Calculus Chapter 8 Review Name Solutions

1) $\lim_{x\to\infty} \frac{\ln x}{x^2} = \frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{\frac{1}{x}}{2x} = \lim_{x\to\infty} \frac{1}{2x^2} = 0$

2) $\lim_{x\to 0} \frac{x}{\tan^{-1}(4x)} = \frac{0}{0} \quad \lim_{x\to 0} \frac{1}{\frac{4}{1+16x^2}} = \frac{1+16x^2}{4} = \frac{1}{4}$

3) $\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \frac{\infty}{\infty} = \lim_{x\to\infty} \frac{2(\ln x)\frac{1}{x}}{1} = \frac{2\ln x}{x} = \frac{\infty}{\infty}$

$= \frac{2/x}{1} = 0$

Evaluate the given integrals.

4) $\int_0^2 \frac{xdx}{\sqrt{4-x^2}} = \lim_{b\to 2} \int_0^b \frac{x\,dx}{\sqrt{4-x^2}}$ $\quad u = 4-x^2 \quad du = -2x\,dx \quad x=0 \; u=4 \quad x=b \; u=4-b^2$

$\lim_{b\to 2} -\int_4^{4-b^2} \frac{\frac{1}{2}du}{\sqrt{u}} = -\sqrt{u}\Big]_4^{4-b^2} = -\sqrt{4-b^2} + \sqrt{4}$

$= \lim_{b\to 2} -\sqrt{4-b^2} + 2 = 2$

5) $\int_0^{3\sqrt{3}} \frac{dx}{9+x^2}$ $\quad x = 3\tan\theta \quad dx = 3\sec^2\theta\,d\theta$

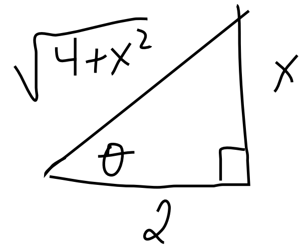
$x=0 \Rightarrow \theta = 0$

$x = 3\sqrt{3} \Rightarrow \theta = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$\int_0^{\frac{\pi}{3}} \frac{3\sec^2\theta\,d\theta}{9+9\tan^2\theta} = \int_0^{\frac{\pi}{3}} \frac{3\sec^2\theta\,d\theta}{9\sec^2\theta} = \int_0^{\pi/3} \frac{1}{3}d\theta = \frac{1}{3}\theta\Big]_0^{\pi/3}$

$= \frac{\pi}{9}$

$$x = 2\tan\theta$$



6) $\int \frac{dx}{x^2\sqrt{4+x^2}}$ $\qquad dx = 2\sec^2\theta \, d\theta$

$$\int \frac{2\sec^2\theta \, d\theta}{4\tan^2\theta \sqrt{4+4\tan^2\theta}} = \int \frac{2\sec^2\theta}{4\tan^2\theta \; 2\sec\theta} \, d\theta$$

$$\frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta} \, d\theta$$

$u = \sin x$

$du = \cos x \, dx$

$$\frac{1}{4}\int u^{-2} \, du = \frac{1}{4} \frac{1}{\sin\theta} + C$$

$$\frac{1}{4} \frac{1}{\frac{x}{\sqrt{4+x^2}}} + C = \frac{\sqrt{4+x^2}}{4x} + C$$

7) $\int \frac{3x^3 - 5x^2 + x - 1}{(x^2+1)(x-1)^2} \, dx$

See Next Page for solution

7) $$\int \frac{3x^3-5x^2+x-1}{(x^2+1)(x-1)^2}\,dx = \int \frac{Ax+B}{x^2+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}\,dx$$

$$(Ax+B)(x-1)^2 + C(x-1)(x^2+1) + D(x^2+1) = 3x^3-5x^2+x-1$$

$$Ax^3 - 2Ax^2 + Ax + Bx^2 - 2Bx + B + Cx^3 + Cx - Cx^2 - C + Dx^2 + D = 3x^3-5x^2+x-1$$

$Ax^3 + Cx^3 = 3x^3$ $\rightarrow$ ① $A + C = 3 \Rightarrow C = 3 - A$

$-2Ax^2 + Bx^2 - Cx^2 + Dx^2 = -5x^2$ $\rightarrow$ ② $-2A + B - C + D = -5$

$Ax - 2Bx + Cx = x$ $\rightarrow$ ③ $A - 2B + C = 1$

$B - C + D = -1$ $\rightarrow$ ④ $B - C + D = -1$

$$A - 2B + (3-A) = 1$$
$$-2B + 3 = 1$$
$$B = -1$$

Equation ④ becomes $-1 - (3-A) + D = -1 \rightarrow A + D \quad 3$

② becomes $-2A - 1 - (3-A) + D = -5$ $\quad -A + D = -1$

$D = 1 \Rightarrow C = 1$

$A = 2$

$$\int \frac{2x-1}{x^2+1} + \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx = \int \frac{2x}{x^2+1} - \frac{1}{x^2+1} + \frac{1}{x-1} + \frac{1}{(x-1)^2}\,dx$$

$u = x^2+1$, $du = 2x\,dx$ $\quad$ $\tan^{-1}x$ $\quad$ $\ln|x-1|$ $\quad$ $\frac{(x-1)^{-1}}{-1}$ (power rule)

Answer:

$$\boxed{\ln(x^2+1) + \tan^{-1}(x) + \ln|x-1| - \frac{1}{(x-1)} + C}$$