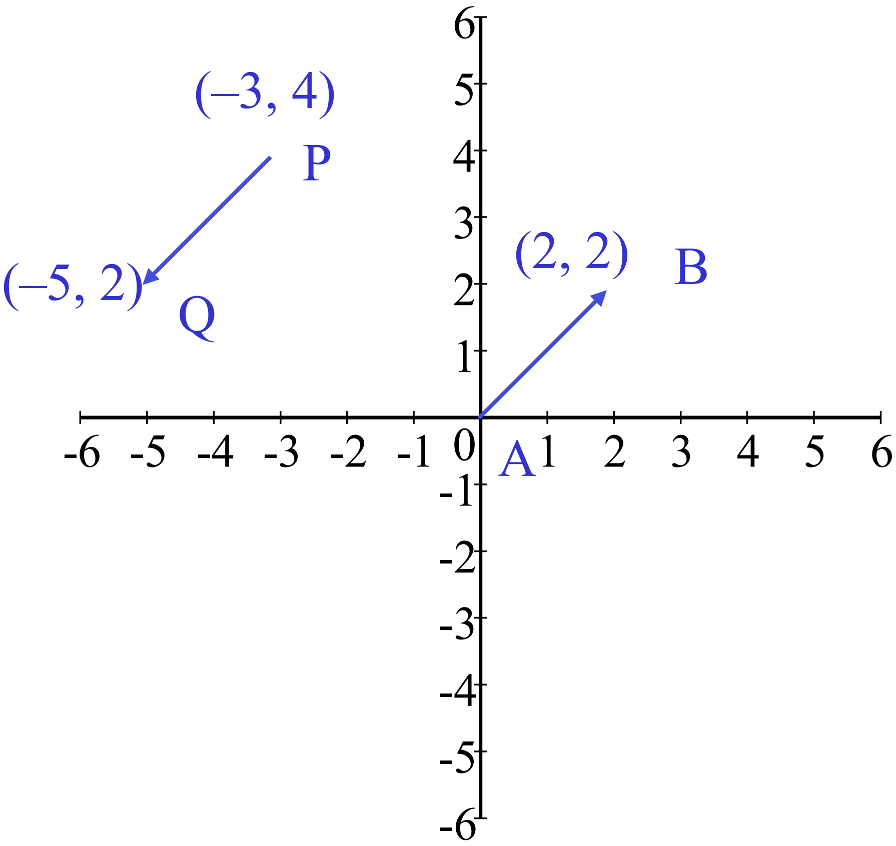
$$\overrightarrow{PQ} = -2i - 2j \; or \; \langle -2,-2 \rangle$$

$$\left|\overrightarrow{AB}\right| = \sqrt{(2)^2 + (2)^2}$$

$$\left|\overrightarrow{PQ}\right| = \sqrt{(-2)^2 + (-2)^2}$$

$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{PQ}\right| = \sqrt{8} \;= 2\sqrt{2}$$

Find the component form of each vector



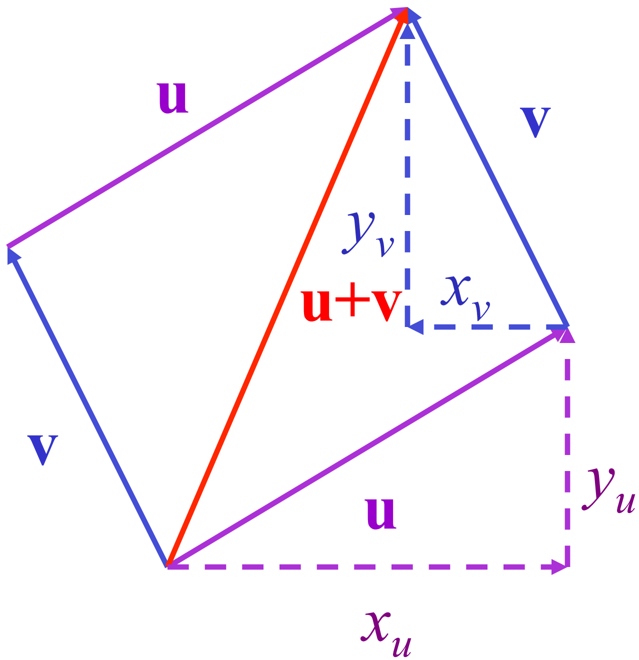
$$\overrightarrow{AB} = 2i + 2j \ \ or \ \left\langle 2,2 \right\rangle$$

$$\overrightarrow{PQ} = -2i - 2j \ or \ \left\langle -2,-2 \right\rangle$$

$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{PQ}\right| = \sqrt{8} \ = 2\sqrt{2}$$

Notice that the vectors are pointed in opposite directions

but have the same length.



Vector Addition:

u + v

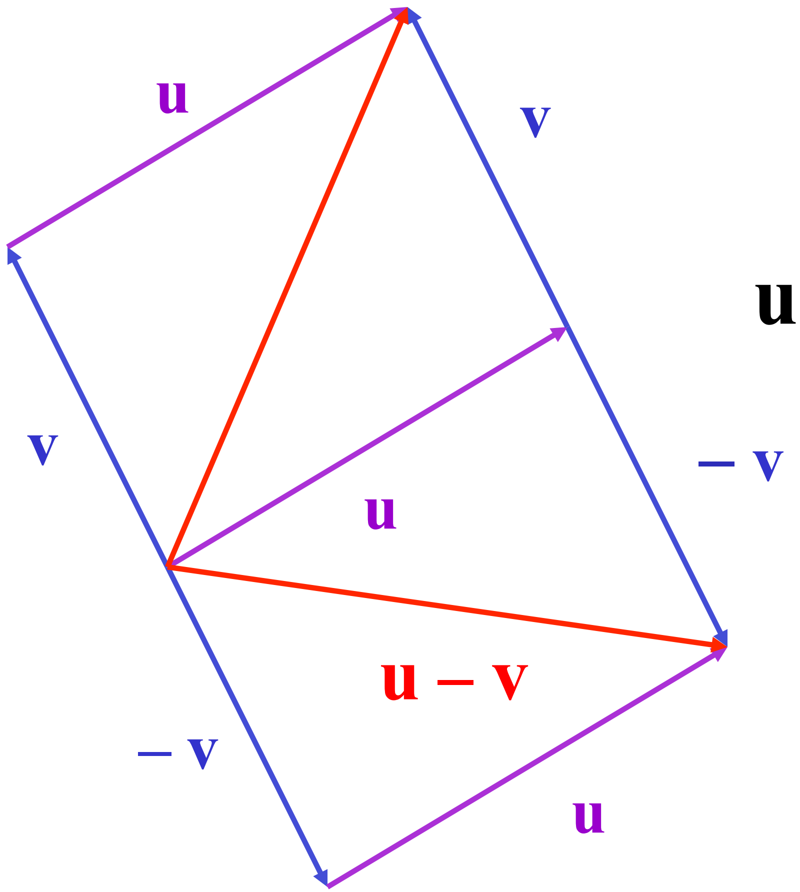
u + v is the resultant vector.

$$\mathbf{u} = x_u i + y_u j$$

$$\mathbf{v} = x_v i + y_v j$$

$$\mathbf{u} + \mathbf{v} = (x_u + x_v)i + (y_u + y_v)j$$

(Add the components.)

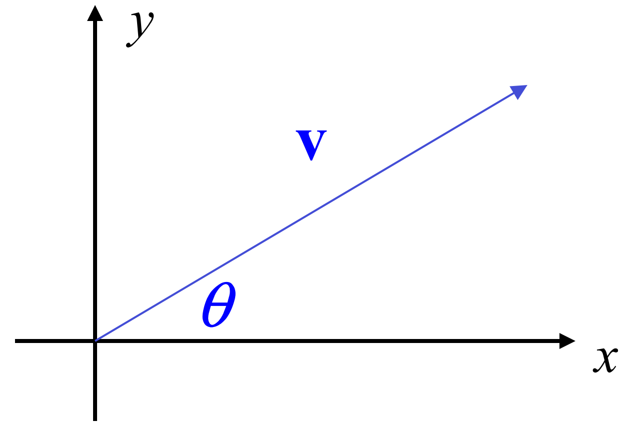


u − v

(Subtract the components.)

$$\mathbf{u}-\mathbf{v}=(x_u-x_v)i+(y_u-y_v)j$$

A vector is in <u>standard position</u> if the initial point is at the origin.



What if we only knew the $\underbrace{\text{length}}_{|\mathbf{v}|}$ of the vector and the $\underbrace{\text{angle}}_{\theta}$?

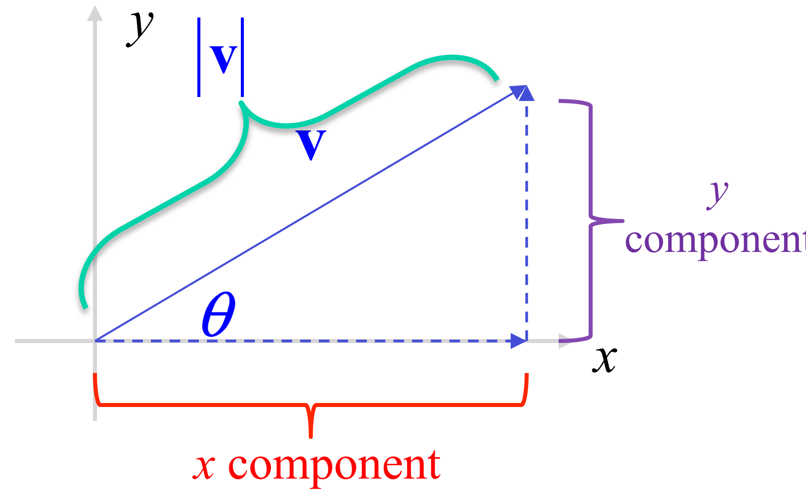
The <u>component form</u> of this vector is:

$$\mathbf{v} = \left(|\mathbf{v}|\cos\theta\right)i + \left(|\mathbf{v}|\sin\theta\right)j \quad \text{or} \quad \mathbf{v} = \left\langle |\mathbf{v}|\cos\theta, |\mathbf{v}|\sin\theta \right\rangle$$

Before anyone panics, this is just SOHCAHTOA…

Just watch…

A vector is in standard position if the initial point is at the origin.



$$\frac{adj}{hyp} = \frac{x \text{ component}}{|\mathbf{v}|} = \cos\theta$$

Remember what this really means: $|\mathbf{v}|$ ← Think of it as a hypotenuse of the right triangle above because it's the length of the vector

$$\mathbf{v} = \underbrace{\left(|\mathbf{v}|\cos\theta\right)}_{x \text{ component}} i + \underbrace{\left(|\mathbf{v}|\sin\theta\right)}_{y \text{ component}} j \quad \text{or} \quad \mathbf{v} = \left\langle \underbrace{|\mathbf{v}|\cos\theta}_{x \text{ component}}, \underbrace{|\mathbf{v}|\sin\theta}_{y \text{ component}} \right\rangle$$

See? Just SOHCAHTOA

If it's the angle that you need to find, then you need to know this:

The direction of a vector **u** is found this way:

$$\theta = \pm\cos^{-1}\left(\frac{x_u}{|\mathbf{u}|}\right)$$

The sign follows the sign of the *y* component

This is just the *x* component divided by the magnitude

Unit Vectors

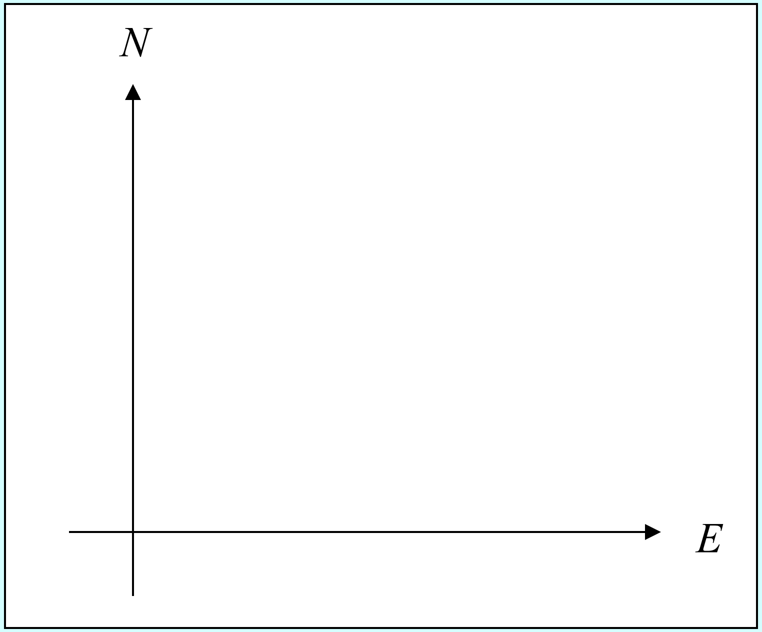
Unit Vectors, just like r in the unit circle, have a length of **1**.
The formula for finding any given unit vector is simple:

The unit vector for $\mathbf{u} = x_u i + y_u j$ is $\dfrac{\mathbf{u}}{|\mathbf{u}|} = \dfrac{x_u}{|\mathbf{u}|} i + \dfrac{y_u}{|\mathbf{u}|} j$

Now we're going to try a few homework problems…

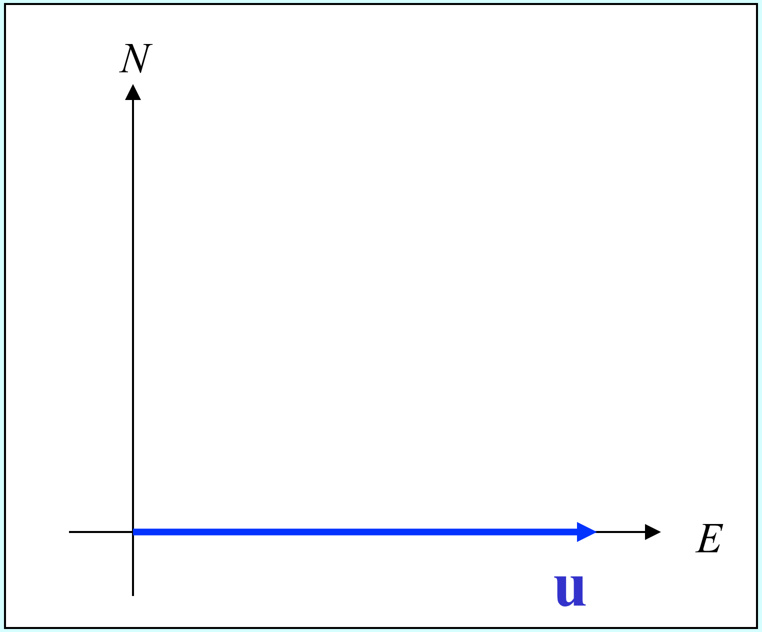
Application:

A Boeing 727 airplane, flying due east at 500mph in still air, encounters a 70-mph tail wind acting in the direction of 60^o north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?



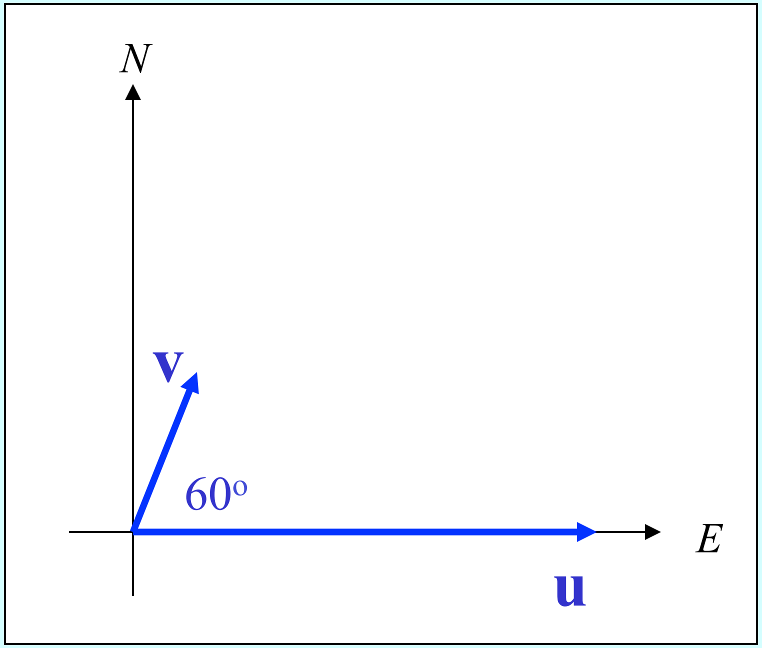
Application:

A Boeing 727 airplane, flying due **east at 500mph** in still air, encounters a 70-mph tail wind acting in the direction of 60^o north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?



Application:

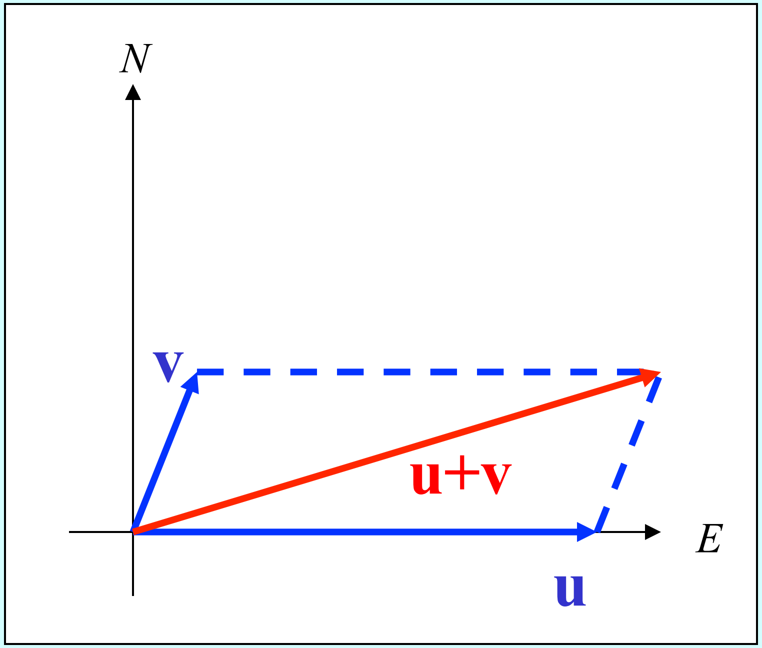
A Boeing 727 airplane, flying due east at 500mph in still air, encounters a **70-mph** tail wind acting in the direction of **60^o north of east**. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?



Application:

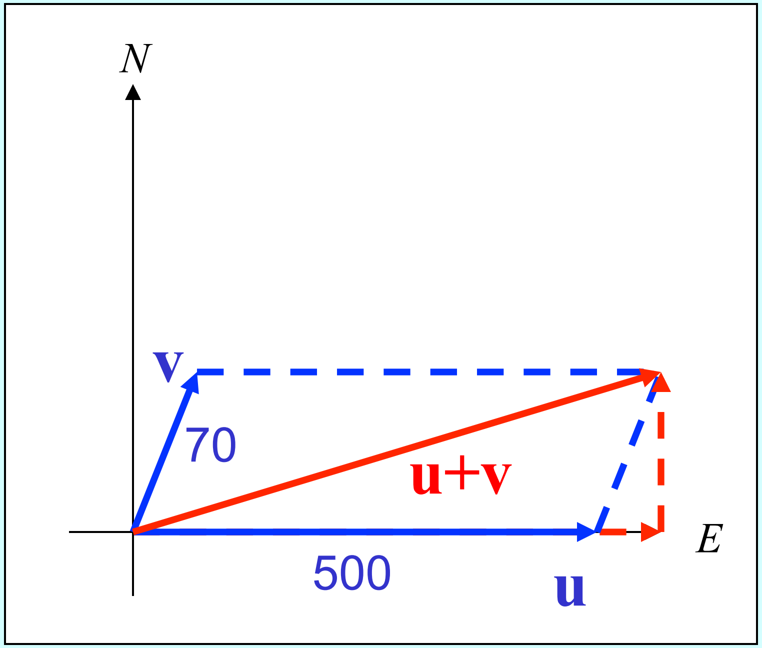
A Boeing 727 airplane, flying due east at 500mph in still air, encounters a 70-mph tail wind acting in the direction of 60^o north of east. The airplane holds its compass heading due east but, because of the wind, acquires a new ground speed and direction. What are they?

We need to find the magnitude and direction of the resultant vector **u + v.**



The component forms of **u** and **v** are:

$$\mathbf{u} = \langle 500, 0 \rangle$$



$$\mathbf{v} = \langle 70\cos 60^{\circ}, 70\sin 60^{\circ} \rangle$$

$$\mathbf{v} = \langle 35, 35\sqrt{3} \rangle$$

Therefore: $\mathbf{u} + \mathbf{v} = \langle 535, 35\sqrt{3} \rangle$

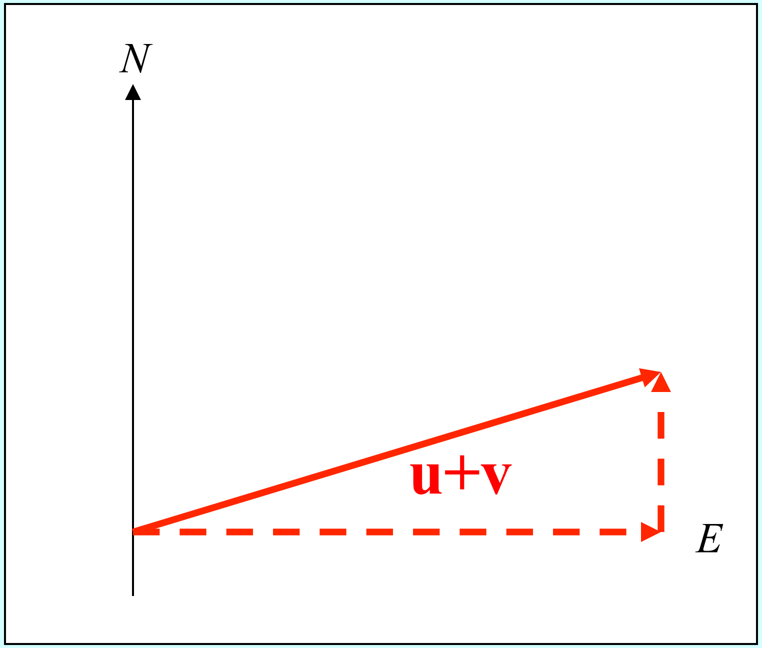
$$|\mathbf{u} + \mathbf{v}| = \sqrt{535^2 + \left(35\sqrt{3}\right)^2} \approx 538.423625$$

and:

$$\theta = \pm\cos^{-1}\left(\frac{x_{\mathbf{u}+\mathbf{v}}}{|\mathbf{u} + \mathbf{v}|}\right) \qquad \theta = \cos^{-1}\left(\frac{535}{538.423625}\right) \approx 6.5^{\circ}$$

The component forms of **u** and **v** are:

$$\mathbf{u} = \left\langle 500, 0 \right\rangle$$



$$\mathbf{v} = \left\langle 70\cos 60^{\circ}, 70\sin 60^{\circ} \right\rangle$$

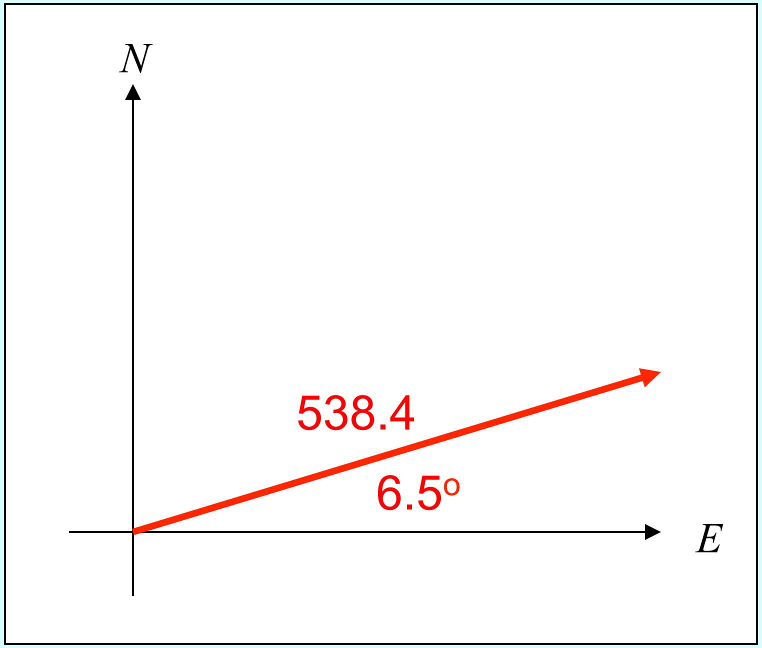
$$\mathbf{v} = \left\langle 35, 35\sqrt{3} \right\rangle$$

Therefore: $\mathbf{u} + \mathbf{v} = \left\langle 535, 35\sqrt{3} \right\rangle$

Remembering that $\tan\theta = \dfrac{y}{x}$

You can also use $\tan^{-1}$

$$\theta = \tan^{-1}\frac{35\sqrt{3}}{535} \approx 6.5^{\circ}$$



The new ground speed of the airplane is about 538.4 mph, and its new direction is about 6.5° north of east.