Extreme Values of Functions

Name <u>Solutions</u>

Find the critical values of the functions given and indicate which are maxima or minima and determine if it is relative or absolute. Sketch the graph of the function on the axes provided. Find your answers using the methods in Section 4.1 and *show the work* that leads to your answers. You may use your calculators *only* to check your answers.
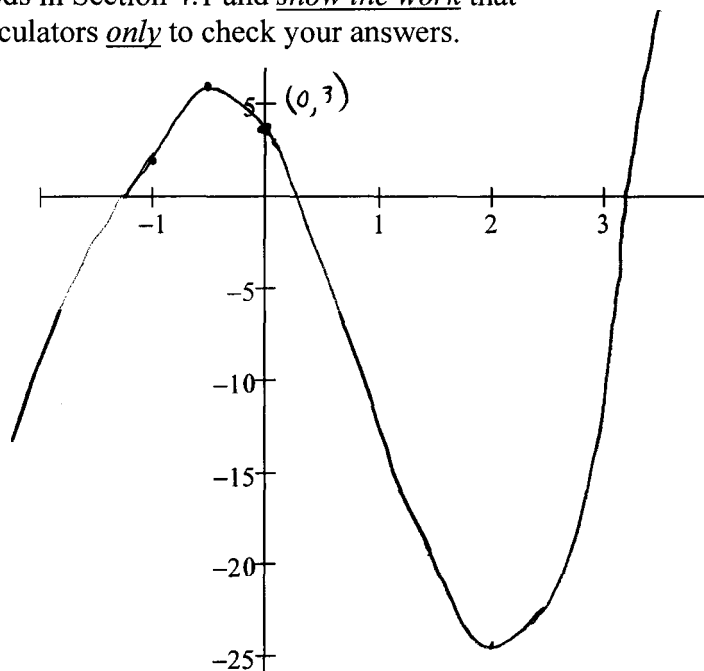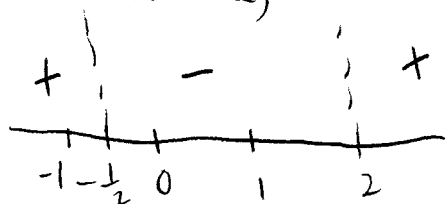
1) $y = 4x^3 - 9x^2 - 12x + 3$

$$y' = 12x^2 - 18x - 12 = 0$$

$$6(2x^2 - 3x - 2) = 0$$
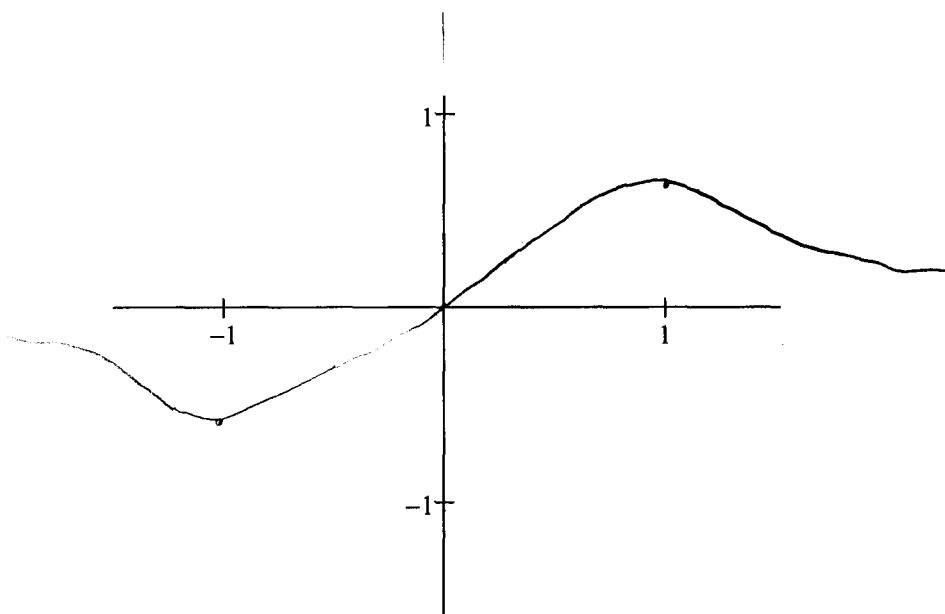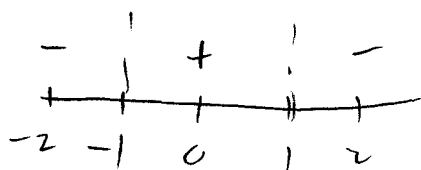
$$6(2x + 1)(x - 2) = 0$$

$$x = -\tfrac{1}{2}, 2$$

2) $y = \dfrac{x}{x^2 + 1}$

$$y' = \dfrac{x^2 + 1 - 2x^2}{(x^2 + 1)^2}$$

$$= \dfrac{1 - x^2}{(x^2 + 1)^2} = 0$$

$$x = \pm 1$$

3) $y = \sqrt{t}(1-t) = t^{1/2} - t^{3/2}$
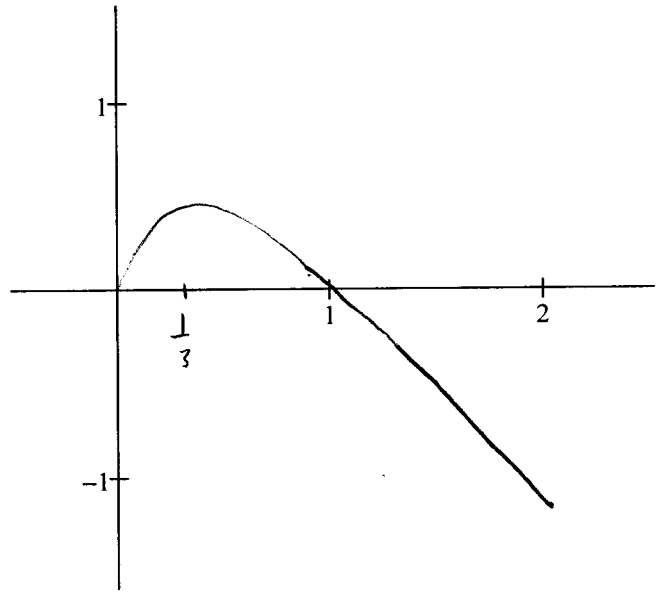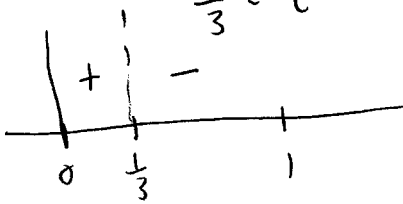
$y' = \frac{1}{2}t^{-\frac{1}{2}} - \frac{3}{2}t^{1/2}$

$= \frac{1}{2\sqrt{t}} - \frac{3\sqrt{t}}{2} = 0$

$t \neq 0$

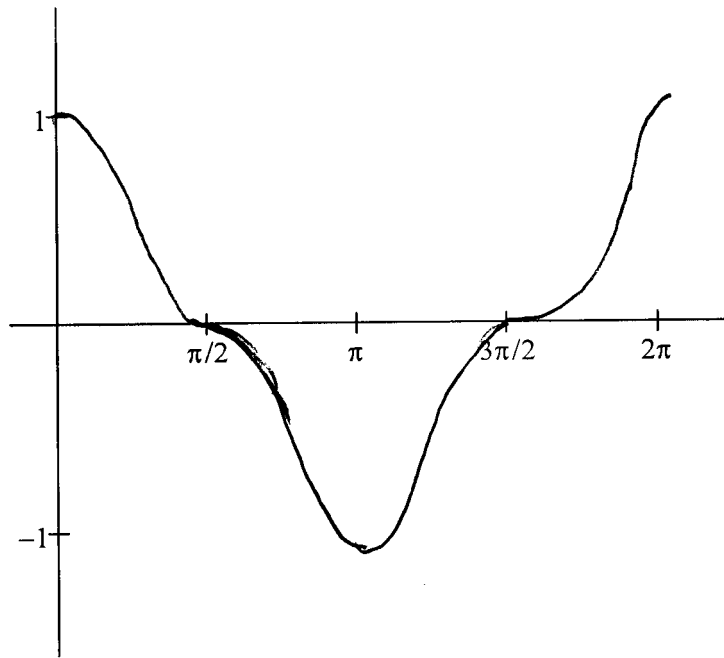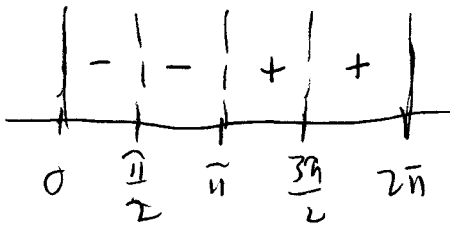$\frac{1}{2\sqrt{t}} = \frac{3\sqrt{t}}{2}$

$1 = 3t$

$\frac{1}{3} = t$

$\begin{array}{c} \uparrow \quad | \quad | \\ + \quad | \quad - \\ \hline 0 \quad \frac{1}{3} \quad 1 \end{array}$

4) $y = \cos^3 x \quad [0, 2\pi]$

$y' = -3\cos^2 x \sin x = 0$

$\cos x = 0 \qquad \sin x = 0$

$\Downarrow \qquad \qquad \Downarrow$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad 0, \pi, 2\pi$

$\begin{array}{c} | \quad | \quad | \quad | \quad | \\ - \quad | \quad - \quad | \quad + \quad | \quad + \\ \hline 0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi \end{array}$

f'(x)
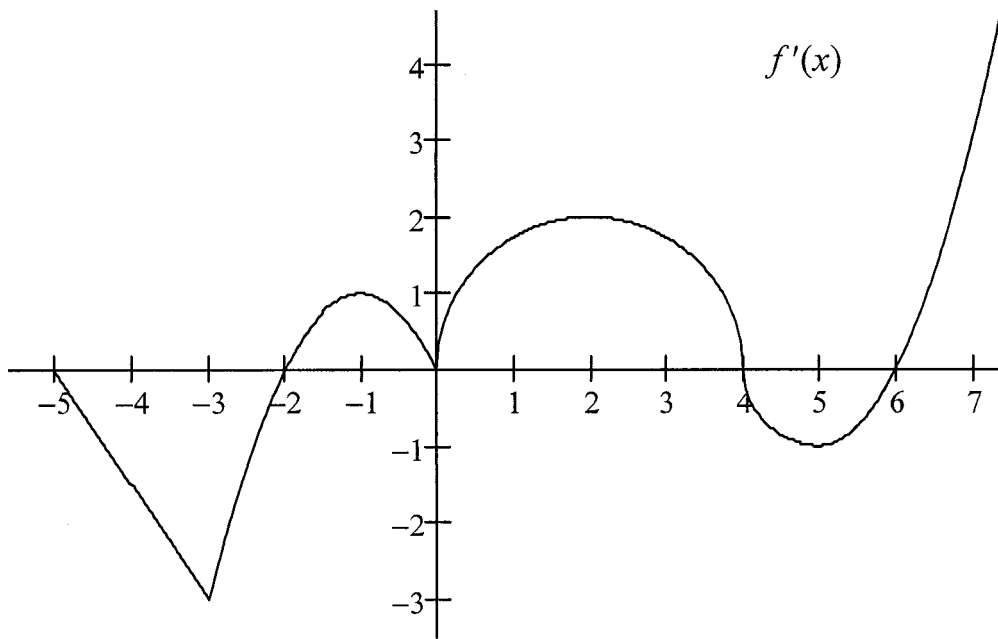
5) The graph above represents $f'(x)$ over the interval $[-5 , 7]$. The graph has a vertical tangent line at $x = 4$.

a) Find all values of x for which f is increasing?

$$(-2, 0) \quad (0, 4) \quad (6, 7)$$

b) Find all values of x for which f has a point of inflection? Give the value of f'' at each of these points.

$$X = -3, -1, 0, 2, 5$$

$$f''(X) = \text{undefined}, 0, \text{undef.}, 0, 0$$

c) Over which interval(s) of x is the graph of f concave down?

$$(-5, -3) \quad (-1, 0) \quad (2, 4) \quad (4, 5)$$

f has no concavity at $X = 4$ because f' has a vertical tangent line. Therefore, $f''(4)$ is undefined. But since concavity does not change at $X = 4$ it is not a point of inflection