FTC "Part 3" Name Solutions

Your calculator will be required to complete problem #1. Be sure to show your analytic work before your numeric answer. For this problem, give each of your answers to three decimal places.

1) A particle starts at $x = 2$ and moves along the x-axis with velocity $v(t) = \cos(t^2)$ over the interval $0 \le t \le 3$ where t is measured in seconds.

(a) Over what intervals is the particle moving to the left?

$v(t) = \cos(t^2) < 0$ Graph on calculator and find where it intercepts the t axis (horizontal axis)

$1.253 < t < 2.171$

$2.802 < t < 3$

(b) Find the total distance traveled by the particle over the interval $0 \le t \le 3$.

Option 1: $\int_0^{1.253} v(t)\,dt - \int_{1.253}^{2.171} v(t)\,dt + \int_{2.171}^{2.802} v(t)\,dt - \int_{2.802}^{3} v(t)\,dt$

Option 2: $\int_0^3 |v(t)|\,dt = \int_0^3 |\cos(t^2)|\,dt \approx 2.054$

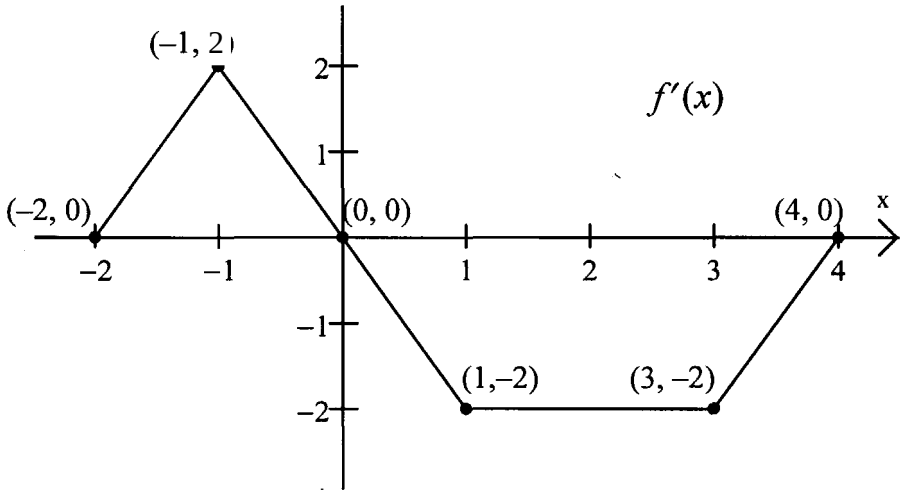
(c) What is the final position of the particle on the x-axis? "final" $\Rightarrow$ at $t = 3$

We want to find $x(3)$

$$x(3) = x(0) + \int_0^3 \cos(t^2)\,dt$$

$$= 2 + 0.703 \approx 2.703$$

2) Let f be a twice-differentiable function over the interval $-2 \le x \le 4$ and containing the point (1, 3). The graph of its derivative $f'(x)$, consisting of four line segments, is shown below.



(a) Write the equation for the line tangent to f at the point (1, 3)

$f'(1) = -2$

$y - 3 = -2(x-1)$

(b) Find $f(-2)$, $f(0)$, and $f(4)$

$f(-2) = f(1) + \int_1^{-2} f'(x)\,dx = 3 - \int_{-2}^{1} f(x)\,dx = 3 - 1 = 2$

$f(0) = f(1) - \int_0^1 f'(x)\,dx = 3 - (-1) = 4$

$f(4) = f(1) + \int_1^4 f'(x)\,dx = 3 + (-5) = -2$

(c) Given that $f(1) = 3$, find the value for x other than 1 over the interval $-2 \le x \le 4$ for which $f(x) = 3$. Justify your answer.

$f(x) = 3 + \int_1^x f'(t)\,dt = 3$

$\int_1^x f'(t)\,dt = 0 \qquad x = -1$ because area above and below are equal