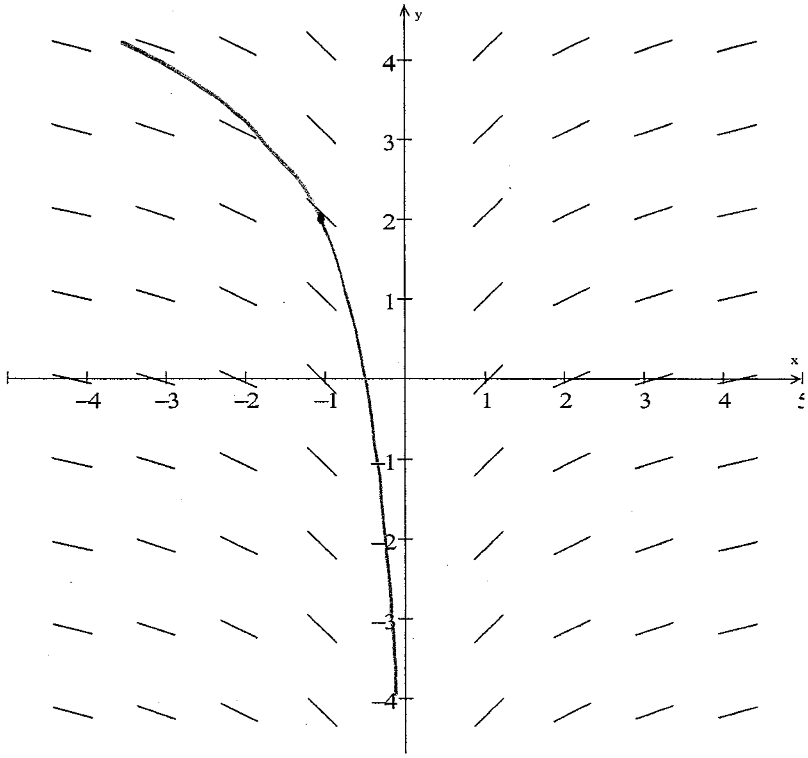
Name Solutions

Separable Differential Equations

For each of the given initial value problems, solve the differential equations for y. Then trace the curve on the given slopefield to confirm that the solution fits. State the domain of y in each solution.

1) $\frac{dy}{dx} = \frac{1}{x}$, $y(-1) = 2$.



$y = \ln|x| + C$

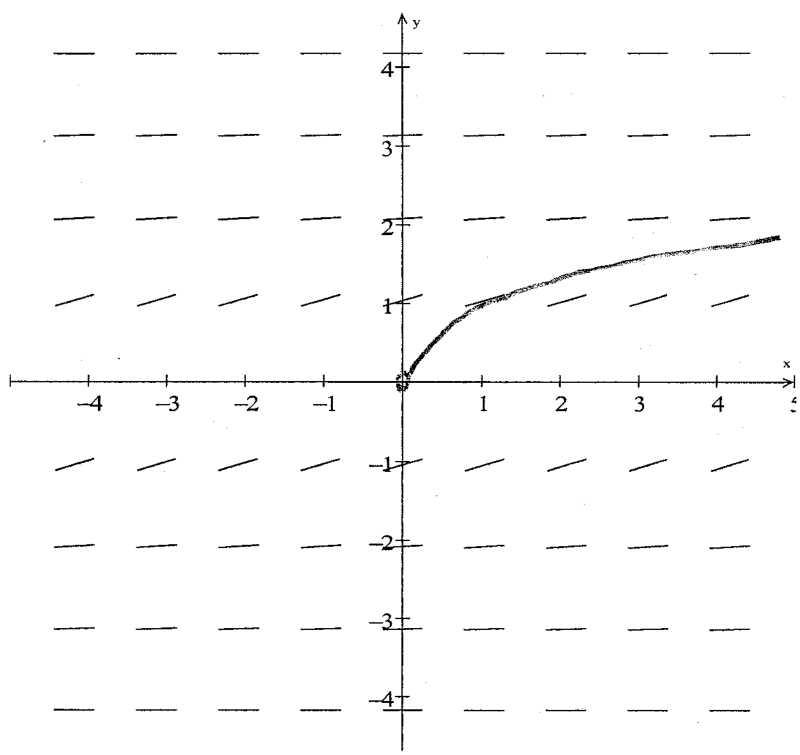
$2 = \ln|-1| + C$

$2 = C$ so $\boxed{y = \ln(-x) + C}$

Since there is an asymptote at $x = 0$ the domain must be

$-\infty < x < 0$

2) $\frac{dy}{dx} = \frac{1}{3}y^{-2}$, $y(1) = 1$.



$\int 3y^2\,dy = \int dx$

$y^3 = x + C$

$y = \sqrt[3]{x + c}$

$1 = \sqrt[3]{1 + c}$ $\quad c = 0$

$y = \sqrt[3]{x}$ $\quad x > 0$

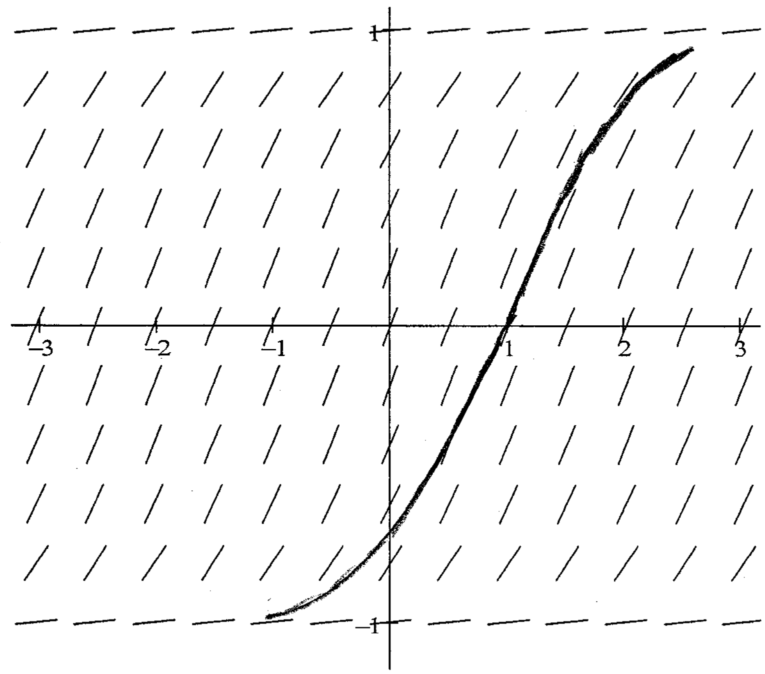
Since $\frac{dy}{dx}$ is undefined for $y = 0$ the domain ends at $x = 0$

So the domain here is

$x > 0$

3) $\frac{dy}{dx} = \sqrt{1-y^2}$, $y(1) = 0$.

$\int \frac{dy}{\sqrt{1-y^2}} = \int dx$ ← because of the left side integral, $\sqrt{1-y^2} \neq 0$



$\sin^{-1} y = x + C$

$y = \sin(x+c)$

$0 = \sin(1+c)$ $\quad c = -1$

$y = \sin(x-1)$ but $\frac{dy}{dx} > 0$ so

we need only an interval that has $\underbrace{\text{positive slopes}}_{}$ which happens over the interval

$-\frac{\pi}{2} < x-1 < \frac{\pi}{2}$

$1-\frac{\pi}{2} < x < \frac{\pi}{2}+1$ ← Domain $\quad$ (remember that $\sqrt{1-y^2} \neq 0$)

4) $\frac{dy}{dx} = (y+2)^{\frac{3}{2}}$, $y(0) = -1$.

$\int (y+2)^{-\frac{3}{2}} = \int dx$

$-2(y+2)^{-\frac{1}{2}} = x + C$

$\frac{2}{(y+2)^{\frac{1}{2}}} = x + C$

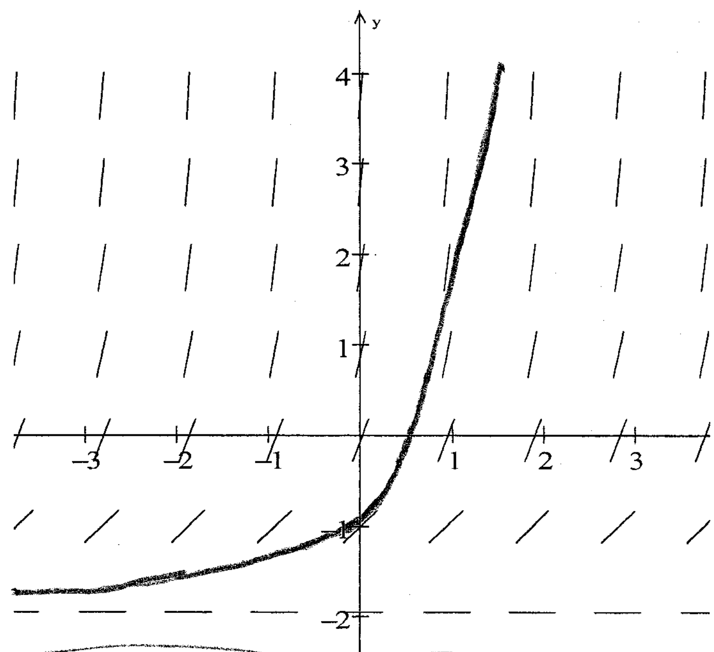
$(y+2)^{\frac{1}{2}} = \frac{2}{x+C}$

$y+2 = \frac{4}{(x+C)^2}$

$y = -2 + \frac{4}{(x+C)^2}$

$-1 = -2 + \frac{4}{c^2}$

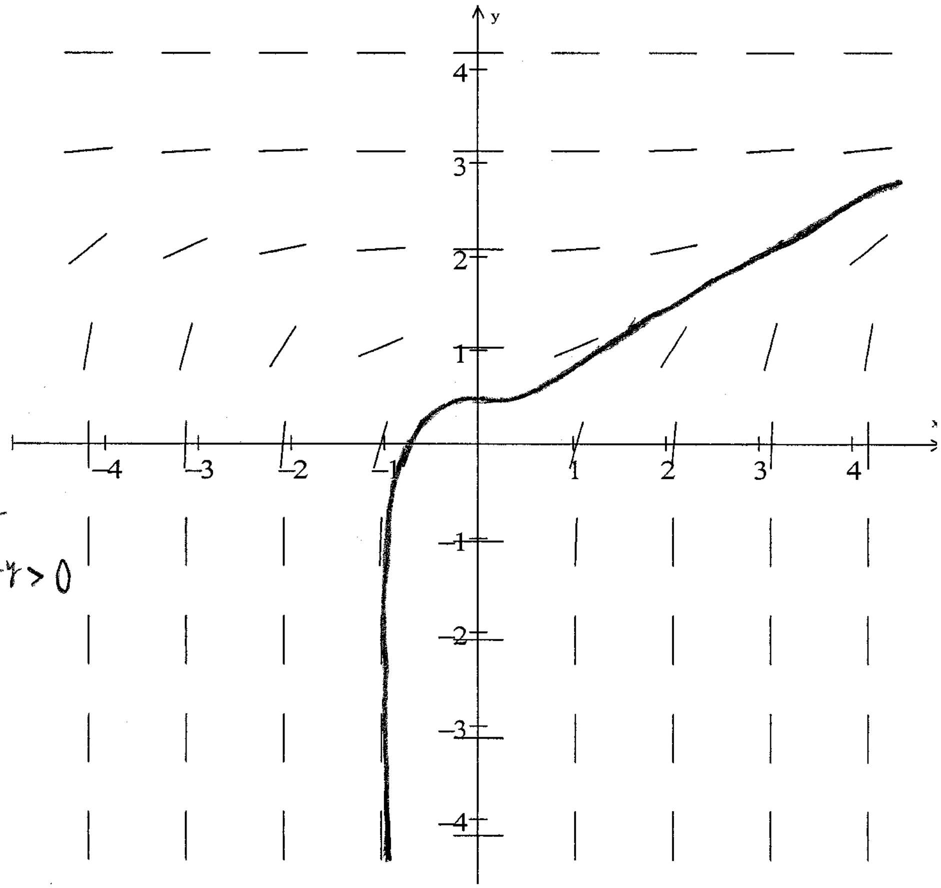
$C = \pm 2$ but only $C = -2$ satisfies positive values for $(y+2)^{\frac{1}{2}}$



$y = -2 + \frac{4}{(x-2)^2}$ $\quad x < 2$ ←Domain

note also that for $x > 2$, $\frac{dy}{dx} < 0$ so this would contradict our conditions for $\frac{dy}{dx} = (y+2)^{3/2}$

5) $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$, $y(0) = \frac{1}{2}$



$$\int e^{2y}\,dy = \int 3x^2\,dx$$

$$\frac{1}{2}e^{2y} = x^3 + C$$

$$e^{2y} = 2x^3 + C$$

$$2y = \ln(2x^3 + c)$$

note that because $e^{2y} > 0$, $2x^3 + c > 0$

$$y = \frac{1}{2}\ln(2x^3 + c)$$

$$\frac{1}{2} = \frac{1}{2}\ln(c) \qquad c = e$$

$$y = \frac{1}{2}\ln(2x^3 + e)$$

$2x^3 + e > 0$ ← recall from above

$$2x^3 > -e$$

$$x^3 > -\frac{e}{2}$$

$$x > \sqrt[3]{-\frac{e}{2}}$$