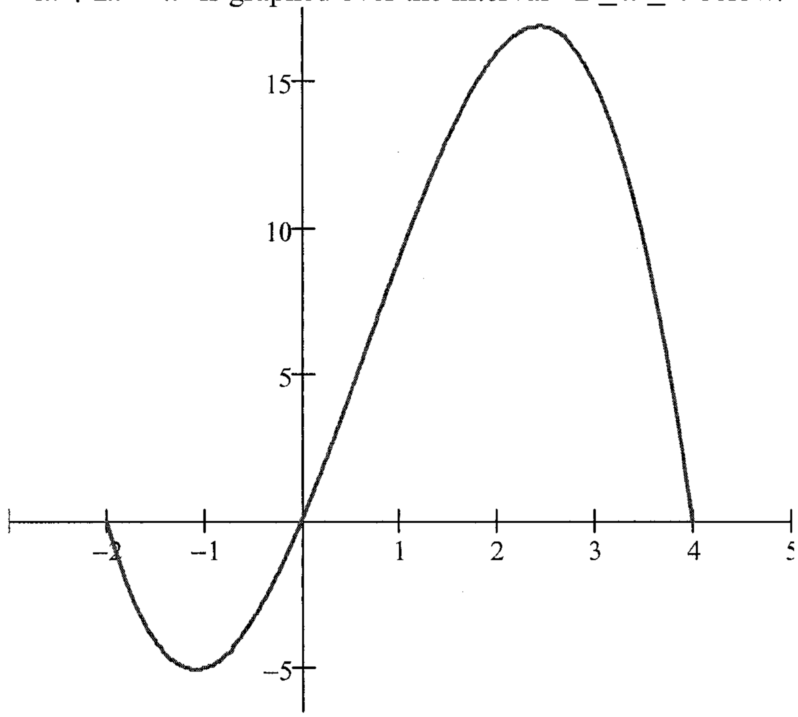
Name Solutions

Chapter 5 Review 1

Complete the given problems on a separate sheet of paper. Attach this sheet when you turn in your work. Your graphing calculator may be needed for parts of these problems.

1) The function $f(x) = 8x + 2x^2 - x^3$ is graphed over the interval $-2 \le x \le 4$ below.



(a) Approximate the value of $\int_0^4 f(x)\,dx$ using a left Riemann sum with four subintervals.

(b) Approximate the value of $\int_0^4 f(x)\,dx$ using a midpoint Riemann sum with four subintervals.

(c) Use the definite integral to find the exact value of $\int_{-2}^4 f(x)\,dx$

(d) Use the definite integral to find the exact value of the total area between the curve and the x-axis over the interval $[-2, 4]$

2) A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \ge 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when $t = 1$ is $x(1) = 3$.

(a) Write an expression for the velocity $v(t)$ of the particle at any time $t \ge 0$.

(b) Find the values of t for which the particle is at rest.

(c) Write an expression for the position $x(t)$ of the particle at any time $t \ge 0$.

① $f(x) = 8x + 2x^2 - x^3$ Left hand method $n = 4$

a) $\Delta x = 1$ $f(0) = 0$ $f(1) = 9$ $f(2) = 16$ $f(3) = 15$

$A_L = 1(0 + 9 + 16 + 15) = 40$

b) Midpoint Method $n = 4$

$\Delta x = 1$ $f\left(\frac{1}{2}\right) = \frac{35}{8}$ $f\left(\frac{3}{2}\right) = \frac{105}{8}$ $f\left(\frac{5}{2}\right) = \frac{135}{8}$ $f\left(\frac{7}{2}\right) = \frac{77}{8}$

$A_m = 1\left(\frac{35}{8} + \frac{105}{8} + \frac{135}{8} + \frac{77}{8}\right) = \frac{352}{8} = 44$

c) $\int_{-2}^{4} 8x + 2x^2 - x^3 \, dx = 4x^2 + \frac{2x^3}{3} - \frac{x^4}{4}\Big]_{-2}^{4}$

$= 4(16) + \frac{128}{3} - \frac{256}{4} - \left[16 - \frac{16}{3} - \frac{16}{4}\right]$

$= 48 + \frac{144}{3} - \frac{240}{4} = 48 + \frac{144}{3} - 60$

$= 36$

d) $\int_0^4 f(x)\,dx - \int_{-2}^{0} f(x)\,dx = $ total Area

$\left[4x^2 + \frac{2x^3}{3} - \frac{x^4}{4}\right]_0^4 - \left[4x^2 + \frac{2x^3}{3} - \frac{x^4}{4}\right]_{-2}^{0}$

$\left[\frac{128}{3}\right] - \left[-\frac{20}{3}\right] = \frac{148}{3} = 49\frac{1}{3}$

(2)

a) $v(t) = 4t^3 - 4t + C$ $\quad v(0) = 0$ ("initially at rest")

$C = 0$

$\boxed{v(t) = 4t^3 - 4t}$

b) $v(t) = 0 = 4t^3 - 4t = 4t(t^2 - 1)$

$t = 0, 1$

c) $x(t) = t^4 - 2t^2 + C$ $\quad x(1) = 3$

$x(1) = 1 - 2(1) + C = 3$

$C = 4$

$\boxed{x(t) = t^4 - 2t^2 + 4}$

3) Given $\int_{-1}^{2} f(x)\,dx = 1$, $\int_{4}^{-1} f(x)\,dx = 5$, and , $\int_{2}^{6} f(x)\,dx = -3$, find

(a) $\int_{2}^{4} f(x)\,dx = \int_{-1}^{4} f(x)\,dx - \int_{-1}^{2} f(x)\,dx = -5 - 1 = -6$

(b) $\int_{4}^{6} f(x)\,dx \quad \int_{2}^{6} f(x)\,dx - \int_{2}^{4} f(x)\,dx = -3 - (-6) = 3$

(c) $\int_{-1}^{6} f(x)\,dx \quad \int_{-1}^{2} f(x)\,dx + \int_{2}^{6} f(x)\,dx = 1 + (-3) = -2$