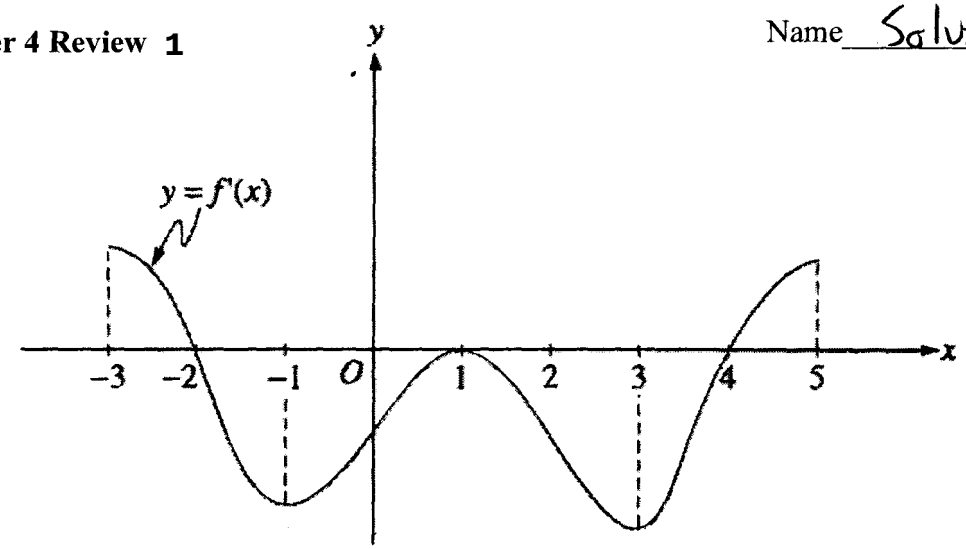
Chapter 4 Review 1

Name Solutions



1) The figure above shows the graph of f', the derivative of a function f over the interval $-3 < x < 5$.

(a) For what values of x does f have a relative maximum? Explain.

$x = -2$ f' goes from + to −

(b) For what values of x does f have a relative minimum? Explain

$x = 4$ f' goes from − to +

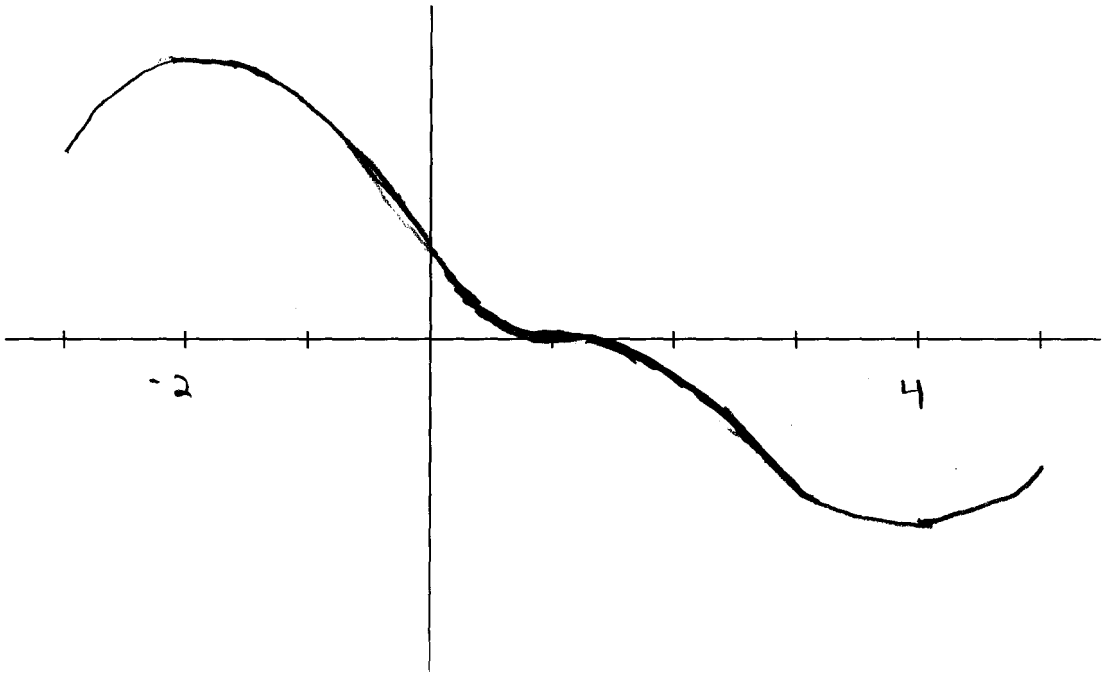
(c) On what intervals is the graph of f concave upward? Use f' to justify your answer.

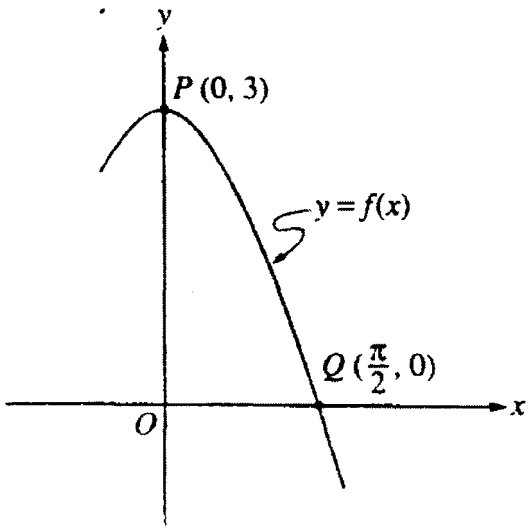
$-1 < x < 1$

$3 < x < 5$

f' is increasing which implies that $f'' > 0$

(d) Suppose that $f(1) = 0$. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the interval $-3 < x < 5$.





2) Let f be the function given by $f(x) = 3\cos x$. As shown above, the graph of f crosses the y-axis at the point P and the x-axis at point Q.

(a) Write an equation for the line passing through the points P and Q.

$m = \frac{3}{-\frac{\pi}{2}} = -\frac{6}{\pi}$

$y - 3 = -\frac{6}{\pi}(x - 0)$

(b) Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to the line PQ.

Mean Value Theorem M.V.T.

$f'(x) = -\frac{6}{\pi} = -3\sin x$

$\frac{2}{\pi} = \sin x$

$\sin^{-1}\left(\frac{2}{\pi}\right) = x \approx 0.690$

(c) Write an equation for the tangent line at the x-coordinate found in part (c).

$y - 3\cos\left(\sin^{-1}\left[\frac{2}{\pi}\right]\right) = -\frac{6}{\pi}\left(x - \sin^{-1}\left(\frac{2}{\pi}\right)\right)$

or $\rightarrow$ $y - 2.314 = -\frac{6}{\pi}(x - 0.690)$

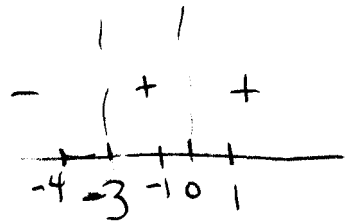
3) Given the function $y = \frac{x^4}{12} + \frac{x^3}{3}$ for all real numbers,

(a) Find all critical points and identify each as a relative maximum, minimum, or neither.

$y' = \frac{x^3}{3} + x^2 = 0$

$= x^2\left(\frac{x}{3} + 1\right) = 0$

$x = 0, -3$



(b) Identify any absolute extrema and give both their x and y coordinates

$x = -3$ absolute minimum

$\left(-3, -\frac{9}{4}\right)$

(c) On which intervals is y increasing? Decreasing?

increasing $(-3, 0)(0, \infty)$

decreasing $(-\infty, -3)$

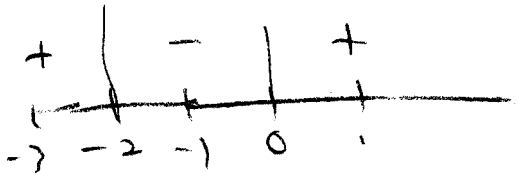
(d) Find all points of inflection.

$y'' = x^2 + 2x = 0$

$x(x+2) = 0$

$x = 0, -2$

(e) On which intervals is y concave up? Concave down?



concave up $(-\infty, -2)(0, \infty)$

concave down $(-2, 0)$

4) A particle moves along the x-axis with position $s(t) = t^2 - 4t + 3$ over the interval $0 \le t \le 4$.

(a) When is the particle moving to the left?

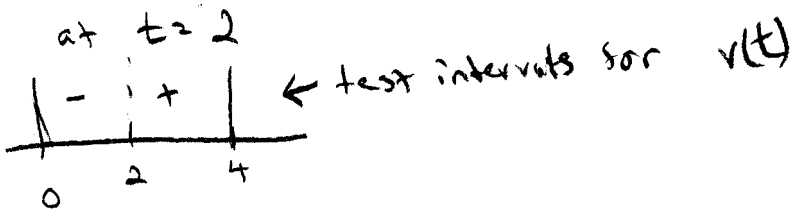
$v(t) = s'(t) = 2t - 4 < 0$

$2t < 4$

$t < 2$

$0 < t < 2$

(b) When does the particle change directions?



(c) What is the particle's location when it changes directions?

$s(2) = 2^2 - 4(2) + 3 = -1$

(d) At what time is the particle's average velocity over the interval $0 \le t \le 4$ equal to its instantaneous velocity? What is the value of this velocity?

M.V.T. $\quad \text{avg velocity} = \frac{s(4) - s(0)}{4 - 0} = \frac{3 - 3}{0} = 0$

$s'(t) = 0 = 2t - 4 = 0$

$t = 2$