(−1, 2)   2   (1, 2)

                        Graph of f

1   (0, 1)

                                             (5, 0)

−2   −1   1   2   3   4   5

−1

(−2, −2)   −2   (3, −2)
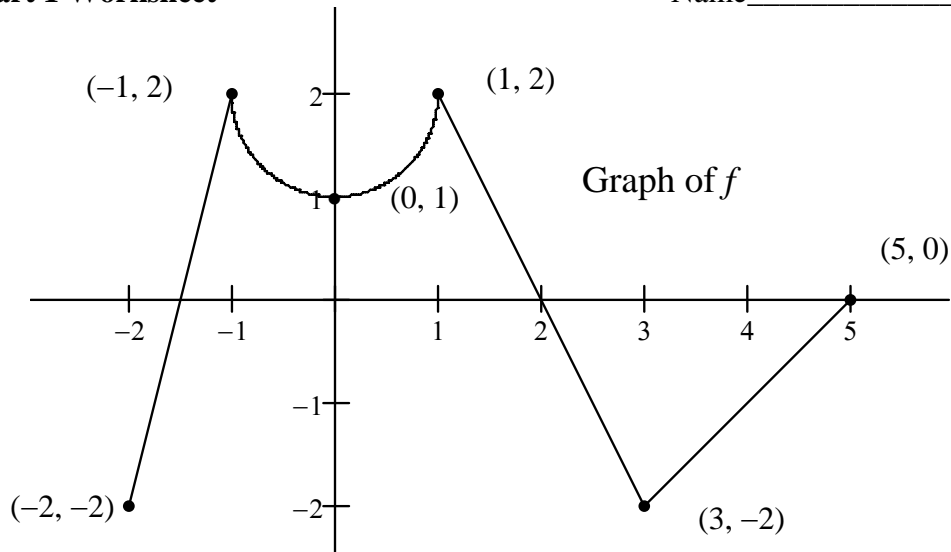
The graph of f shown above is made up of three line segments and a semi-circle of diameter 2.

1) Let $g(x) = \int_1^x f(t)\ dt$ be a differentiable function on the interval [−2, 5] with the graph of f shown above.

Find
    a) $g(5)$
    b) $g(0)$.

<u>Answer for a):</u> $g(5) = \int_1^5 f(t)\ dt$ so find the area under the curve f over the interval [1, 5]. You can do this by finding the areas of the triangles as shown below.

Area of this triangle is 1

                                  1   2   3   4   5

Area of this triangle is −3
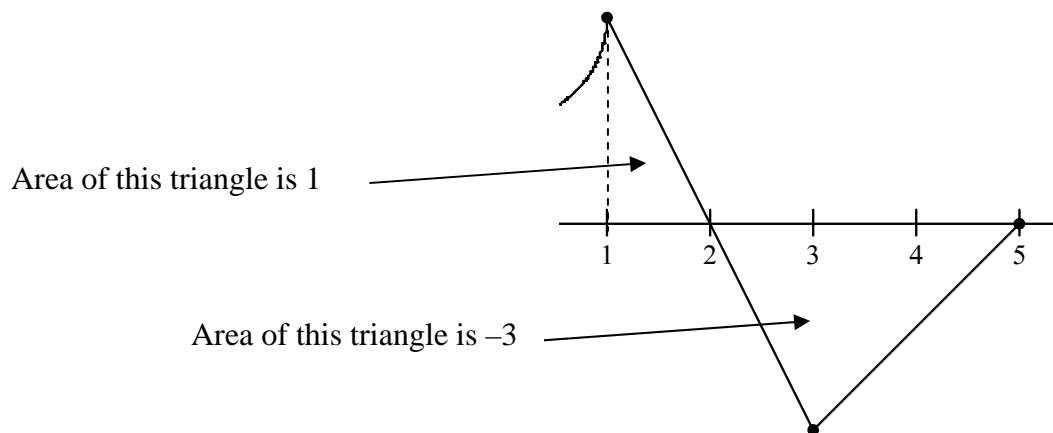
So $g(5) = \int_1^5 f(t)\ dt = 1 + (-3) = \mathbf{-2}$

Now find b) for yourself.

2) Let $h(x) = \int_{-1}^{x} f(t)\, dt$ be a differentiable function on the interval $[-2, 5]$. (f still refers to the graph on the previous page)

a) Find $h(-1)$, $h(3)$, and $h(5)$

b) Where does h have a relative maximum? Why?

c) Find $h''(2)$

d) Over what intervals is h concave down? Why?