1) Lauren is solving calculus problems at a rate of 1.5 problems every minute. Meanwhile, right next to her, Adrienne is telling Samantha about all of her grade drama. She is talking at a rate of 120 words for every problem that Lauren solves. How fast is Adrienne talking in words per minute?

$$\frac{dW}{dP} = 120$$

$$\frac{dW}{dt} = \text{How fast Adrienne is talking in words per minute?}$$

$$\frac{dP}{dt} = 1.5$$

$$\frac{dW}{dt} = \frac{dW}{dP} \cdot \frac{dP}{dt} = 120 \cdot 1.5 = \boxed{180 \text{ words}/\text{minute}}$$

2) Mussman and Hamidou are airline pilots who would like to take off but can't because Matthew is in charge of fueling their plane and there is a leak that he is too distracted to notice. The amount of fuel in the tank in gallons at this time is given by $Q(t) = 2t^3 - 39t^2 + 180t + 170$ where t is measured in minutes from the time that they began fueling.

a) When did the fuel level first begin to drop? → when $Q'(t) = 0$ (Hint: Graph $Q(t)$)

$$Q(t) = 2t^3 - 39t^2 + 180t + 170$$

$$Q'(t) = 6t^2 - 78t + 180 = 0 = 6(t^2 - 13t + 30) = 0$$

$$= (t-10)(t-3) = 0$$

$$t = 3, 10 \qquad \boxed{t = 3} \text{ minutes}$$

b) When did the fuel level finally start to level off and rise again?

$\boxed{t = 10 \text{ minutes}}$ ← find the other point where $Q'(t) = 0$

c) What was the fuel level at this time?

$$Q(10) = 70 \text{ gallons}$$

Find the derivative of each expression.

3) a) $\cos^2\left(\dfrac{1}{\sqrt{x^3-2x}}\right) = \left[\cos\left(\dfrac{1}{\sqrt{x^2-2x}}\right)\right]^2$

b) $\cos(e^{x^3-2x}) \Rightarrow \left[-\sin(e^{x^3-2x})\right](e^{x^3-2x}) \cdot$

$(3x^2-2)$

$\Rightarrow 2\left[\cos\left(\dfrac{1}{\sqrt{x^3-2x}}\right)\right]\left[-\sin\left(\dfrac{1}{\sqrt{x^2-2x}}\right)\right]\left(-\dfrac{1}{2}\right)(x^3-2x)^{-\frac{3}{2}}(3x^2-2)$

$\Rightarrow \cos\left(\dfrac{1}{\sqrt{x^3-2x}}\right)\sin\left(\dfrac{1}{\sqrt{x^3-2x}}\right)(3x^2-2x)(x^3-2x)^{-\frac{3}{2}}$

$\left[-\sin(e^{x^3-2x})\right](3x^2-2)e^{x^3-2x}$

See last page
for more on #3

4) $\tan^{-1}(\sec x)$

$\Rightarrow \dfrac{1}{1+\sec^2 x}\sec x \tan x$

$= \left[\dfrac{\sec x \tan x}{1+\sec^2 x}\right]$

5) $\sin^{-1}(5^{3x})$

$\Rightarrow \dfrac{1}{\sqrt{1-5^{6x}}}(5^{3x})(3\ln 5)$

$\dfrac{5^{3x}(3\ln 5)}{\sqrt{1-5^{6x}}}$
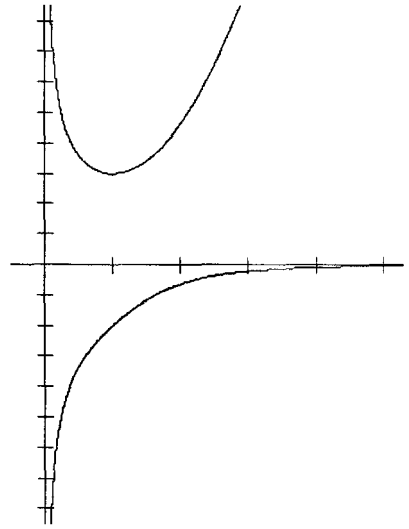
6) Given the equation $xy^2 - x^3y = 6$ graphed below

a) Find the slope of the line tangent to the graph at the point (1, 3)

$$y^2 + 2yy'x - 3x^2y - x^3y' = 0$$

$$9 + 6y' - 9 - y' = 0$$

$$5y' = 0$$

$$\boxed{m = 0}$$

b) Find the other point on the curve where the x coordinate is 1
   and find the equation of the tangent line there.

plug in 1 for X

$$y^2 - y = 6$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3, -2 \implies (1, -2) \text{ is the other point}$$

plug $(1, -2)$ into the derivative $\implies 4 - 4y' + 6 - y' = 0$

$$-5y' = -10$$

$$y' = 2$$

$$\boxed{y + 2 = 2(x-1)}$$

7) $y = x^{\frac{1}{x}}$ Find y'

$$\ln y = \ln x^{\frac{1}{x}} = \frac{1}{x} \ln x = \frac{\ln x}{x}$$

$$\frac{1}{y} y' = \frac{\frac{1}{x} x - \ln x}{x^2}$$

$$\frac{1}{y} y' = \frac{1 - \ln x}{x^2}$$

$$y' = y \frac{1 - \ln x}{x^2}$$

$$y' = x^{\frac{1}{x}} \left(\frac{1 - \ln x}{x^2} \right) \quad \text{or}$$

$$x^{\frac{1}{x} - 2} (1 - \ln x)$$

$$\cos^2\left(\frac{1}{\sqrt{x^3-2x}}\right) =$$

$$\left[\cos\left(x^3-2x\right)^{-\frac{1}{2}}\right]^2$$

$$2\left[\cos\left(x^3-2x\right)^{-\frac{1}{2}}\right]\cdot\left[-\sin\left(x^3-2x\right)^{-\frac{1}{2}}\right]\cdot$$

$$-\frac{1}{2}\left(x^3-2x\right)^{-\frac{3}{2}}\cdot\left(3x^2-2\right)$$

$$= \cos\left(\frac{1}{\sqrt{x^3-2x}}\right)\sin\left(\frac{1}{\sqrt{x^3-2x}}\right)\left(\frac{3x^2-2}{2\left(x^3-2x\right)^{3/2}}\right)$$